MATHEMATICS AND OTHER DISCIPLINES
The Impact of Modern Mathematics in other Disciplines

Verdiana Grace MASANJA
University of Dar es Salaam
Mathematics Department
P.O. Box 35062
Dar es Salaam, Tanzania
vmas anja@maths.udsm.ac.tz

ABSTRACT

The impact of modern mathematics and its application in other disciplines is presented from the 20th century historical perspective. In the period l930's to 1970's mathematics became more inward looking, and the distinction between pure and applied mathematics became much more pronounced. In the 1970s, there was a return to more classical topics but on a new level and this resulted in a new convergence between mathematics and physics. The 20th century approach to mathematics resulted in a more developed mathematical language, new powerful mathematical tools, and inspired new application areas that have resulted in tremendous discoveries in other applied sciences. Towards the end of the 20th Century, mathematicians were making a re-think on the need to bridge the division lines within mathematics, to open up more for other disciplines and to foster the line of inter-discipline research. The current cry is that this interaction will be further strengthened in the 21st Century.

1. Introduction

Mathematics has been vital to the development of civilization. From ancient to modern times mathematics has been fundamental to advances in science, engineering, and philosophy. Developments in modern mathematics have been driven by a number of motivations that can be categorised into the solution of a difficult problem and the creation of new theory enlarging the fields of applications of mathematics. Very often the solution of a concrete difficult problem is based on the creation of a new mathematical theory. While on the other hand creation of a new mathematical theory may lead to the solution of an old classical problem, (Monastyrsky, 2001). This paper is discussing the current role of mathematics in other disciplines.

The presentation is in four parts. Section 2 is dealing with trends of application areas of mathematics at the wake of the twentieth century, Section 3 looks at the changes in mathematics application as a result of the modern approach to mathematics and discoveries in other scientific fields, section 4 addresses the current (21st century) thinking of collaborative and inter discipline mathematics and the section 5 gives some examples of application areas where mathematics is emerging as a vital component with great opportunities for inter discipline research.

2. Trends of Applications in the 20th Century

The 20th century made a rethink on the foundations of mathematics, it was characterised by a new approach to mathematics, fuelled by David Hilbert's (1862-1943) famous set of "mathematical problems" in the 1990 International Congress of Mathematicians. Hilbert's vision was to analyse axioms of each subject and state results in their full generality. This vision became concrete in the 1930's through the development of the axiomatic approach to algebra, pioneered by E. Artin and Edith Noether. Parallel trends took place in functional analysis with Banach Spaces. This spread rapidly to algebraic topology, harmonic analysis and partial differential equations. In addition to this axiomatic approach, the Bourbaki group introduced the idea that there was one universal set of definitions, which once learnt, would be the foundations of everything more specialised (Mumford, 1998). In the drive to seek generality, 20th century mathematics became more diverse, more structured and more complex.

2.1 Divergence of Mathematics from Physics

In the 18th and 19th century mathematical language was vague and did not allow much interaction among mathematicians of different fields. In the period 1950's to 1970's Mathematicians concentrated around problems of algebraic topology, algebraic geometry and complex analysis and they developed new concepts and new methods. New powerful mathematical tools were developed and the language of mathematics became highly developed and very powerful. This has had great impact on diverse fields such as number theory, set theory, geometry, topology and partial differential equations. This new approach to mathematics resulted in greater abstraction. Mathematicians spent years of apprenticeship in a full set of abstraction before doing their own thinking. When the basics were clear enough there was a search for powerful tools that allowed for development and expansion of the geometric intuition into new domains. Examples are topology, homological algebra and algebraic geometry. These new developments made it possible for great breakthroughs in solving several difficult problems that were stuck. For example the Deligne's proof of Weil conjectures, Faltings' proof of Mordell conjecture and Wiles' proof of Fermat's theorem could not have been done in the 19th century just because mathematics was not developed enough. Mathematics of the 20th century has started the path for harmonising and unifying diverse fields. The unification of mathematics started with a common language that has greatly simplified the interaction between mathematicians. This language became the basis for development of new technical tools for the solution of old problems and the formulation of research programmes.

As a consequence of the new approach to mathematics, pure mathematicians drifted away from applications and saw no need to collaborate with other scientists, even their traditional neighbours, and the physicists. On the other hand, application of the highly abstract modern mathematics could not be easily visualised by the traditional users of mathematics. The period l930's to 1970's saw a divergence within mathematics itself and between mathematics and other applied sciences. Mathematics became more inward looking, and the distinction between pure and applied mathematics became much more pronounced. The diversification of mathematics was first of all connected with external social phenomenon, the rapid growth of the scientific community and the breaking discoveries in physics.

The traditional area of application of mathematics is physics. Within this area the deepest mathematics and success stories have been achieved. For example, Einstein's general theory of relativity was based on classical differential geometry of Riemannian spaces, the Hilbert spaces, the theory of linear operators, and spectral theory. In the 1930's the connection of mathematics and other sciences, especially physics was broken. Physicists got interested in solving more concrete problems that could be solved without the application of sophisticated and abstract modern mathematics. The developments of pure mathematics in the post World War II period became weakly connected with applied sciences especially physics. Mathematicians' could not view how physics could assist modern mathematics while physicist could not imagine the application of new abstract mathematical concepts such as sheaf, cohomolgy, J- functor and the like in their fields (Monastyrsky, 2001).

2.2 Re-Convergence of Mathematics with Physics

From the beginning of 1970s, there was a return to more classical topics but on a new level. These developments resulted in the new convergence between mathematics and physics. Some modern mathematicians (e.g. S. Novikon, S.T. Yau, A. Connes, S. Donaldson and E. Witten) quickly saw new opportunities and challenges hidden in the new physics. Examples of mathematical results that got inspired by physical ideas include Donaldson's proof of the existence of different differential structures on simply connected 4-dimensional manifolds. This has very deep consequences for quantum gravity and the gauge theory on strong and weak interactions and resulted in the revisit of the Yang-Mills equations of elementary particles, which had been developed by physicists C. N. Yang and R. Mills almost twenty years earlier in 1954. The Yang-Mills equations had been considered non-physical and had attracted very little attention of physicists. Structures in the elementary particles are described by highly nonlinear equations with deep topological properties. Donaldson's proof inspired physicists to do a deeper study of the Yang-Mills equations. In the 1970's information flow between mathematicians and physicists resumed and led to new and deeper connections between modern mathematicians and physicists. Basing on this new union, theoretical physicists have made substantial progress in uncovering the principles governing particle interaction. The new conservation laws developed in the last part of the 20th Century are believed to be the most fundamental in physics. Most success stories of application of pure, most abstract mathematics are in physics. The application of modern abstract mathematics in physics has resulted in astounding discoveries of the 20th Century in the physical sciences, the life sciences and technology.

The new approach to mathematics resulted in a more developed mathematical language, new powerful mathematical tools, and inspired new application areas that have resulted in tremendous discoveries in other applied sciences including computer science and computer technology. The new mathematical tools and the developments in computer technology, the development of algorithms, mathematical modelling and scientific computing have led to remarkable new discoveries is physics, technology, economics and other sciences in the last half of the 20th century. This has also enabled mathematicians to use modern mathematical tools to solve deep classical problems left by the previous generation of mathematicians.

3. New Application Areas

The branch of mathematics traditionally used in the applications in physics is analysis and differential geometry. Most of the advances in pure mathematics were propelled by problems in physics. In the last part of the 20th century researchers in many other sciences have come to a point where they need serious mathematical tools. The tools of mathematical analysis and differential geometry were no longer adequate. For example a biologist trying to understand the genetic code will need tools of graph theory than differential equations because the genetic code is discrete. Issues of information content, redundancy or stability of the code are more likely to find tools of theoretical computer science useful than those of classical mathematics are. Even in physics discrete systems such as elementary particles need use of combinatorial tools and statistical mechanics need tools of graph theory and probability theory. Traditionally economics is a heavy user of applied mathematics toolbox. Now economics utilises sophisticated mathematics in operations research such as linear programming, integer programming and other combinatorial optimisation models, (Lovasz, Laszlo, (1998)).

3.1 Bridging the Division Lines

Developments in computer technology have re-activated some areas in the fields of discrete mathematics, formal logic and probability that were otherwise dormant for a long time. Examples include the vast and rapid developments in the areas of algorithms, databases, formal languages, as well as cryptography and computer security. Just about 25 years ago questions in number theory that seemed to belong to the purest, most classical and completely in applicable mathematics now belong to the core of mathematical cryptology and computer security.

Towards the end of the 20th Century, mathematicians were making a re-think on the need to bridge the division lines within mathematics, to open up more for other disciplines and to foster the line of inter-discipline research. The current cry is that this interaction will be further strengthened in the 21st Century. Many believe it is better to view pure and applied mathematics as a continuum rather than as two competing and hostile camps.

Efforts being undertaken in other scientific communities will bring the full range of mathematical techniques to bear on the great scientific challenges of our time. It is quite obvious that in this century, the need for mathematics to enrich other scientific disciplines, and vice versa, is most urgent. Currently there is a sense of readiness among mathematicians to interact with the world around them. Currently there is a sense of readiness among mathematicians to interact with the world around them. This is in addition to continuing the pursuit of mathematics for internal motivations such as revealing its inherent beauty and understanding its coherent symmetries.

Being the language of sciences, mathematics has a great potential to make tremendous contributions to the other sciences. The current move is to breakdown barriers that still exist between mathematicians and other scientists. For example, there is still a large gap in the knowledge of physics. The two main pillars of 20^{th} century physics, quantum theory and Einstein's general theory of relativity are mutually incompatible. It is speculated whether string theory and other most abstract mathematics areas will provide the solution. Mathematicians and theoretical physicists are busy working to bridge this gap.

3.2 Potential Contribution to Other Fields

As evidenced by the discoveries of the last half of the 20^{th} century, mathematics can enrich not only physics and the other physical sciences, but also medicine and the biomedical sciences and engineering. It can also play a role in such practical matters as how to speed the flow of traffic on the Internet or sharpen the transmission of digitised images, how to better understand and possibly predict patterns in the stock market, how to gain insights into human behaviour, and even how to enrich the entertainment world through contributions to digital technology.

Through mathematical modelling, numerical experiments, analytical studies and other mathematical techniques, mathematics can make enormous contributions to many fields. Mathematics has to do with human genes, the world of finance and geometric motions. For example, science now has a huge body of genetic information, and researchers need mathematical methods and algorithms to search the data as well as clustering methods and computer models (among others) to interpret the data. Finance is very mathematical; it has to do with derivatives, risk management, portfolio management and stock options. All these are modelled mathematically, and consequently mathematicians are having a real impact on how those businesses are evolving. Motion driven by the geometry of interfaces is omnipresent in many areas of science from growing crystals for manufacturing semiconductors to tracking tumours in biomedical images. The convergence of mathematics and the life sciences, which was not foreseen a generation ago, is a tremendous opportunity for application.

4. Inter-Discipline Mathematics

Currently, efforts are being undertaken to facilitate collaborative research across traditional academic fields and to help train a new generation of interdisciplinary mathematicians and scientists. Also similar efforts are slowly being introduced in undergraduate and postgraduate mathematics curricula and pedagogy. Disciplines that hitherto hardly used mathematics in their curricula are now demanding substantial doses of knowledge of and skills in mathematics. For example the pre-requisites for mathematical knowledge and skills for entry in into biological and other life sciences as well as the mathematics content in the university curricula of these programmes is becoming quite substantial. Curricula for the social sciences programmes now include sophisticated mathematics over and above the traditional descriptive statistics. Curricula of some universities in the developed countries have inter-disciplinary programmes where mathematics students and students from other sciences (including social sciences) work jointly on projects. The aim is to prepare graduates for the new approaches and practices in their fields and careers.

4.1 Examples of Inter-Discipline in Research

Complexity theory is an example of inter-discipline and is the new focus on research in mathematics (Hoyningen-Huene, *et al* 1999). Certain essential details of complexity have been known for quite some time. At the end of the 19th century, the first source of a general idea of complex systems was research in dynamical systems, in the context of classical mechanics. It is an interdisciplinary approach fuelled by sophisticated mathematics, mathematical modelling and computer simulation, inspired by observations made on complex systems in the most diverse fields including meteorology, climate research, ecology, economics, physics, embryology, computer networks and many more. Examples are systems that adapt to changes in their environment in an extremely surprising way. They include Economics (economy of a country), Biodiversity (ecosystem of a pond), Biology (the immune system of an organism) and Artificial Intelligence (Computer Networks).

Probability theory seems to bridge most of the division lines within mathematics. The importance of probabilistic methods in almost all areas of mathematics is exploding. Probability theory is one illustration of the unity of mathematics that goes deeper than just using tools from other branches of mathematics. With probability theory, many basic questions can be modelled as discrete or as continuous problems.

4.2 Illustration of Current Needs Of Mathematics in University Curricula

The role of mathematics in other disciplines has become clearer. I will illustrate this by making quotations from a public reaction to a decision by the Rochester University to reduce the size of mathematics faculty.

Below are quotations from an article titled "Demotion of mathematics meets groundswell of protest" by Arthur Jaffe, Harvard University, President-elect, American Mathematical Society (AMS), Salah Baouendi, University of California at San Diego, Past Chair, AMS Committee on the Profession and Joseph Lipman, Purdue University, Chair, AMS Committee on the Profession presents the statements from different people. The article dated February 1, 1996, is available on the Internet http://www.ams.org/committee/profession/rochester.html and it appeared in Notices of the American Mathematical Society. "In 1996, the University of Rochester planed to downgrade its mathematics program by reducing faculty size and closing down some postgraduate programmes. University of Rochester's plan met with outright protest not only from mathematicians but also from well-known scientists both in universities and in business. Strong protest statements were made by at least six Nobel laureates, by dozens of members of the

National Academy of Sciences, as well as by other leaders in science and industry. The outpouring came from many fields, including biology, chemistry, computer science, economics, geology, mathematics, philosophy, physics, and sociology".

Below are verbatim quotations of some of the statements:

31 professors in the Harvard physics department (including 3 Nobel laureates) wrote: "Recent history confirms the interaction between fundamental mathematical concepts and advances in science and technology. We believe that it is impossible to have a leading university in science and technology without a leading department of mathematics".

Norman Ramsey, Nobel laureate in physics, remarked: "If you had only one science department at a university, it would be mathematics, and you build from there".

All members of the Harvard chemistry department, including one Nobel laureate wrote: "For centuries, mathematics has rightly been termed "the queen of the sciences," and this is just as apt today. In particular, chemistry has benefited more and more from mathematical developments and concepts. A university that aims to have a worthy program in science and technology simply must have a genuine department of mathematics pursuing original research"

Steven Weinberg, University of Texas, Nobel laureate in physics stated the following: "I am not a mathematician, but I regard mathematics as the core of any research program in the physical sciences. If you do not have a graduate program in mathematics, then eventually you will have no research mathematicians, which will make Rochester far less attractive to theoretical physicists. Experimental physicists may not feel the loss of the mathematics program directly, but with fewer first-rate theoretical physicists you will begin to lose your best experimentalists as well. You will also be weakened in your ability to compete for good students; both graduate and advanced undergraduate physics students need to take advanced courses in mathematics, which can only be taught well by active research mathematicians. I imagine that similar effects will eventually be felt in your chemistry and optics departments. I would not advise any prospective undergraduate or graduate student who wishes to concentrate on the physical sciences to go to a university that did not have a graduate program in mathematics".

Joel Moses, a computer scientist and provost at MIT, wrote: "I for one cannot imagine operating a school of engineering in the absence of a strong and research-oriented mathematics department. The same can be said for a school of science. I am also dismayed at the prospect of covering a substantial portion of the teaching load in mathematics with adjunct faculty".

George Backus, research professor of geophysics at the University of California at San Diego and a member of the National Academy of Sciences, wrote: "At UCSD, the Institute of Geophysics and the Scripps Institute of Oceanography often recommend that our Ph.D. students take graduate courses in the UCSD Department of Mathematics. Modern theoretical geophysics and physical oceanography simply cannot be done without sophisticated modern mathematics. To teach these [advanced mathematical subjects] with sophistication and insight requires people for whom they are the primary research interest".

Neil A. Frankel, manager, Advanced Components Laboratory at the Xerox Corporation expressed the following industrial point of view: 'It is evident that neither [Kodak nor Xerox] is well served by the elimination of two technology-related [graduate] departments [chemical engineering and mathematics]. To stay ahead of the very significant competition from Japan and elsewhere, [Kodak] will need all the quality engineering talent it can find. The availability of a quality university in Rochester enhances our ability to attract the very best people to our company. If graduate mathematics is eliminated, I really don't see how UR can support first-rate programs in the sciences and in engineering, and I fear that all of these will decline".

Professor Sir Michael Atiyah, director of the Newton Institute in Cambridge, England; also the past president of the Royal Society wrote: "Increasingly the complex problems that scientists now face require more sophisticated mathematical understanding. The advent of more powerful computers has in no way decreased the fundamental relevance of mathematics. I can illustrate the scope of mathematical interaction with other fields by listing just a few of the inter-disciplinary programmes that we have run at the Newton Institute in the past few years: computer vision, epidemics, geometry and physics, cryptology, financial mathematics, and meteorology".

Edward Dougherty, editor of the Journal of Electronic Imaging, wrote: " While at first this might appear to most people as simply one major research university deciding to restructure itself into a not-so-major university, for those of us in the imaging community there is much more at stake. Because it is home to both Kodak and Xerox, Rochester is one of the major imaging centers in the world, and therefore the future of imaging is closely tied to significant imaging events in Rochester. Suspension of graduate research and teaching in two key foundational imaging disciplines is not insignificant. Chemical engineering plays a role in imaging materials, toners, and numerous other staples of digital imaging. The case for mathematics is even more compelling when it comes to digital imaging. Simply put, there is no scientific phenomenology without mathematics. The kind of mathematics graduate courses necessary for contemporary research in image processing might simply cease to exist in the city of Kodak and Xerox".

Marvin L. Goldberger, dean of the Division of Natural Sciences in the University of California at San Diego wrote: "Not only is mathematics an exciting and vital intellectual endeavour, but from a number of standpoints, plays an exceptional educational role at both the undergraduate and graduate levels. Advanced mathematics is essential in all areas of applied science; economics; technological risk analysis; to an increasing extent in fundamental and applied biology (e.g., drug design); in national security issues involving communication, cryptanalysis, satellite reconnaissance--the list is endless, but one more example is particularly relevant: in recent years topology has played a central role in elementary particle physics where string theory is a candidate for "Theory of Everything." This is another case of the remarkable and mysterious relationship between mathematics and the physical world. Topology is one of the strengths of the Rochester Mathematics Department".

These public reactions illustrate the ever-expanding interrelationship between mathematics and other disciplines, today and in the immediate future.

5. Examples of Key fields where Mathematics is emerging vital

Friedman, A., 1998, presented three examples of key fields in science and technology to the 1998 Berlin International Congress of Mathematicians. The examples are from the disciplines of materials sciences, the life sciences, and digital technology. Also recently, Hu, J.J and Wang, H. 2001, presented to a conference a brief outline of a perspective from the USA army research office on trends in army funding for mathematics research. Below are summaries of the four examples:

5.1 Mathematics in Materials Sciences

Materials sciences is concerned with the synthesis and manufacture of new materials, the modification of materials, the understanding and prediction of material properties, and the evolution and control of these properties over a time period. Until recently, materials science was primarily an empirical study in metallurgy, ceramics, and plastics. Today it is a vast growing body of knowledge based on physical sciences, engineering, and mathematics.

For example, mathematical models are emerging quite reliable in the synthesis and manufacture of polymers. Some of these models are based on statistics or statistical mechanics and others are based on a diffusion equation in finite or infinite dimensional spaces. Simpler but more phenomenological models of polymers are based on Continuum Mechanics with added terms to account for 'memory.' Stability and singularity of solutions are important issues for materials scientists. The mathematics is still lacking even for these simpler models.

Another example is the study of composites. Motor companies, for example, are working with composites of aluminium and silicon-carbon grains, which provide lightweight alternative to steel. Fluid with magnetic particles or electrically charged particles will enhance the effects of brake fluid and shock absorbers in the car. Over the last decade, mathematicians have developed new tools in functional analysis, PDE, and numerical analysis, by which they have been able to estimate or compute the effective properties of composites. But the list of new composites is ever increasing and new materials are constantly being developed. These will continue to need mathematical input.

Another example is the study of the formation of cracks in materials. When a uniform elastic body is subjected to high pressure, cracks will form. Where and how the cracks initiate, how they evolve, and when they branch out into several cracks are questions that are still being researched.

5.2 Mathematics in Biology

Mathematical models are also emerging in the biological and medical sciences. For example in physiology, consider the kidney. One million tiny tubes around the kidney, called nephrons, have the task of absorbing salt from the blood into the kidney. They do it through contact with blood vessels by a transport process in which osmotic pressure and filtration play a role. Biologists have identified the body tissues and substances, which are involved in this process, but the precise rules of the process are only barely understood. A simple mathematical model of the renal process, shed some light on the formation of urine and on decisions made by the kidney on whether, for example, to excrete a large volume of diluted urine or a small volume of concentrated urine. A more complete model may include PDE, stochastic equations, fluid dynamics, elasticity theory, filtering theory, and control theory, and perhaps other tools.

Other topics in physiology where recent mathematical studies have already made some progress include heart dynamics, calcium dynamics, the auditory process, cell adhesion and motility (vital for physiological processes such as inflammation and wound healing) and bio-fluids. Other areas where mathematics is poised to make important progress include the growth process in general and embryology in particular, cell signalling, immunology, emerging and re-emerging infectious diseases, and ecological issues such as global phenomena in vegetation, modelling animal grouping and the human brain.

5.3 Mathematics in Digital Technology

The mathematics of multimedia encompasses a wide range of research areas, which include computer vision, image processing, speech recognition and language understanding, computer aided design, and new modes of networking. The mathematical tools in multimedia may include stochastic processes, Markov fields, statistical patterns, decision theory, PDE, numerical analysis, graph theory, graphic algorithms, image analysis and wavelets, and many others. Computer aided design is becoming a powerful tool in many industries. This technology is a potential area for research mathematicians. The future of the World Wide Web (www) will depend on the development of many new mathematical ideas and algorithms, and mathematicians will have to develop ever more secure cryptographic schemes and thus new developments from number theory, discrete mathematics, algebraic geometry, and dynamical systems, as well as other fields.

5.4 Mathematics in the Army

Recent trends in mathematics research in the USA Army have been influenced by lessons learnt during combat in Bosnia. The USA army could not bring heavy tanks in time and helicopters were not used to avoid casualty. Also there is need for lighter systems with same or improved requirement as before. Breakthroughs are urgently needed and mathematics research is being funded with a hope to get the urgently needed systems. These future automated systems are complex and nonlinear, they will likely be multiple units, small in size, light in weight, very efficient in energy utilisation and extremely fast in speed and will likely be self organised and self coordinated to perform special tasks.

Research areas are many and exciting. They include: (i) Mathematics for materials (Materials by design - Optimisation on microstructures; Energy Source - compact power, Energy efficiency; Nonlinear Dynamics and Optimal Control). (ii) Security issues (needs in critical infrastructure protection, mathematics for Information and Communication, Mathematics for sensors, i.e. information/ data mining and fusion, information on the move i.e. mobile communication as well as network security and protection). (iii) Demands in software reliability where mathematics is needed for computer language, architecture, etc. (iv) Requirements for automated decision making (probability, stochastic analysis, mathematics of sensing, pattern analysis, and spectral analysis) and (v) Future systems (lighter vehicles, smaller satellites, ICBM Interceptors, Hit before being Hit, secured wireless communication systems, super efficient energy/ power sources, modelling and simulations, robotics and automation.

During the last 50 years, developments in mathematics, in computing and communication technologies have made it possible for most of the breath taking discoveries in basic sciences, for the tremendous innovations and inventions in engineering sciences and technology and for the great achievements and breakthroughs in economics and life sciences. These have led to the emergency of many new areas of mathematics and enabled areas that were dormant to explode. Now every branch of mathematics has a potential for applicability in other fields of mathematics and other disciplines. All these, have posed a big challenge on the mathematics curricula at all levels of the education systems, teacher preparation and pedagogy. The 21st Century mathematics thinking is to further strengthen efforts to bridge the division lines within mathematics, to open up more for other disciplines and to foster the line of inter-discipline research.

REFERENCES

- Monastyrsky, Michael, 2001, "Some trends in Modern Mathematics and the Fields Medal", *NOTES-de la SMC*, Volume **33**, Nos. 2 and 3.

- Mumford, David, 1998, "Trends in the Profession of Mathematics: Choosing our Directions", *Berlin Intelligencer*, **ICM August 1998**, pp. 2-5.

- Hoyningen-Huene, P., Weber M. and Oberhem, E. (1999), "Science for the 21st century: A new commitment", *Background Paper Presented at The World Conference on Science*, **26th June - 1st July 1999**, Budapest.

- Avner Friedman, 1998, "Reflections on the future of Mathematics: We are on the threshold of ever expanding horizons", *Berlin Intelligencer*, **ICM August 1998**, pp. 6-8.

- Lovasz, Laszlo, 1998, "One Mathematics: There is no natural way to divide mathematics", *Berlin Intelligencer*, **ICM August 1998**, pp. 10-15.

- Jaffe, A., Baouendi, S., and Lipman, J, 1996, "Demotion of mathematics meets groundswell of protest" http://www.ams.org/committee/profession/rochester.html, This draft is dated February 1, 1996, appeared in Notices of the American Mathematical Society.

- Hu, J.J and Wang, H., 2001, " US national research trends in mathematics and intelligent Control: Trends In DOD/Army Funding In Mathematics Research", http://www.arofe.army.mil/Conferences/Intelligent_Abstract/1AWu.pdf