"MODELLING AND SPREADSHEET CALCULATION"

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ABSTRACT

A. The process of modelling is a constant oscillation between various levels of abstraction. This can be divided into the following phases:

- get to grips with the problem (define the key questions)
- formulate a mathematical model
- generate solutions (from the mathematical model)
- validate the model (and if necessary re-formulate the model until it fits with the real-world-context)

B. Especially pupils in lower secondary schools have problems formulating their ideas or assumptions in a mathematical (algebraic) way. The transfer from the spoken language into the mathematical language becomes a difficulty as well, as it's not easy for them to generate solutions from the mathematical model. With **spreadsheets** these effects could be reduced in some fields because it is not urgently necessary to define variables and formulate equations. Furthermore the possibility of intuitive use and the splitting into modules appear as an advantage.

C. We have a classification for models.

- Models which process large quantities of data in an elementary fashion.
- Models which solve by using systematic testing.
- Models which are based on iteration and recursion.
- The qualitative and quantitative evaluation of data which requires only functional relations.
- The simulation of operations from which a mathematical solution model can be deduced.

D. Examples of the classification of models and using spreadsheets in the modelling process.
The **gas problem** (How far is it worse to drive for gas?) is an example of the class of models which are based on **systematic testing**. After formulating the model, the pupils try to solve the task by manipulating the in-data until the out-data fits the problem. The **Fermi task** (How many piano players exits in Chicago?) fits into the class of models which are based on the **evaluation** of data. Because there is no "true" answer to the question, the task is to find criterions for evaluation. To check the assumptions and the numbers in the model the use of spreadsheets provides advantages. The **bathroom problem** is an example of models which are based on **iteration and recursion**. In this model one can split of the whole process of filling up an wash-basin into modules and iterate single parts like water inflow, water outflow, total volume, water depth and outflow velocity. Other examples of the remaining classes of models can be found in the literature, listed in the bibliography. These are the **parachute jump problem** for the class of **simulation of operations** and the **financing problem** for the class of models which deals with **large quantities of data**. Of course these examples not only fit into one of the above mentioned classes.

E. Conclusions.
The earlier one begins with the concept of modelling, the better ones abilities became during the time at school and the better one recognises mathematics as a part of our world. In the modelling process spreadsheets could be used with all classes of models.

1. Introduction

This paper is based on the assumption that there is a great need for real world problems and modelling activities in schools. The paper is also based on the assumption that the use of spreadsheets in the modelling cycle offers advantages at different stages (of the cycle) and in different classes of models.

To start with modelling activities in lower secondary schools (or already in primary schools) means to start with uncomplicated but exemplary real-world problems and increase the complexity in lower and upper secondary schools, until final examinations achieve one of the most important goals of mathematical education: the arrangement of abilities to handle oncoming problems from different parts of life with mathematical methods.

More than 60% of about 100 pupils (14 to 16 year olds from a grammar school in Magdeburg) asked to solve word-problems like: *The schools solar panels collect 15kW of energy every hour. How many hours a day must the sunshine in order to collect 180kW?* "solved" the tasks without any critical reflection, only by manipulating the given numbers. More than 60% gave the "simple answer": *The sun must shine for 12 hours.* Afterwards when analysed, the pupils knew about the complexity of the tasks and were also aware of the knowledge that the "simple results" cannot be true. In summary, most of the pupils said: In *mathematics* I would find these results (because I have to calculate it with the given numbers), but in *reality* there is a great difference. That led us to our first assumption, that there is a great need of "real" real world problems.

2. The process of modelling

The process of modelling is an constant oscillation between various levels of reality. The modelling cycle can be divided in the following phases.

- get to grips with the problem (define the key questions)
- formulate a mathematical model
- generate solutions (from the mathematical model)
- validate the model (and if necessary re-formulate the model until it fits with the real-world-context)

The figure 1 shows the modelling cycle.

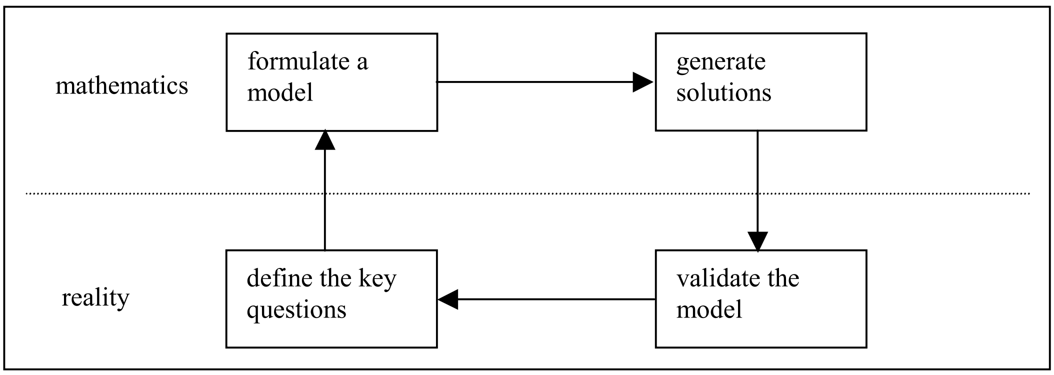

Fig. 1: modelling cycle

The process of modelling continues until the modeller accepts the validation criterion. In the cycle, one uses different kinds of languages (in "reality" one uses the spoken language and in "mathematics" one uses the mathematic language).

All four parts or steps of the modelling process can be a single component of an task for pupils or can be a topic for discussion in class-rooms as well as the whole process. For example, it's possible to concentrate on the formulation of the model without solving it or one can give a problem/task and a mathematical solution to the model and the pupils have to find and discuss evaluation criterions.

3. Modelling with spreadsheets

Especially pupils in lower secondary schools have problems formulating their ideas or assumptions in a mathematical (algebraic) way. It's not easy for them to define variables, formulate equations or inequalities.

The transfer from spoken language into mathematical language (fig. 1) becomes difficult as well, as it's not easy for the pupils to generate solutions. Often the mathematical model is unsolvable with school-skills. (For example differential equations.) Often difficulties occur and as a result displeasure or failure.

With spreadsheets these effects can be reduced in some fields because it is not immediately necessary to define variables and formulate equations. There is the possibility for a more intuitive or "real-life-use" of mathematical language.

Especially for "modelling beginners" this could be an advantage. Furthermore, the possibility of intuitive use and the splitting into modules appear as an advantage. That means one does not have to formulate a whole equation or a whole set of equations. With spreadsheets one "only" has to formulate ones ideas in very small steps. To fit the steps in the whole model becomes more and more the task of the spreadsheet. But not in a blackbox way, because every single step is visible (in the real meaning of the word). Last but not least, the algebraic equations have not disappeared, they are only hidden and for analytic discussion able to derive but with one big benefit: motivation (the pupils know, that their model is good).

Spreadsheets have further features, which are useful in the modelling process, mainly concerning modelling in schools:

- spreadsheets are simple-to-handle tools (without necessarily advanced computer knowledge)
- concentration on the modelling process without dealing with software-skills
- spreadsheets handle large quantities of data
- iterations and recursions are simple to implement
- visualisation of data, relations and functions in an easy way
- systematically testing (manipulate in-data and observe the resulting out-data)
- spreadsheets provide tools like sum, mean, max, min, etc.
- the table describes the problem, represents the model and often its solution
- spreadsheets are useful in almost every field of real-world problems

In summary of the mentioned features, spreadsheets are an educational equipment which are useful to teach efficiently the modelling process and better still, may be used in "real" real-world problems, not only in schools.

4. Classification for models

We found a classification for models if, in the modelling cycle, computers and software are used. We defined the classification as follows:

- Models, which process large quantities of data in an elementary fashion.
- Models, which solve using systematic testing.
- Models, which are based on iteration and recursion.
- The qualitative and quantitative evaluation of data, which requires only functional relations.
- The simulation of operations from which a mathematical solution model can be deduced.

Of course there are real world problems or tasks that fit into more then one of the above mentioned classes. But for pedagogical reasons the classification is useful and one can find problems or sub-problems (like the Fermi-Task) which only fit into one of these categories.

5. Examples

The first example is the so-called gas-problem. It can be solved by using a model, which calculates using systematic testing.

In Germany no week goes by without public discussion about the price of gas (and its increase). One can find in local newspapers and car magazines tables with price comparisons and a lot of people try to fill up their cars for a low price even if they don't use the nearest gas station, instead the gas station with the lowest price. Under these circumstances you have to ask the question:

Is it worth to tank up at a other gas station then the nearest?

This open problem can be modelled under different viewpoints! It depends on one understood under WORTH. At first almost everyone considers the price as a factor. Secondly one can think about factors like time or environment. Of course the assumption, one only have to go to the gas station with the lowest price doesn't work because the car use gas on the way. Now one is able to formulate the problem more precisely. How far is it worth driving to fill up the car? Or on a higher level: To what extent do the factors price difference, distance to the nearest gas station and the gas station with the cheapest petrol connect with each other? Normally one would now define variables and build up an equation and try to answer these questions. If you use a spreadsheet one can formulate the model in a more intuitive way and be able to look at the problem from different sides.

Gas-Problem						
price at station A	2,04	DM		total costs A	81,6	DM
price at station B	2,00	DM		travelling expenses	1,40	DM
gas consumption	0,07	litre/km		gas expenses B	80	DM
tank capacity	40	litre				
distance A <-> B	5	km		total costs B	81,40	DM

Table 1

Table 1 shows a simple model of the problem only considering the price, distance and tank capacity. But even at this level the pupils can check their assumptions, play with the numbers and answer questions like: How far is it worth travelling at a given price difference? How much do I have to put in the tank? What influence does the gas consumption have? Comparing this table with the 'equivalent' inequality $p_B \cdot c + 2 \cdot d \cdot g \cdot p_B < p_A \cdot c$ ($p_{A/B}$ … price; c … tank capacity; d … distance; g … gas consumption) the table is much more "real". Of course it's possible to derive the same results by manipulating the inequality, but the table is much more vivid (especially considering lower secondary schools). After looking at the model at this first level it's possible to re-formulate the model having considered of the actual position between A and B, remaining gas and on another level time costs and so on. This problem is didactically interesting because it is an example of iterative modelling (every model level fits the reality more).

The second example, mainly for lower secondary school, shows another advantage of using spreadsheets. The problem deals with the known FERMI-Task:

"How many piano tuners exist in Chicago?"

In this example we concentrate on a particular section of the modelling cycle (modelling process): the validation of the model. In this open problem one has to define a criterion to evaluate the model because there is no true answer. (Of course one could try to ask the trade corporation and hope they have the true numbers.)

Table 2, which was created by pupils of the 9th form, starts the process from their view of the world (school world).

How many piano tuners exist in Chicago?				
			number of pianos	
Population	3000000			
schools (each 12. --> pupil; 900 pupil each school)	280		280	(1 piano each)
households (4 persons each)	750000		187500	(each 4. household)
theaters/opera houses	50		100	(2 pianos each)
			187880	**total number**
piano tuner				
Pianos each day	2			
working days a year	300			
frequency per piano (once in ... years)	8			
piano tuners in Chicago	39,1			

Table 2

The interesting point is the answer to the question: Could this be true? Or better: Is it probable that this is a good estimation? Or more mathematical: Do I have a good model? Pupils have big problems with the evaluation of their solution because they are used to getting a right or wrong answer. They are trained to calculate the right numbers and not to get a good solution.

One possibility of validating a model is the method of parameter variation. (If the model reacts under small changes to the given numbers with only small or without any change to the model-solution then one has the indication that the model could be good.) In the mentioned example one can double the number of schools or theatres and find that the number of piano tuners doesn't change. On the other hand if one divide the number of working days by two the number of piano tuners double. It shows that the formulation of the given numbers and the model probably good. And it shows the potentials of the spreadsheet too. Pupils would lose the interest if they had to work out the numbers with paper and pencil (even if they used a calculator). They cannot fail and do miscalculations. They can play with the numbers and verify their ideas extremely quickly.

The last example deals with an interesting phenomenon in the bathroom and shows the potentials of spreadsheets handling large quantities of data and models, which are based on iteration. The phenomenon is:

If you turn on the tap the washbasin only fills up to a certain level then doesn't continue to rise if the outlet is open.

If the pupils consider this problem it's not as easy as it seems to begin with, because it is obvious that the water stops if there is an equivalent inflow and outflow but the outflow velocity is not constant and develops in a non-linear in comparison to the water depth. There are questions about the relation between radius of the outlet and water depth or the relation between inflow and water depth. In schools there is no possibility to set up a model with differential equations or put respect on effects like swirls and so on. Table 3 shows a model which was formulated by pupils after analysing the problem verbally, especially talking about the dependence of velocity ($v = \sqrt{2 \cdot g \cdot h}$; g … gravitational pull of the earth; h …height) and the outflow volume in time ($V_O = \pi \cdot r^2 \cdot v \cdot \Delta t$; r … radius of the outlet) and the assumption that the basin is a cuboid.

wash-basin problem					
inflow	10	litre / min	166,7	cm³ / s	
base area	900	cm³			
outlet radius	0,7	Cm			
time interval	10	S			
time / s	V_{Inflow} / cm³	$V_{Outflow}$ / cm³	total volume / cm³	depth / cm	velocity / cm s⁻¹
0,0	0,0	0,0	0,0	0,0	0,0
10,0	1666,7	0,0	1666,7	1,9	60,3
20,0	1666,7	927,9	2405,4	2,7	72,4
30,0	1666,7	1114,7	2957,4	3,3	80,3
⋮	⋮	⋮	⋮	⋮	⋮
620,0	1666,7	1666,6	5377,0	6,0	108,3
630,0	1666,7	1666,7	5377,0	6,0	108,3

Table 3

With this spreadsheet model the pupils are now able to gain play with the numbers and assumptions again without any need to do calculations by hand.

6. Conclusion

Spreadsheets provide a powerful, multipurpose tool to teach mathematical modelling. They are useful in almost all kinds of real world problems especially in connection with models, which deal with iterations, recursions, visualisation, simulation and mathematical experimentation. Pupils are almost free from technical problems (model building or model solving) and able to build a model in a very intuitive way.

The modelling process (cycle) is one didactical way of putting more emphasis on real world problems in schools even at lower levels.

If we want our pupils to develop the ability and skill to solve complex real-world problems at the end of their secondary school life, I believe we have to implement modelling activities in our classroom as early as possible. We have to start at primary school level with uncomplicated but exemplary problems and increase the complexity in the following years, in lower and upper secondary schools, until final examinations thus achieving one of the most important goals of mathematical education (besides the acquisition of elementary techniques): the ability to handle recurring problems from different parts of life with mathematical methods and to use mathematics in order to understand real-world-problems and real-world phenomena better and solve them in an (intelligent) mathematical way.

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