

TEACHING MATHEMATICS TO ENGINEERING STUDENTS WITH HAND-HELD TECHNOLOGY

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ABSTRACT

The École de technologie supérieure is an engineering school specializing in applied engineering and technology. Since 1999 fall semester, graphic calculators TI-92 Plus or TI-89 have been a compulsory purchase for every new student. This paper will give two examples of how we use these symbolic tools and will show how the calculator has changed the type of questions we ask our students. We will focus on Calculus and Differential Equations. We have been using computer algebra systems (like *Derive*) for 10 years now. But our students have to go to the computer labs if they want to use it. This is one of the major reasons for choosing hand-held technology like the TI-92 Plus. It can be used by the teacher and the student in order to teach and learn mathematics in a very original manner. One of our goals is to continue to teach "classical maths" with innovative approaches. This is possible if you can make use of technology in the classroom, when you need it, when you want it, without having to wait to go to the computer labs. And, when the teacher thinks that technology should not be used in some parts of an exam, students are not allowed to use the calculator! Finally, for some problems, hand-held technology can not compete with fast computers. There is no problem to switch from one to the other, when systems are close together.

1. Introduction

In this article, we will attempt to demonstrate, using two specific examples, that it is difficult to teach mathematics to future engineers if they don't have access to a symbolic calculator **at all time in the classroom**. We don't think it is absolutely impossible to efficiently teach mathematics without a symbolic calculator, and that such a calculator should be allowed during all exams. We only think that engineering students would better understand and appreciate different mathematical results if such a calculator is always available. Obviously, learning the use of the calculator is compulsory to the course. For this reason, it is essential for the system to be easy to manipulate by students and not be an hindrance for the teacher in order to realize the course syllabus. We must admit that TI-92 Plus and TI-89 are powerful calculators, they nevertheless remain user friendly. As for computer algebra systems, the *Derive* system is also a relatively easy tool to use and is among the current computer algebra systems most resembling TI symbolic calculators.

Another aspect we want to emphasize is the following: several teachers are still very reluctant in using this technology in their teaching, or simply don't realize the need. Indeed, they are most of the time very well prepared and competent teachers, and highly appreciated by their students. So why should they change their way of teaching and introduce this new technology? Our two examples will address this question. In fact, the use of technology not only allows answering more complex and general problems, but it also permits to do more mathematics. Let us be clear: the use of a symbolic portable system is more than scaffolding for weaker students! It's use can and must become a part of teaching and learning mathematics.

2. Taming a general result, graphically and symbolically

Our first example comes from our ODE course. Our students attend this course after their first single variable calculus course. Before the advent of computer algebra systems, ODE courses were generally focused on resolution techniques. Students didn't have to produce graphical solutions, nor did they have to use numerical methods and the emphasis wasn't concentrated on problem solving. The advent of computer algebra systems allowed teachers to introduce computing projects to their students. Introduction of symbolic calculators created a new dynamic in the classroom. Students can work on these projects **without leaving the classroom**. This is what we will demonstrate in the following example. When studying mechanical vibrations spring-mass problem in our ODE courses, we have to solve the damped forced oscillation problem.

Example 1 : let m, b, k, F be fixed positive constants such that $0 < b^2 < 4mk$, let ω be a non negative real number. Students must show that the amplitude A (function of ω) of the particular solution of the ODE

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F \cos(\omega t)$$

is given by

$$A(\omega) = \frac{F}{\sqrt{(k - m\omega^2)^2 + b^2\omega^2}}.$$

Then, they have to show that, if $b^2 \geq 2mk$, then the preceding function A decreases from F/m to 0 when ω goes to infinity, whereas if $b^2 < 2mk$, then the preceding function reaches a maximum value at

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}.$$

This constitutes a type of project students were often asked to solve (Nagle & Saff, 1993, give all the details on pages 254 to 258). Nevertheless, it would have been much more interesting and gratifying to solve this problem in the classroom using first concrete numerical values. But this was practically impossible before. Because since September 1999, all our students have to buy a TI-92 Plus or TI-89 symbolic calculator, they now have the possibility to find a particular solution of the ODE considering fixed values for the parameter, before attempting to prove the general result. In this way, the « experimental aspect » of mathematics is emphasized. It is important to recall that, too often, in mathematics courses for engineers, we forget that the students prefer concrete examples. It will be easier to « sell » mathematics to students if the problem was presented in a concrete way in the first place.

We will fix the value of F to 2 and compare the graphs of A , as a function of ω , for each of the following situations: $m = 3/2$, $b = 1/2$ and $k = 1/16$ for the first case and $m = 1$, $b = 1/4$ and $k = 2$ in the second case. Each situation yields the underdamped case ($b^2 < 4mk$) but only the second one satisfies $b^2 < 2mk$. But even considering such numerical values, it remains quite laborious and boring to find the particular solution using only pencil and paper techniques. In order to obtain these results, we think that students should use their calculator. This way, the teacher can encourage students to use the method of undetermined coefficients. We can divide the classroom in two groups, each one working on a case. Both groups have to do the following steps:

a) They have to define a differential operator (and, in order to do this, they have to understand what it means). They have to think about the candidate for the particular solution (the teacher can remind the students that there is no way to get mechanical resonance because we have a damped oscillation, so a linear combination of sine and cosine will do the job). Students will ask the following questions: does the differential operator need to be a function of one or two variables? If they are satisfied by the independent variable t , they can simply set

$$op(y) = m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky$$

and let the calculator perform $op(a \cos(\omega t) + b \sin(\omega t))$. Figure 1 at the end of the paper shows this for the second case.

b) When the system simplifies the expression $op(a \cos(\omega t) + b \sin(\omega t))$, we still have a linear system of equations to solve. Here again, it is a good opportunity for the teacher to remind students that sine and cosine functions are linearly independent and the teacher should ask the students to solve the following *linear* system of equations with two unknowns a and b using matrix approach and not the « solve » function of the system. For the second case, we obtain the following system:

$$\begin{bmatrix} 2 - \omega^2 & \omega/4 \\ -\omega/4 & 2 - \omega^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

c) Finally, they have to find the amplitude A , which is $\sqrt{a^2 + b^2}$ and maximize this function, so students have to use results from the first calculus course. Even if they can plot the graph of $A(\omega)$,

we think that it is important that they find, using exact arithmetic, the coordinates of the maximum. As figure 2 shows (for the second case), the function $A(\omega)$ is

$$A(\omega) = \frac{8}{\sqrt{16\omega^4 - 63\omega^2 + 64}}.$$

Here something strange happens. Many students will use their symbolic calculator in order to find the critical points of the above function and will check if these points are maximum values. Just a few will simply minimize the expression under the radical, and this does not require the use of the calculator! But, we don't have to forget that, when students have access to a symbolic calculator, you have to let them work with it, even if, sometimes, its use is not appropriate. And, finally, a plot of the function $A(\omega)$ is always possible and we encourage our students to do so (figure 3).

Some students will try to find a particular solution of our ODE using, instead of the method of undetermined coefficients, the method of variation of parameters. They will just be amazed by the complicated trigonometric expressions that the calculator will produce! Others will want to make use of the Laplace transforms methods. It is important, as a teacher, to tell them that, if they use this method, they will find the entire solution, provided they know the initial conditions; and even if they know it, why should they obtain the transient solution?

When students have experimented with different values of the parameter, we can ask them to **prove** the general result: that is to show that, if $b^2 < 2mk$, then the function

$$A(\omega) = \frac{F}{\sqrt{(k - m\omega^2)^2 + b^2\omega^2}}$$

reaches a maximum value of

$$\frac{F}{b\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

at the point

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}.$$

3. Connecting Multivariable Calculus with Single Variable Calculus

Our second example will try to show that, too often, students don't make all the connections they are supposed to do in order to get a good understanding of a problem. For instance, they are solving Lagrange multipliers problems with no idea of what is going on. One of the major reasons for this is that they rarely have to produce a graphical analysis of the problem. If a computer is a must for beautiful 3D graphs, we can't underestimate the graphing calculator for many 2D graphs related with the situation.

Example 2 : we have to use the method of Lagrange multipliers in order to find the closest and the farthest points from the origin on the curve of intersection of two surfaces. Students are asked to verify their answers by finding extreme values of a one variable function. The surfaces will be

the paraboloid $z^2 = x^2 + y^2$ and the plane $z = 1 + x + y$; secondly, the cone $z = x^2 + y^2$ and the plane $x + y + z = 10$. What are the pedagogical interests of such of a question? Let us mention the following:

a) Our students are told that the condition $\nabla f = \lambda \nabla g$ (or, here, $\nabla f = \lambda \nabla g + \mu \nabla h$) is a necessary but not sufficient condition in order to get an extreme value of f , subject to the constraints g and h . For the cone and the plane, one could check that we find two critical points, no one being a maximum value (these two points are both minimum, one being a global minimum). For the paraboloid and the plane, we find again two critical points; one of it corresponds to to maximum value and the other one is the minimum. Regardless of the nature of the critical points, there is a challenge in solving the system of equations arising from the equality $\nabla f = \lambda \nabla g + \mu \nabla h$ where f is the square of the distance, $f = x^2 + y^2 + z^2$ and where g and h are the two constraints. One of the advantages of using a symbolic portable calculator consists of the possibility of solving such systems by pushing on a « solve » button. The system's solver will make use of the lexical Gröbner/Buchberger elimination method and, most of the times, will find, in exact arithmetic, the candidates for extreme values. We have to admit that solving the system of equations $\nabla f = \lambda \nabla g + \mu \nabla h$ is not easy and, before the era of symbolic calculator, the chapter about min/max problems for functions of several variables was not receiving all the attention it deserved. Students were getting lost in solving systems of equations instead of thinking about what the situation should be. As teachers, we cannot blame them for their poor ability in solving such kind of problems. Their attention was focused on algebraic skills instead of problems solving.

b) There is a very important interest for the graphs of the surfaces. The «basic surfaces» should be recognized by the students. But, with the computer and an appropriate software, we can plot, on the same window, more than one surface at the time. So we can plot both surfaces and see the curve of intersection. As teachers, we find important to ask our students to do this. They appreciate to see the ellipse of the intersection of the paraboloid and the plane and the hyperbola of intersection of the cone and the plane! They now understand why there is a point on the ellipse closest and farthest from the origin and why there is no point on the hyperbola farthest from the origin!

c) There is an important link with parametrized 3D curves, and even 2D curves. Students should be able to find parametric equations for the curve of intersection of two surfaces. If we take the case of the paraboloid and the plane, they need to complete a square and use their first trigonometric identity in order to obtain the parametric equations for the ellipse of intersection. Students love to see the projection of this ellipse, onto the xy -plane, which gives the circle they parametrized before (see figures 4 and 5). After, they are surprised to note that they can produce, using single variable calculus, the graph of the square of the distance. They don't always think that the norm of the vector describing the ellipse is a one variable function!

Unfortunately, we rarely see such approaches in our textbooks. People seem to think that basic concepts are not so important when we deal with multiple variable calculus. And we have to admit that, without appropriate technology, it will remain good intentions but practically impossible to propose many solutions for that kind of problem. Solving the system of equations generated by the vector equality $\nabla f = \lambda \nabla g + \mu \nabla h$ remains difficult and/or long to do by hand, plotting nice 3D surfaces without computer is quite difficult and finding extreme values of a single variable function without a graphic calculator is tedious and not quite interesting! These are reasons why solution to the above problems **should be done using technology**. Let us look at the problem

involving the plane and the paraboloid. So, let the three functions be $f = x^2 + y^2 + z^2$, $g = z - x^2 - y^2 = 0$ and $h = x + y + z - 10 = 0$. This leads to the following system of 5 equations in 5 unknowns

$$2x = -2\lambda x + \mu$$

$$2y = -2\lambda y + \mu$$

$$2z = \lambda + \mu$$

$$z = x^2 + y^2$$

$$x + y + z = 10$$

and we ask the symbolic calculator to solve this system. Of course, students have to learn how to use the « solve » or « zero » function of the calculator. This requires them to correctly write the syntax. One could find the following vector function for the curve of intersection:

$$\vec{r}(t) = \left[\frac{\sqrt{42} \cos t - 1}{2}, \frac{\sqrt{42} \sin t - 1}{2}, 11 - \frac{\sqrt{42} \cos t}{2} - \frac{\sqrt{42} \sin t}{2} \right] \quad (0 \leq t \leq 2\pi).$$

We ask our students to check first if the above vector function is correct, simply by substitution into the equations of both surfaces. And we want our students to understand that the (x, y, z) solutions of the system of equations are connected with the extreme values of the function $\|\vec{r}(t)\|$ (see figure 6). Speaking of parametric curves, finding the above vector remains a good exercise for students: they are told, in multiple variable calculus, that parametric curves are important subjects, but they rarely see parametric curves that come from an intersection of surfaces ! This gives the opportunity to recall some concepts, like vector valued function, norm of a vector, total distance, extreme value of a function of one variable (we can even make use of the second derivative test). Without technology, it would be too long to try different approaches. In order to be convinced of this, the reader should perform the calculations involved to get the zeros of derivative of the above function $\|\vec{r}(t)\|$: with the graph of $\|\vec{r}(t)\|^2$, it is so simple!

4. Conclusion

Teaching engineering mathematics with technology constitutes a good opportunity to teach « classical subjects with a new taste ». It allows teachers to adapt their teaching methods to the new technological reality. Most important, technology helps the teacher to present live examples of what mathematics are, how beautiful they are. Students will much more appreciate theorems and general results if they can visualize concrete examples. We have experimented this for the last three years in our single calculus course, our multi-variable calculus course and in our ODE course. Other colleagues are experimenting the same in a probability and statistics course, where both Excel and the TI Statistics package are used. Students don't feel that they get lost in all this technology: having the same kind of calculator surely helps, and being aware that, for some exams or some parts of an exam, they won't be allowed to use it, is a way to remind them that we still want them to learn basic concepts, do some basic manipulations by hand or, even, learn definitions by heart. But they also know that they have to learn how to use their symbolic calculator.

We are now experimenting the teaching of a graduate course of mathematics to engineers, dealing with systems of differential equations, eigenvalues problems, Fourier analysis and complex analysis. Such subjects are much more interesting if many computations involved can be

done in the classroom. If not, the course remains quite theoretical. For example, studying the pointwise convergence of a Fourier series or analyzing the Gibbs' phenomenon should now be investigated first, graphically, and then, we can prove some results, like we were doing before technology. For a final remark, technology is changing the way we teach mathematics, but not so much: it simply gives some teachers the opportunity to continue to teach, year after year, the same subjects without having the impression of «déjà vu».

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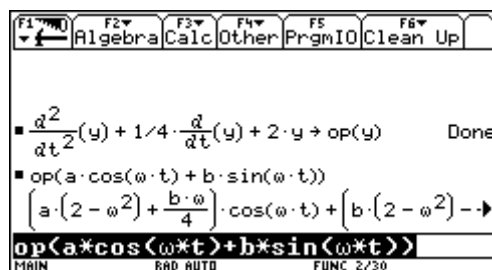


Figure 1. Differential operator with the TI-92 Plus

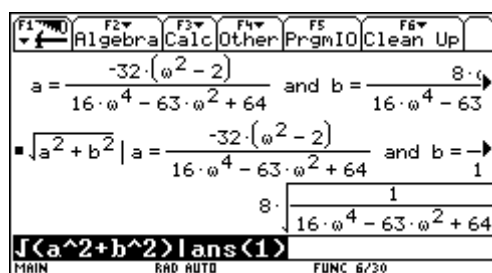


Figure 2. Amplitude $A(\omega)$

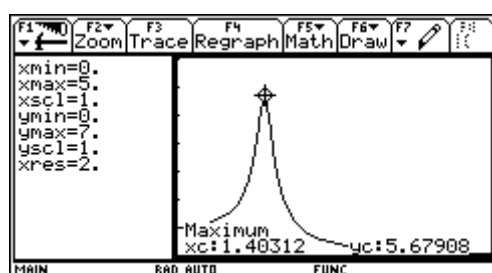


Figure 3. Graph of $A(\omega)$, showing maximum value

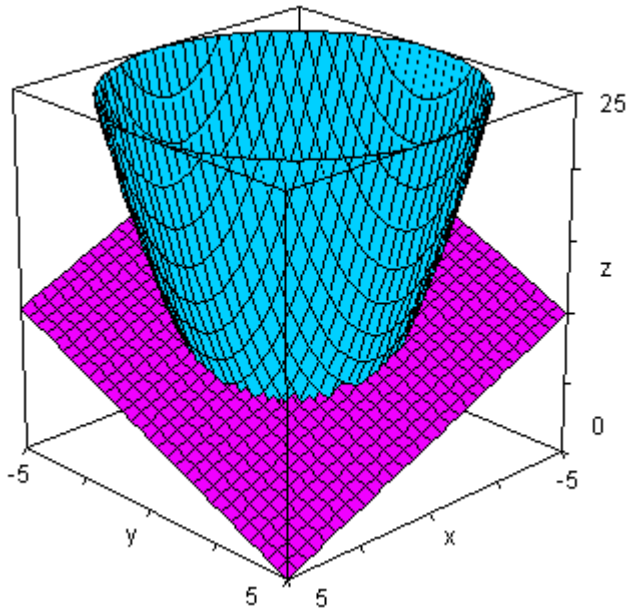


Figure 4. Intersecting surfaces with *Derive 5*

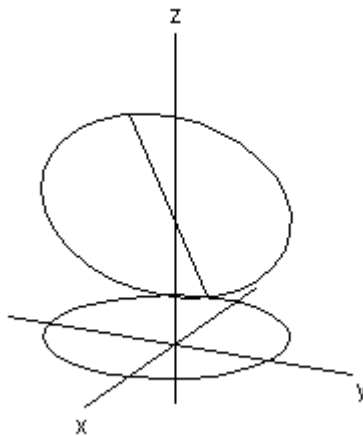


Figure 5. Curve of intersecting surfaces and its projection onto xy -plane. The line segment connects the points we are looking for.

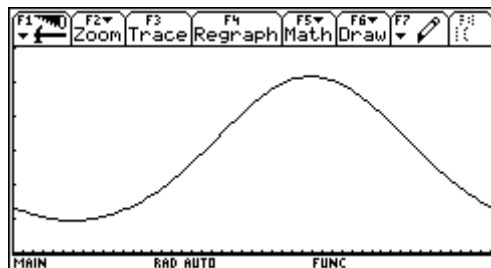


Figure 6. Graph of $\|\vec{r}(t)\|^2$.

If t_0 is any extreme value of this curve, $\vec{r}(t_0)$, when considered as a point in space, is one of the two connected points in figure 5.