# USING WEB-BASED INTERACTIVE GRAPHICS TO ENHANCE UNDERSTANDING OF PARAMETRIC EQUATIONS:
## Lessons Learned

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## ABSTRACT

Like many instructors, we have increasingly employed web-based graphics both for in-class demonstrations and for use by students on out-of-class assignments. Web-based interactive graphics have the potential to enrich learning in ways that print resources lack. Furthermore, web-based interactive graphics offer several advantages over other technological devices: they can be more adaptable than graphing calculators, can use familiar and readily accessible interfaces (web browsers), and (in principle) can be run on any computer using any web browser (either from remote websites or from local secondary storage). In the process of employing such web-based interactive graphics, we have learned some lessons about issues such as: "How do students interact with Web-based interactive graphics?" and "What kinds of activities facilitate learning with such graphics?" In this paper, we will show examples of web-based interactive graphics that we have used and we will offer experience-driven recommendations for future implementation and development.

# 1. Introduction

The use of technology in teaching college mathematics is a pressing issue, currently with more questions than answers. Although some instructors continue to resist the use of technology, for many of us the issue is not whether to use technology but how to use it in efficient ways for effective student learning. The authors of this paper hope to fuel conversations about the development of technology, the evaluation and use of technology, and circumstances under which various media are appropriate for teaching and effective for student learning.

The authors have a history of integrating graphing calculators and computer algebra systems (e.g., *Mathematica*) into calculus courses. These experiences, combined with conversations with other instructors and researchers, have led us to conclude that there are activities for which graphing calculators are insufficient but computer algebra systems are excessive. For example, many standard college algebra and calculus courses introduce curves given by parametric equations x(t) and y(t). Students are often puzzled by the relationship between the parametric equations and the resulting curve. Graphing calculator features such as TRACE provide students some ability to explore relationships dynamically. In addition, graphing calculators can be used to examine the graph of x as a function t and the graph of y as a function of t. However, it is unwieldy on graphing calculators to simultaneously display all three graphs (x, y, and the curve given parametrically by x and y) to examine relationships graphically. On the other hand, students need not go so far as to become proficient with a computer algebra system in order to gain the insight they seek.

Web-based interactive graphics have the potential to fill needs in these areas because they

- can be more adaptable than graphing calculators,
- can use familiar and readily accessible interfaces (web browsers), and
- (in principle) can be run on any computer using any web browser (either from remote websites or from local secondary storage).

A Java applet, for example, is a computer program ("applet" means "small application") that can be run as part of a web page. QuickTime movies are another, perhaps more familiar, example of such programs. The web browser plug-ins needed to run Java applets (and QuickTime movies) are free and, with well-designed applets (see the mathlet review criteria at the JOMA website), users can get started more quickly than they can with computer algebra systems, which tend to have considerably steep learning curves.

The *Journal of Online Mathematics and its Applications* (JOMA) defines a "mathlet" to be a "small, interactive, platform-independent tool for teaching math" (see the JOMA website). Many mathlets are Java applets. In this paper, we restrict our attention to mathlets that are not only interactive but also graphical and that can be run via web browsers. In section 2, we offer an example of how web-based interactive graphics might contribute to teaching and learning in combination with other media. In section 3, we share thoughts on some of the lessons we have learned, including some advantages and disadvantages of web-based interactive graphics. Finally, in section 4, we make recommendations for future work in this area. At the end of the text, we provide a list of related websites.

# 2. A Role for Web-based Interactive Graphics

In June 2000 and 2001, Murphy attended the Mathematical Java Workshops and Conference at Emporia State University in Kansas, U.S.A. Faculty members Joe Yanik and Chuck Pheatt of

Emporia State University (with funding from the National Science Foundation) had written a collection of Java classes, The MathToolKit, with the intention of teaching mathematicians to use the toolkit to produce Java applets for use in teaching mathematics (see the Mathematical Java website). Using the MathToolKit, Murphy wrote the Parametric Curves Applet (currently, version 3.0) shown in Figures 1 and 2, for use in her calculus classes at the University of Oklahoma (OU).

This applet behaves much like a graphing calculator, allowing the user to input parametric equations x(t) and y(t) as well as to adjust the viewing window. Unlike a graphing calculator, the applet can simultaneously display, on separate sets of axes, the graphs of t versus x(t), t versus y(t), and x(t) versus y(t). The user has additional options to (1) show the progression of points being plotted on all three graphs as t increases (Figure 1); (2) trace simultaneously along all three curves; and (3) trace simultaneously along all three curves with tangent lines (Figure 2). More details about revisions inspired by use are offered in the "lessons learned" section below.

Fall 2001 was the first semester during which Murphy used (an earlier prototype of) this applet with her calculus classes. To give some indication of possibilities for using multiple kinds of media (e.g., chalkboard, graphing calculators, applets), each lending itself well to some activities and less well to others, we describe one of the class sessions:

Projected on an overhead screen was the context for the problem (designed by White): "Milo the Mouse is out for a walk. The coordinates of his position at a time t (in minutes after noon) are given by the equations $x(t) = t^2 - 1$ and $y(t) = t^3 - 5t$."

> Reason for choosing overhead projector: keep the context available for the duration of the exercise; save time by not having to write it out during class.

With Murphy as scribe at the chalkboard, the class calculated Milo's position at times t=0, t=1, and t=2, plotting the corresponding points on an xy-coordinate system on the board.

> Reason for choosing chalkboard: build computational understanding of the relationship between the parametric equations and the resulting curve; writing calculations and plotting points all in one convenient area.

Murphy pointed out that x is a function of t and, with the Parametric Curves Applet, graphed that curve (upper left graphing space in Figure 1). She asked the students to describe orally the shape of x(t), emphasizing the use of "increasing" and "decreasing" rather than "up" and "down". After doing the same for y(t), she chose the "plot points" option in the applet. This option shows, as the user types the [→] key, the progression of points being plotted on all three graphs as t increases (Figure 1). As she typed the key, Murphy asked the students to predict, based on the behavior of x(t) and y(t), what the corresponding behavior of Milo's path would be: e.g., as x(t) decreases and y(t) increases, Milo should be heading left/west and up/north. She asked them to warn her when they expected a change in direction to occur: e.g., when x(t) has a minimum, Milo changes from heading in a westerly direction to heading in an easterly direction.

> Reason for choosing Parametric Curves Applet: build graphical understanding of the relationships between the parametric equations and the resulting curve; ability to display all three graphs dynamically and simultaneously.

Murphy then showed the students how to graph parametric curves on their calculators and the students spent the rest of the class session working in groups on examples.

> Reason for choosing graphing calculator: opportunities to analyze graphs dynamically; students will have regular access to graphing calculators more readily than applets or computer algebra systems.

After the in-class demonstration, Murphy felt that using the applet had been effective, but as she had not tracked the participation of individual students, she really had only her perceptions as a basis for her judgment.

Furthermore, the authors had nagging suspicions that students should interact with the applet for maximal learning. An in-class demonstration (rather than a lab) had been chosen as the initial use for the applet, in part because access to suitable computer classrooms at OU is limited, in part because it was not clear that the applet was "ready for prime time," and in part because we had not yet determined what, if any, activities would be effective to enhance student learning. Thus, we wanted to design an activity that would encourage the students to interact with the Parametric Curves Applet without imposing undue frustration. We finally agreed that, under the circumstances, the activity initially should be an out-of-class extra credit assignment to be completed in groups. Figure 3 gives one version of this activity (written by White). Details about the activity and lessons learned are offered in the next section.

# 3. Lessons Learned

In addition to using Java applets (along with graphing calculators) with classes, the authors formally gathered data related to the use of technology (by "formally" we mean with approval from the OU Institutional Review Board to use human subjects in research). Data collection consisted of (1) an instructor journal, (2) written student work, and (3) observations of several students working on an activity related to the applet. Specifically, we wanted to address two questions: (I) How do students interact with web-based interactive graphics (i.e., what expectations do students have for the technology)? and (II) What kinds of activities facilitate learning with such graphics? These data informed the redesign of both the applet and the related activity.

(I) How do students interact with web-based interactive graphics?

(A) Java applets written using the Emporia State MathToolKit require a particular (free) browser plug-in (see Murphy's Calc 3 website for details). Prior to the Fall 2001 semester, Murphy made sure that the student computer labs at OU had the plug-in properly installed so that students could use the applets in these labs. As Murphy was relatively new to Java programming herself, and not inclined to become an expert, she hoped that this effort would be sufficient. The extra credit assignment related to the parametric curves applets specified, "The applets require a special browser plug-in. Your best bet is to use the applets in the computer lab in PHSC 230 (Murphy has tested them there and knows that they work there – she makes no guarantees about getting them to work anywhere else)." Nevertheless, several students indicated that they had tried to use the applets on their own personal computers. Few realized that they needed the specific plug-in. Surprisingly, almost none of the students attempted to use the applets in the specified student computer lab.

LESSON LEARNED: Students want to be able to work on their own personal computers. Instructors need to know enough about the applets (and other browser-based interactive graphics) to ensure that students can access the needed software.

SOLUTION: One of the students who successfully installed the needed plug-in wrote an instruction sheet, now posted on Murphy's Calc 3 website.

(B) Lacking sufficient experience with Java programming and user expectations, Murphy initially enabled all of the trace features to trace only for increasing values of t, thinking that students would also have their graphing calculators accessible if they wanted other options. However, in observation sessions, students did not simultaneously use their graphing calculators and the applets. Also, thinking that simple was best, Murphy originally separated the tracing options into two applets. In watching students work on the activity, however, we were reminded that users prefer not to switch windows more often than necessary.

LESSON LEARNED: Users prefer to have resources available in one convenient, multi-feature module. If designers believe that a feature is important, then it should not be in a separate, isolated spot. Students also expect web-based interactive graphics to include features and behaviors familiar from graphing calculators.

SOLUTION: Murphy combined the options into one applet and enabled more trace features.

(C) As with graphing calculators and computer algebra systems, Java applets expect input to use specific syntax (e.g., in the MathToolKit, multiplication is represented by an asterisk: 3*t). In addition, Java applets can be somewhat intuitive to use but, as with other technology, they need documentation explaining operation procedures. The web page that houses the Parametric Curves Applet includes instructions detailing the expected syntax as well as explaining the features available. During observation sessions, the students consistently bypassed the instructions, instead going straight to the applet, then asking questions when they got stuck on how to use it. On the other hand, when this item came up for discussion, the students indicated that they did want the instructions provided and that they did not want the instructions on a separate web page (see lesson (I)(B) above).

LESSON LEARNED: Students expect to go straight to using the applet without reading instructions first.

SOLUTION: Murphy moved critical instructions to the applet itself and added syntax error dialog boxes, with references to the instructions provided above the applet. In keeping with student requests not to have multiple windows to navigate among (see lesson (I)(B) above), the detailed instructions were left on the same web page that houses the applet, rather than linked from that page. One option that was not discussed by this project team but that has been implemented by other applet designers is to have detailed instructions linked to small pop-up windows that can be viewed at the same time as the primary window.

(II) What kinds of activities facilitate learning with web-based interactive graphics?

(A) As in Figure 3, the original Milo the Mouse problem asked "Where is Milo at 11:59 a.m.?" We wanted this item to help the students think about the meaning and appropriateness of negative values for t (see lesson (II)(B) below). In practice, however, the students that we observed either calculated the point using just the formulas for x(t) and y(t) or did not answer the question at all. This student behavior raises at least two issues: the first relates to helping students think about appropriate domains and is discussed in lesson (II)(B); the second involves the motivation students have to complete a problem. When this item was asked as a part of the extra credit assignment, all of the groups that did the assignment answered this question. Yet when the activity was used just during an observation session, with no stakes attached, students were inclined to skip it, thinking (for the most part, correctly) that they already knew how to answer it and did not need to practice that skill. If the students do not complete the item, then our "hidden agenda" for the item is lost. This raises concerns about using such an activity as an ungraded in-class lab.

LESSON LEARNED: Students may not complete items that they believe they know how to do unless there is a reward for doing so.

SOLUTION: One possible solution is to grade the activity. Another solution is to write problems that students believe will contribute to their learning.

(B) Originally the first Milo the Mouse question did not include parts (b), (c), or (d) as in Figure 3. The Parametric Curves Applet has as a default domain for t the interval from 0 to $2\pi$, as most graphing calculators do (see lesson (I)(B) above). Using this default domain, students believed that we had mis-worded the question because the graph of x(t) did not appear to go from increasing to decreasing since that part of the x(t) curve shows up when the domain includes negative values for t. We had hoped that the question, "Where is Milo at 11:59 a.m.?", which preceded the increasing/decreasing item, would prompt the students to alter the domain from the default to a domain that included negative values. As noted in lesson (II)(A) above, the students calculated x(-1) with the x(t) formula. Apparently, the students did not make the connection from this exercise to the idea of altering the domain for t. A related phenomenon appears to occur when students graph parametric curves on their calculators. For example, if an exercise asks students to find the area enclosed by a loop, but the loop does not show up with the default domain, the students get confused. For instructors, one question triggered by these observations is: What activities will help students to consider altering a domain in order to see a different part of a curve?

LESSON LEARNED: As when they use graphing calculators and other technology, calculus students do not automatically analyze whether they have an appropriate domain.

POTENTIAL SOLUTION: We changed the problem to include parts (c) and (d). As the part of x(t) that goes from decreasing to increasing shows up using the default domain, we hope that students will at least gain confidence that they can answer such questions, before they get confused by the item asking when x(t) goes from increasing to decreasing. We also added part

(b) to increase the number of instances of negative values of t, hoping that this addition will improve the chances that students will consider negative values of t. [Note: we will test this solution early in March.]

# 4. Discussion

We are certainly not saying that graphing calculators and computer algebra systems should be usurped by web-based interactive graphics. Rather we want to emphasize that each tool has advantages. Graphing calculators provide substantial functionality all in one small easily portable machine. Computer algebra systems have powerful graphing and computation abilities. Yet, for narrowly focused activities, web-based interactive graphics may a better choice.

We want to emphasize all three aspects: web-based, interactive, and graphical. We have long believed in the power of visualization for student understanding, especially approaches that interweave multiple representations (symbolic, graphical, numerical). On various occasions we have used static graphs as well as dynamic animations (e.g., animated GIFs) to enhance visualization skills. Yet we are convinced that students learn best by doing rather than by just watching. Thus, we prefer tools that allow students to interact with the graphics. Finally, web-based resources can be readily accessible to students and instructors anywhere through any networked computer (and/or can be available on secondary storage such as CDs). To these ends, web-based interactive graphics have advantages over graphing calculators, computer algebra systems, animated GIFs, chalkboards, and print.

Websites

*Journal of Online Mathematics and its Applications* (JOMA): http://www.joma.org

Mathematical Java website: http://mathcsjava.emporia.edu/

Murphy's Calc 3 website: http://www.math.ou.edu/~tjmurphy/calc3.html

*Figure 1*. Parametric Curves Applet with "plot points" option selected.

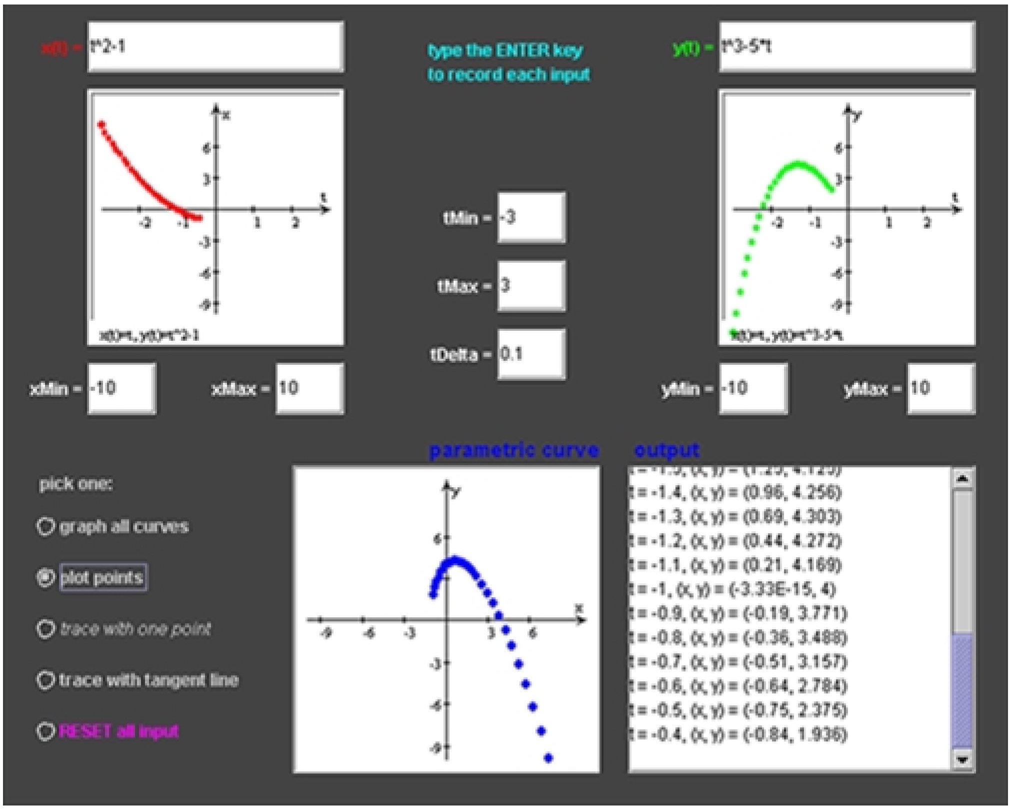


*Figure 2*. Parametric Curves Applet with "trace with tangent line" option selected.

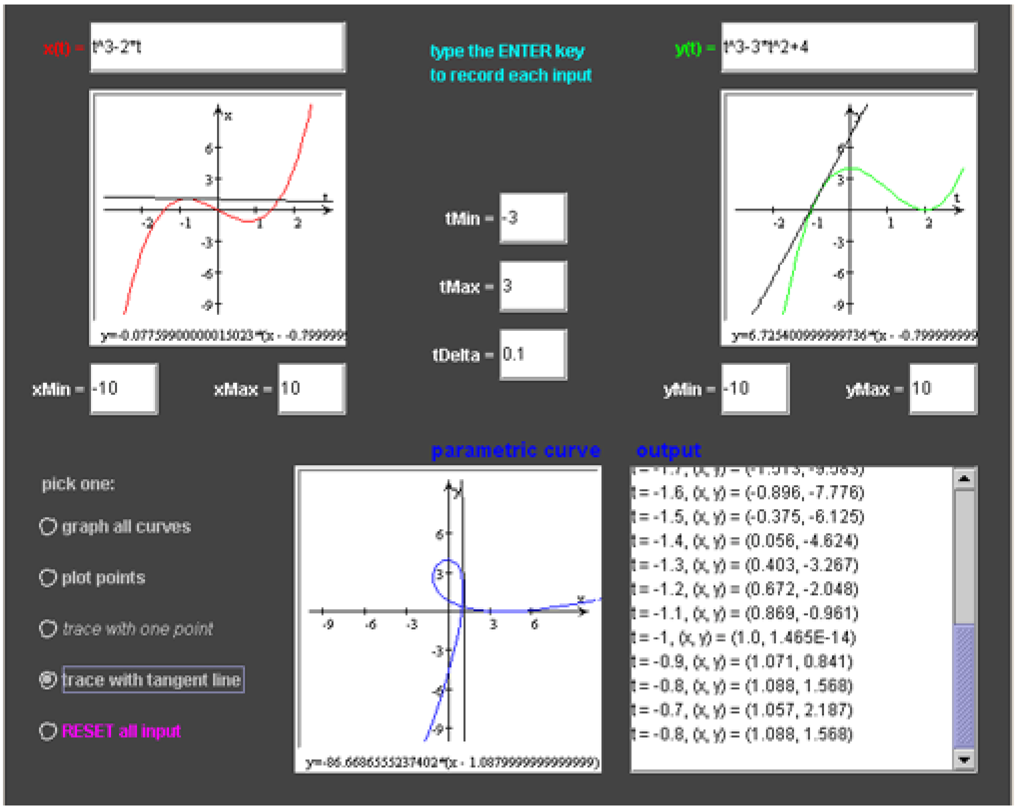

*Figure 3*. An activity for use with the Parametric Curves Applet.

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**Parametric Equations Activity**

For use with the Parametric Curves Applet at http://www.math.ou.edu/~tjmurphy (follow the 2433 link, then follow the link to the applet). If you are using your own computer, be sure that you have installed the required plug-ins.

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For each item below, show and briefly explain your work. Reproduce any graphs you used to think about the questions. Your explanations must include reference to any resources (e.g., people, books, technology) you used and how you used them .

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Milo the Mouse is out for a walk. His path is given by the parametric equations

$$x(t) = t^3 - 2t$$
$$y(t) = t^3 - 3t^2 + 4$$

where t is in minutes after noon (or before noon for negative values of t) and where the positive x direction is East and the positive y direction is North.

(a) Where is Milo at 12:01 p.m.? At 11:59 a.m.? (Note: "where" means "at what point (x,y)".)

(b) When is Milo at the coordinates (-4, 16)? (Note: "when" means "at what time t".)

(c) Look at a graph of t versus x(t). At which value of t does x(t) go from decreasing to increasing?

(d) Look at a graph of x(t) versus y(t) (i.e., look at Milo's path). When does Milo stop heading West-ish and start heading East-ish?

(e) Look at a graph of t versus x(t). At which value of t does x(t) go from increasing to decreasing?

(f) Look at a graph of x(t) versus y(t) (i.e., look at Milo's path). When does Milo stop heading East-ish and start heading West-ish?

(g) What can you say in general about what happens to a parametric graph x(t) versus y(t) at a t value where the graph of t versus x(t) goes from decreasing to increasing? What can you say about t values where the graph of t versus x(t) goes from increasing to decreasing?

(h) What can you say in general about what happens to a parametric graph at a t value where the graph of t versus y(t) goes from decreasing to increasing? Increasing to decreasing?

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