Panel ***ICMI Study on The Teaching and Learning of Mathematics at Undergraduate Level***

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ABSTRACT

A short history of the Study will be given to set the background for a deeper discussion of three of the main areas of the Study.

Educational Research: One of the goals of the Study was to determine what educational research carried out at this level of formal education had to offer; to evaluate the researches potential to help us understand better the observed problems and to offer strategies for tackling these; and to identify the current limitations of research and suggest orientations for its future.

Practice: Recent changes in undergraduate mathematics teaching have been in response to external factors that impinge on the teaching of the discipline, as well as a result of different epistemological views of mathematical learning. Several innovative teaching approaches were highlighted in the Study. These include new approaches to teaching topics of a traditional curriculum, as well as attempts to redefine the nature of undergraduate mathematics teaching and learning.

Technology: Innovations in this area affect both curriculum and pedagogy. Much of the Technology area of the Study centred on the use of technological tools for supporting students learning, particularly via visualisation, computation, and programming both during and after formal lecture time. Consideration was given to technologies potential to foster more active learning, to motivate explanations of surprise feedback, to foster co-operative work and to open a window on students thought processes.

Members of the Panel:

- ***Michèle Artigue***, Université Paris 7, Paris, France
- ***Derek Holton***, University of Otago, Dunedin, New Zealand
- ***Joel Hillel***, Concordia University, Montreal, Canada
- ***Alan Schoenfeld***, University of Berkeley, California, USA

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1. Introduction

The Study began in 1997 with the first meeting of the International Programme Committee. Their Discussion Document appeared in the ICMI Bulletin of December 1997. Somewhat surprisingly, we completed on time, all the Study goals outlined on the timeline and, in addition, produced an extra publication (marked with an asterisk below). The main items on the timeline were

- December 1998: Study Conference, Singapore;
- Special issue of the International Journal of Mathematical Education in Science and Technology, Volume 31, No. 1, 2000*;
- Presentation of main findings 2000, ICME-9, Makuhari, Japan;
- Study Volume, The Teaching and Learning of Mathematics at Undergraduate Level, Kluwer Academic Publishers, Dordrecht, 2001.

We list below some of the main questions raised in the Discussion Document.

- What research methods are employed in mathematics education? What are the major research findings of mathematics education?
- Are the educational theories that are relevant at school level, relevant at university level as well?
- What do we know about the learning and teaching of specific topics such as calculus and linear algebra?
- What alternative forms of assessment exist? How can assessment be used to promote better learning and understanding?
- What are the effects of the use of technology in the teaching and learning of mathematics?
- To what extent do potential teachers of school mathematics, scientists, engineers, etc., need specially designed courses?
- What changes are, or should be, taking place in the curriculum?

Most of the questions raised were discussed in the two publications that have arisen form the Study. We take up issues related to mathematical research, practice and technology for this panel.

2. Educational Research

Some of the goals of the Study were to determine what educational research carried out at this level of formal education had to offer; to evaluate the research's potential to help us understand better the observed problems and to offer strategies for tackling these; and to identify the current limitations of research and suggest orientations for its future.

Research in mathematics education carried out at the university level helps us better understand the learning difficulties our students have to face, the surprising resistance of some, and the limitations and dysfunction of some of our teaching practices. Moreover, in various cases, research has led to the production of teaching designs that have been proved to be effective, at least in experimental environments. It has also been the source of specific theoretical frames. This is well evidenced by the section 3 of the ICMI Study Book and elsewhere. But the Study also shows that the research carried out up to now has been restricted in its cover. For instance, efforts have been concentrated on a few areas of the subject and on the training of future mathematicians or teachers. The Study also shows that, up to now, the influence of research on university teaching remains quite limited. This phenomenon cannot only be explained by the limitations of current research noted above and the Study allows us to better understand this limited impact. For instance, it shows us to that we are unlikely to get substantial gains without more engagement and expertise from teachers and significant changes in practices. One essential reason is that what has to be reorganised is not only the content of teaching but more global issues such as the forms of students' work, the modes of interaction between teachers and students, and the form and content of assessment. This is not easy to achieve and is not just a matter of personal good will. Another crucial point is the complexity of the systems in which learning and teaching take place. Because of this complexity, the knowledge that we can infer from educational research is necessarily partial. The models research can elaborate are necessarily simplistic ones. We can learn a lot even from simplistic models but we cannot expect that they will give us the means to really control didactic systems. As evidenced by the Study, the current links between research and practice do not allow research to play the role it could play. Improving these links is a necessity but has not to be considered as the sole responsibility of researchers. It is the common task of the whole mathematical community.

3. Practice

Recent changes in undergraduate mathematics teaching have been in response to external factors that impinge on the teaching of the discipline, as well as a result of different epistemological views of mathematical learning. Several innovative teaching approaches were highlighted in the Study. These include new approaches to teaching topics of a 'traditional' curriculum, as well as attempts to redefine the nature of undergraduate mathematics teaching and learning.

A fairly accurate picture of current undergraduate mathematics is that, by and large, it is still dominated by the 'chalk-and-talk' paradigm, a carefully selected linear ordering of course content, and assessment which is heavily based on a final examination. Even the highly publicised 'computer revolution' has not really made a sweeping impact on mathematics. The agenda is still basically defined by pure mathematics and one can reasonably claim that as long as the primary goal of mathematics education is conceived in terms of preparing future professional mathematicians, existing curricula function optimally if they just keep abreast of new developments within mathematics. Nevertheless, there are many calls from the general scientific community and professional associations of mathematicians and users of mathematics, to overhaul undergraduate mathematics education. This overhaul might include: goals, epistemology, learning styles, motivational issues, technology, and breadth of training.

In practice, it turns out that actual trends tend to be more modest and depend very much on the contexts and goals of the institutions involved. Changes are most discernible in departments that consider the goal of training future mathematicians as being too narrow, too expensive, or simply unrealistic in terms of who is actually enrolled in their programmes. Rather, they see their goals nowadays as being both academic and vocational. Certain trends however, can be seen. These include:

- Some departments are becoming more explicit about their aims and objectives for courses and for programmes as well as in describing a desired 'profile' of a student completing each of their programmes.
- There is a general trend towards reducing the mathematical content of courses, both for programme and client students.
- There is also an increased emphasis on applications and computer simulations both in main-stream mathematics courses and in courses targeted for client students.
- The transition problem from secondary to tertiary level has led to the appearance of bridging courses aiming to facilitate students' entry into university mathematics.
- The one-maths-course-for-all model is giving way to customised courses for different clientele.
- Though assessment is still dominated by the end-of-year exams there is a move towards a more varied assessment based on projects, weekly tests, essays, report writing, and seminar presentation, and group projects.
- Joint degrees, traditionally in mathematics and physics, have now given way to degrees such as mathematics and finance, mathematics and ecology, mathematics and information technology.

4. Technology

Innovations in this area affect both curriculum and pedagogy. Much of the Technology area of the Study centred on the use of technological tools for supporting students' learning, particularly via visualisation, computation, and programming both during and after formal 'lecture' time. Consideration was given to technology's potential to foster more active learning, to motivate explanations of 'surprise' feedback, to foster co-operative work and to open a window on students' thought processes.

A range of questions was raised by the working group on technology. Some of these questions are listed below. They were discussed to various degrees in the Study volume.

- How can you use technology to teach theoretical concepts?
- Does current literature make convincing arguments for using technology?
- How should the curriculum be reorganised to make effective use of technology?
- How does technology change mathematics (what is considered mathematics, how it is done)?
- How do we characterise teacher-student interactions with technology (the Internet, calculators, computers)?
- Should we focus on the current curriculum and how to integrate technology into it or should we consider what the mathematics curriculum could be now we have technology?
- How do we manage computers and calculators efficiently in the classroom?
- What strategies (e.g., starting with a black box and exploring) do we have for using technology to teach mathematics?
- How do we design technology and build it into the curriculum?