CALCULUS MACHINA: AN INTELLIGENT TUTOR PROVIDING COMPUTER BASED SUPPORT FOR TEACHING UNDERGRADUATE CALCULUS.

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ABSTRACT

Students arriving at University are far from homogeneous and there is a growing need to assess their active mathematical ability on entry to any course **and** provide suitable support materials when necessary. This paper explores how emerging technologies can provide an environment for diagnostic testing and follow up support material for such students. In particular, it discusses a new Computer Algebra System, called *Calculus Machina*. Although many Computer Algebra Systems are excellent at "Doing" mathematics they leave something to be desired when it comes to teaching and supporting learning in first year undergraduate mathematics, as many of the intermediate steps involved with basic calculus are not revealed. *Calculus Machina* is capable of solving many of the problems that arise in the standard Calc I and II sequence, but also disclosing the steps and processes by which these results are obtained. *Calculus Machina* can also function in tutorial mode where students are required to take an active role in learning, and where the program can “look over the shoulder” of a student as the steps in a calculation are performed, checking each step, and offer help when required. Finally, there is always a certain element of inertia when considering the adoption of any new teaching material so we conclude this paper with an evaluation of *Calculus Machina* in a teaching environment.

Keywords: Innovative Teaching, Technology, Computer Algebra Systems (CAS), Teaching Calculus, Diagnostic Testing of mathematics skills.

1. Introduction

When students enter Higher Education courses in Science and Engineering, instructors frequently have to make assumptions relating to their ability in a range of topic areas and mathematical skills. (See Kitchen (1996), Hirst (1997), and Lawson (1997).) Such courses also tend to recruit large numbers of students with a rich diversity of intake qualifications and prior experiences. In addition, over the last decade the nature and background of the students who arrive at our universities each September has changed markedly. The structure of a modular A-level curriculum, the main entry vehicle for students in the U.K., and in particular Curriculum 2000, has meant that students have a considerable range of mathematical experience and limited exposure to mathematical ideas that were once taken for granted. (See Porkess (2001). Furthermore, there is substantial evidence to suggest that schools are being selective in which A-level modules they opt for in order to maximise the overall performance of the student cohort. As a consequence of all these factors, students arriving at University are far from homogeneous. The need to assess individual students on entry and assess their current active ability of students to any course is crucial.

In a previous paper, one possible approach that uses technology for diagnostic testing and follow up support was described. (See Quinney (2001)) This paper explores how emerging technologies can provide support material for students at a time when they most need it and in a form that may encourage them to become independent learners.

2. Diagnostic Testing

The need to provide suitable diagnostic testing of mathematical skills is taken for granted in a wide variety of different institutions for two distinct but inter-related reasons.

(i) To provide students with useful individual feedback **before** problems escalate.

(ii) To provide teaching and tutorial staff with a global assessment of the current active ability of each student on a chosen range of topics.

The Heads of Departments of Mathematical Sciences in the UK (HoDoMS) funded a WWW site giving information, contacts and case studies of existing diagnostic tests. http://www.keele.ac.uk/depts/ma/diagnostic/ in 1996. This site contains links to the diagnostic tests used at a number of universities and a selection of case studies which give details of how diagnostic testing is carried out and, just as importantly, how students are supported thereafter.

Diagnostic testing is now being introduced in many universities, some use paper-based tests that are frequently optically marked to minimise the staff overheads, others have opted for computer-based testing often in the form of Multiple Choice Questions (MCQs). MCQs are attractive to those looking for a way of assessing students arising from their ease of marking by providing a computer-based form of assessment. (See Brydges & Hibberd (1994) and Beevers, Bishop & Quinney (1998).) At Keele University we have used a MCQ diagnostic test for a number of years in order to identify any students who may need particular attention. The test consists of 20 MCQs selected randomly from a bank of about 50 questions each of which is randomised. A typical question is shown in figure 2.1.

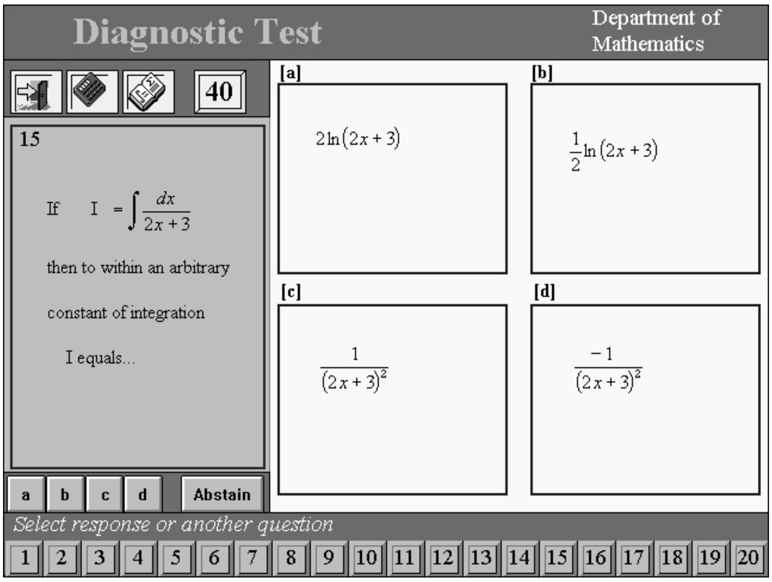
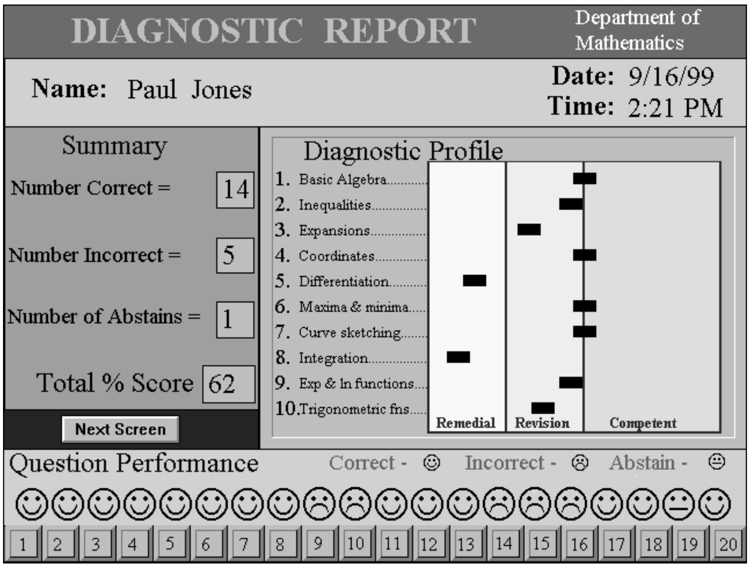


Figure 2.1: Sample Question

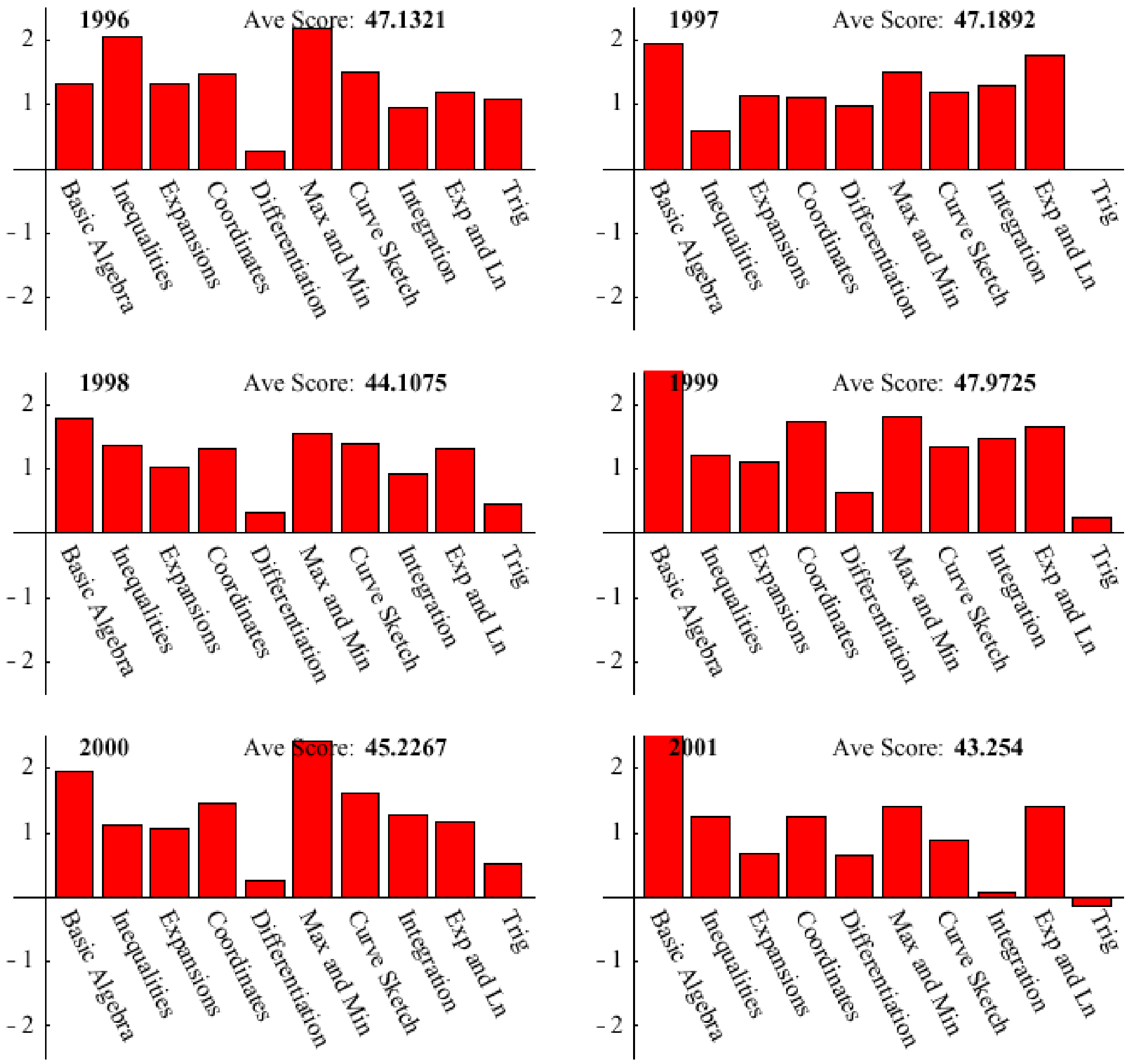
The aim of the test is not simply to return a numerical mark; its primary aim is to identify skills that might be lacking. The test is designed to give partial credit by grading the skills that might lead a student to select one of the incorrect answers and rewarding them accordingly. The student can decide to abstain from a question; in which case they are not penalised for selecting a wrong answer. However, such a decision indicates a deficiency of a particular skill and this is reflected in the final diagnostic report. Each student's responses are analysed to determine the student's capabilities in 10 distinct skills and the results are presented with a diagnostic screen as shown in figure 2.2.



Figures 2.2: Student Diagnostic Report

During the academic year 2000-2001, in an attempt to discover whether the diagnostic test described above provides a realistic indicator of individual students' capabilities, students were asked to take both the diagnostic test and a written paper and the results compared. All 87 students entered Principal Mathematics took both the diagnostic test and completed a written test that involved a large number of problems involving differentiation at various levels of difficulty. A statistical comparison of the written and diagnostic test showed that the scores are highly correlated (r=0.75, p<0.001) and that a simple linear regression model accounts for 55% of the variation of the marks. We conclude that the diagnostic test is a good predictor of individual an individual student's skills in differentiation. (See Quinney (2001).) This is significant, as the reduction in workload required in using the automation provided by the CBL diagnostic test can be significant, but more importantly because the CBL gave immediate feedback to each student.

A diagnostic test described above has been operating in the Mathematics Department at Keele University during 1996-2001; figure 2.3 illustrates results of profile skills for the student cohort in five successive years. The wide discrepancy, year by year, indicates that simply providing common remedial courses will not be suitable. It seems appropriate, therefore, to look at the microscopic scale and try to focus on individual students and attempt to assign each student suitable support material. Providing individualised programmes of study using computer based self-study programmes based on the results of the diagnostic test may provide a solution to this problem.



Figures 2.3: Cohort Profiles 1997-2000

The results of the diagnostic test between 1996 & 2000 were sufficiently encouraging that it was decided to integrate the process of diagnosis and support into the first year programme. The response from students has been exceptionally positive, in that the students have requested similar material to extend the diagnostic process to consider integration in more detail.

3. Online Web Support

At the end of the diagnostic test students were asked to reflect on the result to see if they considered it fair. Many did and excused their poor performance on the grounds that it was several months, over the summer vacation, since they had actually done any mathematics. In order to remedy this in future years, students that have been accepted onto the course at Keele will be given access to WWW-based mathematical quizzes that will enable them to hone up their skills before they arrive at university.

There are a large number of WWW based tutorial systems currently available but we shall be encouraging students to use *eGrade*. (Published by John Wiley (2002).) This system provides a large number of prepared tests but in addition it gives the facilities for instructors to enter their own questions and manage the delivery of both quantitative and technical problems. The questions can be either multiple choice or free text and the software provides facilities for students to preview answers in “pretty print”, i.e. mathematical layout. *eGrade* system has been class-tested for several years at the University of Michigan where in excess of 8000 students have used the system. (See LaRose (2001).) Students can access banks of problem sets and view example problems, which are integrated with some of the better-known texts. The software provides immediate scoring of student work and individualized feedback.

The advantages of such WWW based systems are manifold.

(a) Students can practice their skills and enhance their confidence prior to any formal testing.

(b) The questions are available anywhere and anytime and are therefore more attractive to a generation of students who delight in the availability of the WWW.

(c) The performance of individual students can be tracked and analyzed, though in some cases the latter can be a deterrent if students believe their every mistake is being recorded.

The first of these reasons is by far the most attractive and the availability of a large bank of reliable test problems can be extremely beneficial when coupled with immediate marking and feedback.

4. Computer based support material

Gains made from the implementation of diagnostic testing or the provision of on-line preparatory quizzes is limited without providing suitable learning support material. Such support materials needs to be tailored to each student’s individual needs and yet cover the broad range of core mathematical knowledge at this level. This can be accomplished through human tutors, drop-in clinics, supplementary lectures, and mathematics resource centres, etc. (Lawson, Halpin and Croft, (2001).) However, experience has shown that even though the weaknesses of individual students can be detected using diagnostic testing the restrictions of individual and teaching timetables make it difficult to allot specific times when students can be supervised to ensure that any remedial work is carried out.

During 1996-1999 the mathematics department at Keele University pioneered the use of the TLTP material, *Mathwise*, to provide individual study profiles which were automatically allocated following the diagnostic test. (Hibberd, Looms & Quinney (2001)). However, many students are becoming familiar with computer algebra systems (CAS) such as *Mathematica*, *Maple*, *Derive*, etc. Although these systems are excellent at "Doing" mathematics they leave much to be desired for teaching and learning mathematics. To this end we have been investigating the use of a CAS system that concentrates on teaching and learning, and how such a system can be integrated to provide the student support needed to follow up a diagnostic test.

A new software package called *Calculus Machina* has been developed, which has been designed to have a full range of computer algebraic skills in basic calculus but is also capable of revealing the steps that are required to evaluate derivatives and integrals. Furthermore, the interface between the student and software has been designed to be as simple as possible and yet remain very versatile. Students are able to type in their own expressions and see them displayed immediately in a "pretty print" form, or select and edit the current expression using "point and click". Alternatively, mathematical expressions can be entered using simple templates. (See figure 4.1.)

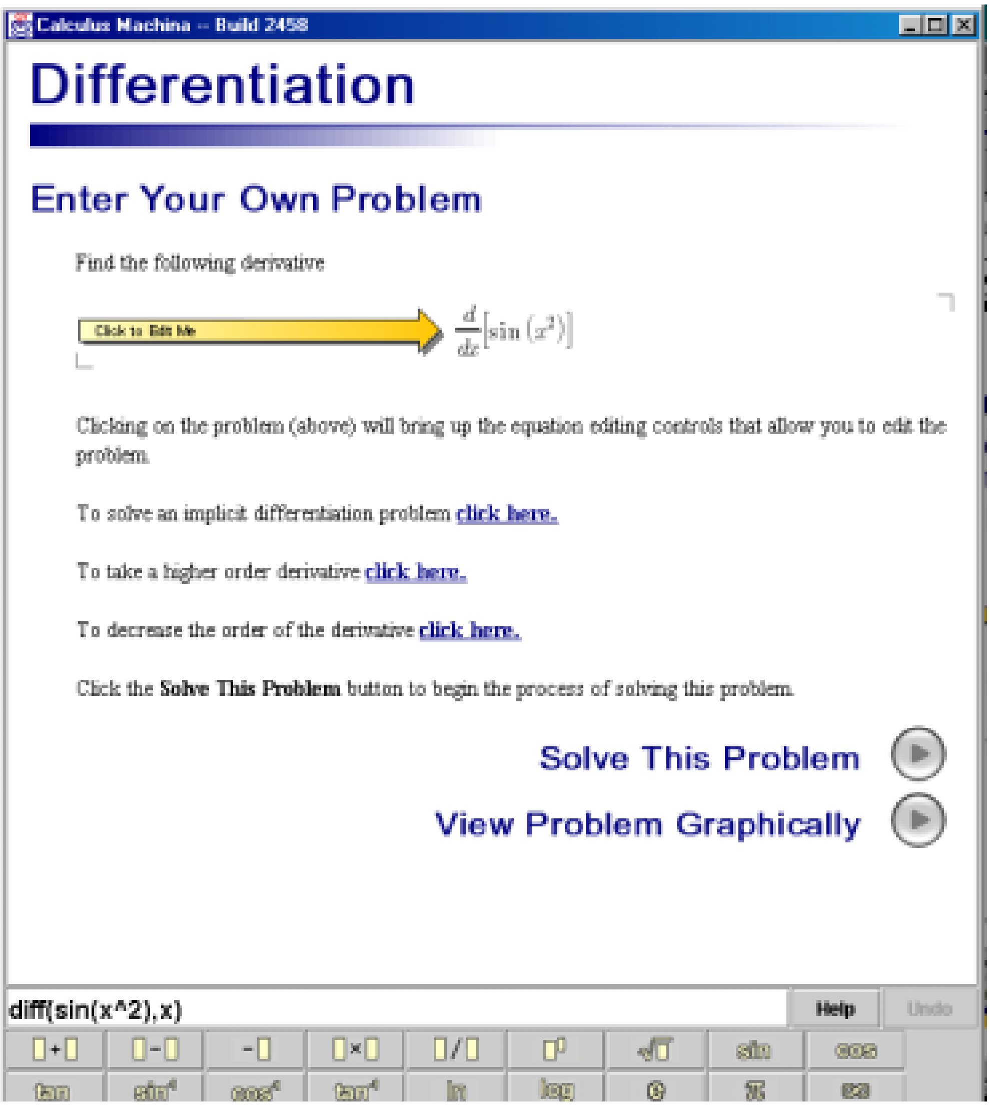


Figure 4.1: ***Calculus Machina's*** **input tool**

Once a function has been defined the software will either display the steps required to determine the derivative, as shown in figure 4.2. In figure 4.3, the *Calculus Machina* has been asked to differentiate sin(x^2). Notice that it recognises that it is necessary to use the Chain Rule (flagged by the text Derivative of Composite Function) and then reveals the steps needed to continue. These flags also provide a hypertext link to context sensitive help that allow the student to "drill down" and gain additional help as shown in figure 4.4. These pages are derived from "*Calculus*", Hughes Hallett, et al (2002) or the "*Calculus*", Anton (2002). Future versions of the software will enable an instructor to add links to alternative texts and additional material. The advantage with *Calculus Machina* is the ability for the students to type in their own problems or for it to generate practise problems for the student to attempt to re-enforce their skills in this topic.

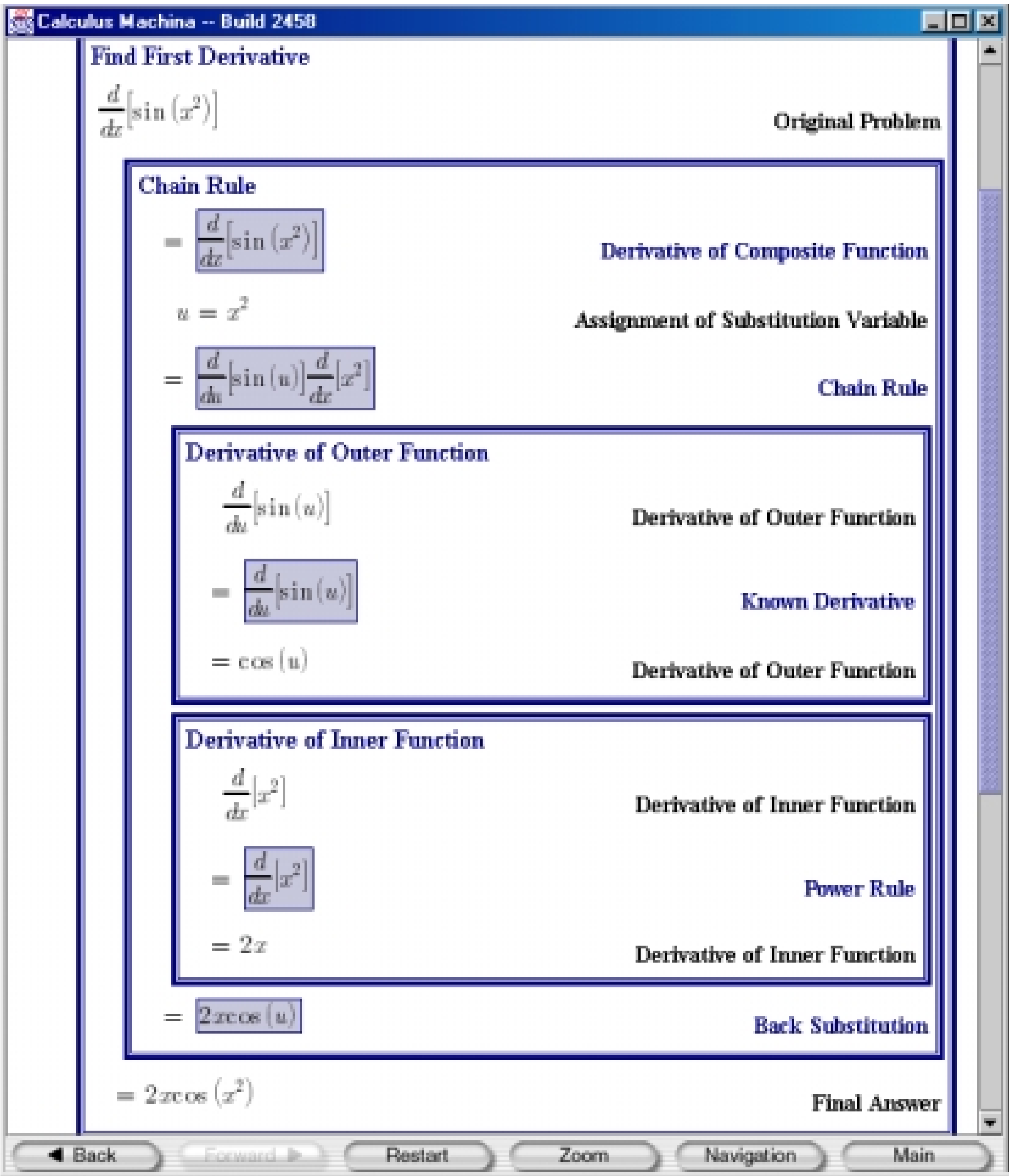


Figure 4.2: *Calculus Machina* output revealing the steps in finding a derivative

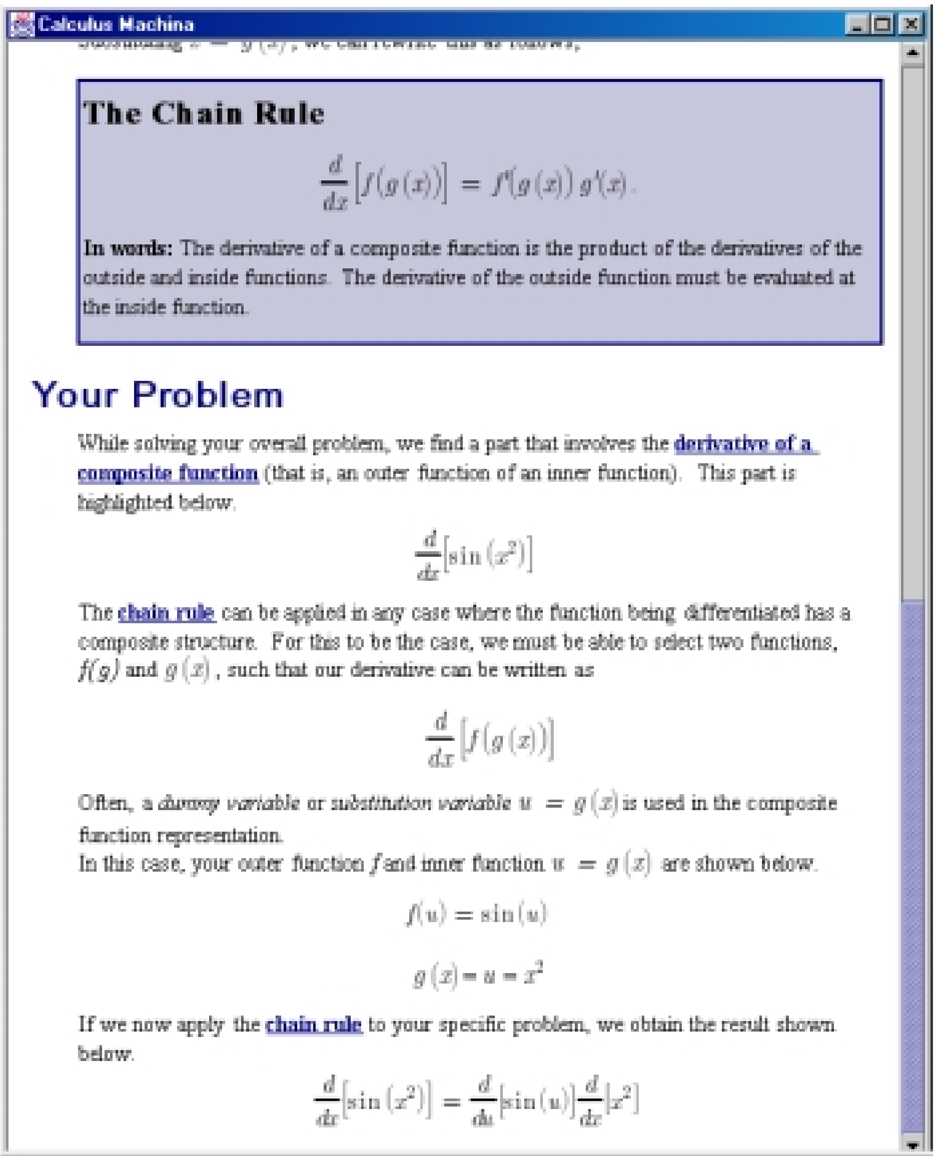


Figure 4.3: Context sensitive help file – note that the example reflects the current problem being solved.

Since *Calculus Machina* is able to differentiate almost all functions met in first and second year mathematics *and documents all the steps involved*, it might be thought that this will encourage students to take a very passive role and allow the computer to do the work. However, *Calculus Machina* has a second, more educational, mode in which the student has to take a much more active part in the process. This mode, called Udo, is illustrated in figure 4.4. Once again *Calculus Machina* has been asked to differentiate sin(x^2) but now the student has to supply the requisite substitution which is then checked before they are permitted to proceed. In this mode *Calculus Machina* can play the part of an individual tutor checking on each step and allowing students as much practise, as they need.

Finally, the software includes the ability to generate further problems that are closely related to the current problem to give further practice.

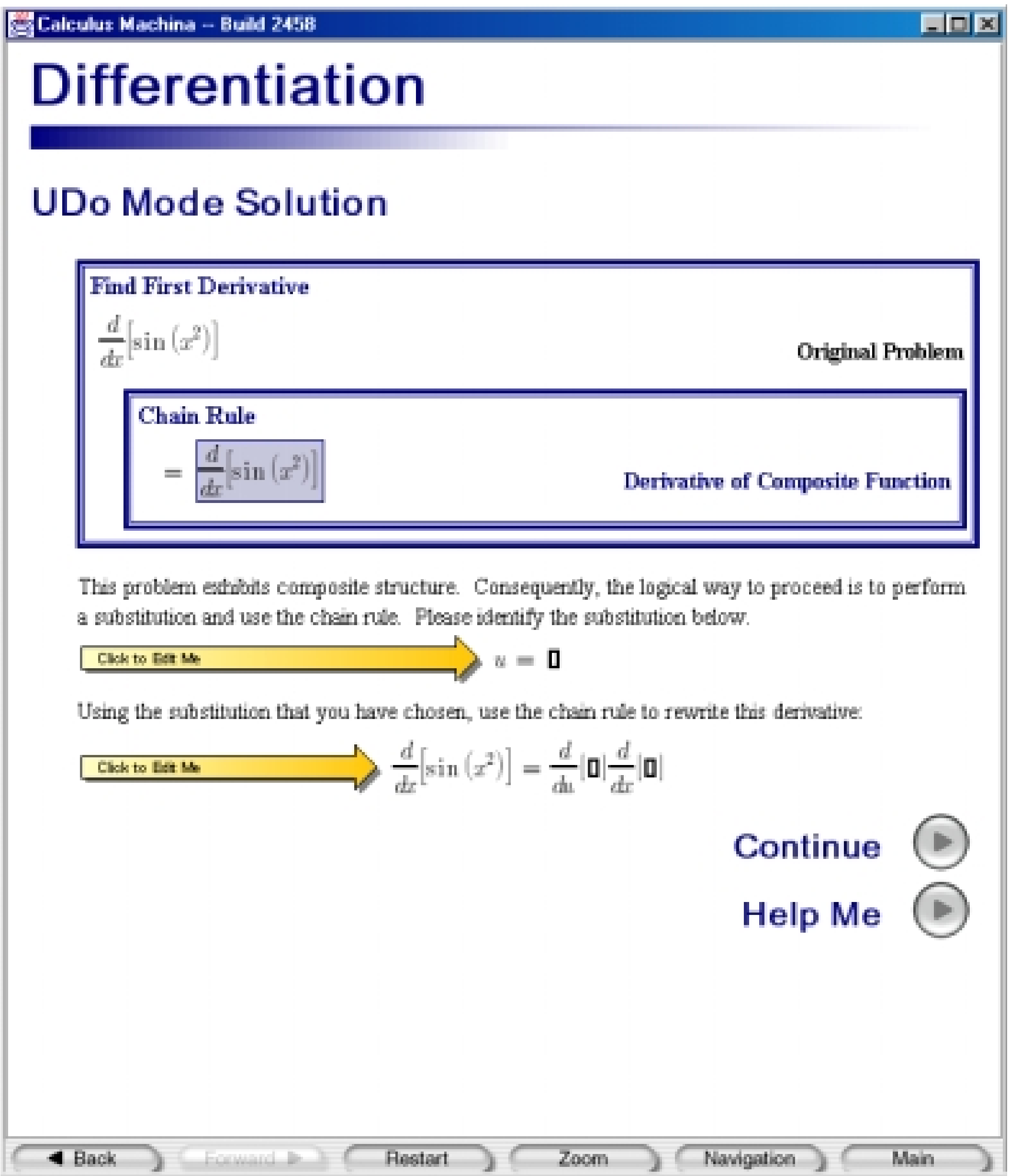


Figure 4.4: ***Calculus Machina* in tutorial (Udo) mode**

5. A Case Study 2000-2001

To investigate the effectiveness of the *Calculus Machina*, the students studying Principal Mathematics at Keele University during the academic year 2000-2001, were divided into two groups. Those scoring in excess of 65%, on the diagnostic test, were asked to look at a *Mathwise* Module called *Applications of Mathematics*. (See Beevers et al, 1998). The remaining students were further randomly sub-divided into two further groups (B1 and B2). Group B1 was asked to study a *Mathwise* Module: *Rules of Differentiation* and Group B2 was asked to use *Calculus Machina.* The aim of the project was to compare the performance of groups B1 and B2 to see if there was any statistical difference in performance of the two groups. To do this Groups B1 and B2 were asked to retake the diagnostic test at the end of their study and also complete a paper-based questionnaire.

5.1 Results

28 students completed the pre and post-diagnostic test though somewhat fewer also completed questionnaire. The students in Group B1 had a mean baseline score of 49.53 whilst those in Group B1 scored slightly less, 43.3 though this difference was not statistically significant, ($p=0.23$ using a t-test). 2 students in Group B2 were not included in the analysis, as they would have skewed the result even further in favour of the *Calculus Machina*. To investigate the effectiveness of the packages allocated to the two groups the mean paired absolute differences of the two groups were analysed.

The results of this trial are given in Table 5.1, and suggest that Group B2 have improved significantly better that Group B1 ($p=0.005$) even though their pre-test score was slightly poorer. Analysing the relative improvement in diagnostic score after using the software gives a similar result. Even though there is substantial variation in the results observed and the sample sizes are relatively small we can conclude that, based on these results, the *Calculus Machina* appears to be the more effective software when used in this context.

Group	Number	Software	Pre-test score	SD	Mean Difference	SD
B1	13	***Mathwise***	49.53	14.61	5.38	10.39
B2	13	***Machina***	43.30	10.94	22.4	17.02

Table 5.1: Results of comparative trials using the *Calculus Machina* and *Mathwise*: Rules of Differentiation.

It must be noted that a direct comparison between the *Calculus Machina* and *Mathwise: Rules Of Differentiation* is a little unfair as they are several generations of software apart and the C*alculus Machina* is designed specifically for the Calculus whereas *Mathwise* covers a wider remit. Nevertheless, the mathematics department at Keele University has invested substantially in its use of *Mathwise* and there is substantial inertia in changing to a new system, however, the evidence of this study provides some credence for changing to *Calculus Machina*. A similar experiment was conducted during the academic year 2001-2002 and the results were very similar. The major advantage of the *Calculus Machina* is its ability to accept problems entered by the student and disclose and document how the derivative or integral is found.

5.2 Questionnaire Results

18 completed questionnaires were returned; 9 from Group B1 and 9 from Group B2. Respondents reported a wide range of reasons for studying Mathematics or Statistics and a wide variety of topics in which they had perceived strengths and weaknesses. Most of the students regarded the diagnostic test as accurate. Students varied widely in their attitudes to the use of computers in teaching and learning. Some appreciated the fact that the computer allows them to work at their own pace, provides instant feedback, and was able to lead them step-by-step through methods; others found the experience somewhat stressful. A similar questionnaire in 2002 found fewer students in the latter category; further investigation has shown that, as might be expected, students are becoming more acclimatised to using courseware.

6. Conclusion

Courseware is now available to help detect areas of mathematical weakness at individual student level, provide individual testing at the convenience of the student and provide individualised support. In particular we have shown:

(1) That the simple diagnostic test that we have used is a good predictor of student performance and may thus be used to support differentiated teaching. Although discussions with course tutorial support staff are vital, the computer-based profiles provide a pro-active mechanism for the early identification of student weaknesses. Of course, the basis of this paradigm is dependent on the development of study skills by individual students and the inclusion of both summative and formative assessment can help re-enforce this. The same software can also be used to gather information on the cohort as a whole and also to track the performance of students on a year-by-year basis.

(2) Although the department has made use of several modules from *Mathwise* over the last 5 years and invested quite heavily in such materials there is sufficient evidence to show that the capabilities of more recent software, *Calculus Machina*, are more beneficial. Accordingly we aim to build it into the week that the Department has set aside for developing the students' skills in Introductory Calculus from the academic year 2002-2003.

REFERENCES

- Anton H., Bivens I, & Davis S. (2002). *Calculus,* 7th Edition. John Wiley and Sons Inc, 0-471-38157-8.
- Beevers C.E., Bishop P, and Quinney D.A. (1998) Mathwise, diagnostic testing and Assessment, Information Services & Uses, Volume 1, 1-15.
- Brydges S & Hibberd S, (1994) *Construction and Implementation of a Computer-Based Diagnostic Test.* CTI Maths and Stats. 5/3 pp9-13.
- Hibberd S, Looms A. & Quinney D.A.. (2001) *Computer based diagnostic testing and support in mathematics*, To appear in Innovative Teaching Ideas in Mathematics, Ed Mohammad H. Ahmadi.
- Hughes Hallett, D, Gleason A, et al. (2002) *Calculus.* 3rd Edition, John Wiley and Sons Inc, 0-471-40827-1
- Hirst K. (1997) *Changes in A-Level mathematics from 1996*. University of Southampton.
- Kitchen A. (1996) *A-Level Mathematics isn't what it used to be; or is it?* Mathematics Today. 32/5 pp. 87-90.
- La Rose G. (2001). http://www.math.lsa.umich.edu/~glarose/projects/gateway/egrade.html
- Lawson D (1967). *What can we expect of A-level mathematics students?* Teaching Mathematics and its Applications 16/4. pp 151-156.
- Lawson D, Halpin M & Croft A, (2001) *After the diagnostic test – what next?* Evaluating and Enhancing the Effectiveness of Mathematics Support Centres, MSOR Connections, Volume 1, Number 3, pp 19-24.
- Porkess R, (2001) *Mathematics in Curriculum 2000: What will students know? What will students not know?* MSOR Connections, Volume 1, Number 3, pp 35-39.
- Quinney D.A. (2001) *Computer based diagnostic testing and student centred support* http://ltsn.mathstore.ac.uk/articles/maths-caa-series/nov2001/index.htm Maths CAA Series: Nov 2001. LTSN Mathematics and Statistics Support Network.
- Wiley *eGrade*. http://jws-edcv.wiley.com/college/egrade/