MATHEMATICS PROBLEM-BASED LEARNING THROUGH SPREADSHEET-LIKE DOCUMENTS

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ABSTRACT

Problem-based learning is particularly suitable in fields like Mathematics. It is quite usual to find abstract descriptions of solving methods to express mathematical knowledge. However, these descriptions happen to be somewhat hard for students. For instance, when a learner faces the integration-by-parts method, one possibility is to try to understand the general formula (by using the abstract description), while on the other hand another possible scenario is to learn by watching particular examples. Both approaches are complementary and, in fact, teachers usually move back and forth in order to manage their students to understand the underlying concepts.

In this paper we introduce *ConsMath* (CONStraint-based MATH teaching), a computer system that includes an authoring tool for the creation by the teacher of interactive Mathematics documents. The teacher can establish spreadsheet-like relations between the different mathematical formulae that appear in the document, and require certain conditions to be held when used by the student. The documents are dynamic and their contents changes depending on the formulae filled in by the student. ConsMath can be used in a particular style of problem-based learning in a context where each document is a problem pattern, and students can work on those problems by practicing with them repeatedly, and deciding in each case about the different steps that are necessary for their resolution. *ConsMath* runs both as a standalone application and in an applet within a web page.

An important feature of *ConsMath* is the possibility for the teacher to create interactive documents starting from static ones. Besides, the student can also ask *ConsMath* to generate specific problem statements from a given problem pattern. Consequently, students can choose between working on a problem posed directly by themselves, or asking the system to generate different problems corresponding to some given problem patterns.

KEYWORDS: Problem-based learning, Mathematics teaching, Spreadsheet, Interactive documents, Distance learning, Authoring tool.

1. Introduction

Learners of Mathematics many times have to solve problems that involve symbolic calculations by performing appropriate sequences of steps. These steps involve the manipulation of mathematical formulae obtained from the ones that appear in the statement of the problem. There are courses on specific subjects, like Integral Calculus and Ordinary Differential Equations, in which a high percentage of the work of the students is devoted to learning different methods of problem resolution of the above kind, and learning when these methods are successful. For instance, a typical course on Ordinary Differential Equations can include methods for the solution of linear equations, solution of equations by separation of variables, homogeneous equations, etc. Similarly, a course where basic integration methods are explained will include integration-by-parts and specific methods for the integration of rational functions, among others; all these methods are processes that consist of steps to be applied to the integrand. In general, Calculus and Algebra are particularly suitable to this approach, as calculations and predefined steps are necessary for the resolution of particular types of problems, although other considerations are also needed such as applicability conditions for certain steps. Textbooks like (Simmons 1981) help students to get insight on the different methods by showing particular examples that can be generalized. The work of the student when solving problems proposed by the textbook or by a teacher consists very often in reasoning about which method can be applied to the problem at hand, and applying it, or trying to adapt a known method to a slightly different situation.

On the other hand, problem-based learning in general, (Dutch & Gron & Allen 2001), is particularly suitable in Mathematics; this is especially relevant in subjects like the ones we have described in the previous paragraph, and textbooks of the type mentioned above help teachers and students to organize the learning process along the lines proposed by this theory. One of the main difficulties students have to overcome is the understanding of abstract descriptions of solving methods that express mathematical knowledge. For instance when a learner faces the integration-by-parts-method, one possibility is to try to understand the general formula (by using the abstract description), while on the other hand another possible scenario is to learn by watching a particular example. Both approaches are complementary and, in fact, teachers move back and forth in order to manage their students to understand the underlying concepts. Problem-based learning can take place in different ways, either through standard presentations by a teacher, or by means of a book or a computer. Even a collaborative approach is possible. Among the advantages of learning in the context of representative problems, probably the most outstanding one is the fact that this method of work allows the student to build a deeper abstract idea out of particular cases. Besides this, students can also recognize different forms of similar problems more easily, and they generate active self learning attitudes that are fundamental as a global goal of the educational process. Moreover, the learning process itself can be more attractive to the student due to the possibility to select the problems to be solved.

In this paper we show how the spreadsheet paradigm can be used in computer assisted tutoring of Mathematics courses of the kind introduced above. Moreover, we also show how these ideas can be used by means of a computerized tool that allows Mathematics teachers, without the need of a specialized technological knowledge, to define sets of problems that cover subjects similar to the ones we have described. The students can work on those problems by practicing with them repeatedly, and deciding in each case about the different steps that are necessary for their resolution.

Before going into more details, we shall see what is needed in order for a computer system to accomplish these goals. Firstly, we must be able to represent math formulae, so we need a powerful language to represent them. Besides, a specific software for rendering the formulae in a convenient manner is necessary, and a WYSIWYG (what you see is what you get) formula editing tool is also necessary. Secondly, the computer system has to support symbolic computations, since much of the work in the areas of Mathematics we are interested about entails transformations of formulae by purely applying arithmetic and symbolic rules, as it happens for example when solving a second-degree equation. Finally, we must remark that both the teacher and the student have to benefit from these possibilities, the teacher using the system as an authoring tool, and the student as a learning tool. In addition, distance learning scenarios would help, since teachers and students would not have to be "at the same time in the same place", and even in such a case a collaborative environment would be thereby possible.

More precisely, in this paper we describe the *ConsMath* (CONStraint-based MATH teaching) computer based authoring tool, which is based on the previous proposals and allows students to learn suitable mathematical concepts and problem solving procedures by means of a specific guided problem-based approach, based on the repetitive practice of particular techniques or processes. Teachers can create documents and sets of problems with *ConsMath* by describing the steps involved in a certain problem-solving method (e.g. solution of differential equations by separation of variables, or differential homogeneous equations) by using constraints between parts of a mathematical text according to a spreadsheet-like fashion. The use of the system in a distance learning environment is also possible, and consequently (this is an ongoing work) in a collaborative framework, (Mora & Moriyon 2001). Moreover, an important feature of the system is the possibility for the teacher to create interactive documents starting from static ones, which shall be explained below.

ConsMath is being developed as a part of the *Encitec* project, (Encitec), that is aimed at the development of tools for the development of distance learning materials and courses in scientific fields, involving symbolic, graphic and simulation components. A first prototype of *ConsMath* has been developed that includes the functionality explained in this paper. It has been tested by Mathematics teachers in order to build practising materials on Calculus and Ordinary Differential Equations, and the use of the corresponding sets of problems by students has started recently. From this point of view the system is suitable for its use according to the initial goals, but a detailed evaluation of its possibilities is still needed. From the technological point of view, the system is completed since the first prototype has been released earlier this year. However, assessment trials are still too limited. The assessment of the technology that has been carried out from the point of view of allowing teachers to incorporate their ideas has been successful, but the result of ongoing tests with students will be essential for the development of a system that can be used by teachers without any help in order to develop interactive materials related to different subjects.

In the following sections, firstly, we describe the technologies available to address these issues (section 2), and after this we describe our approach in the *ConsMath* computer system. Specifically, a first assessment of the system in terms of its use by teachers is discussed (section 3.3), along with considerations related to the technology chosen to implement the system (section 3.2), and a description of the system in terms of its possible use both by document authors (teachers) and by students (section 3.1).

2. Technology

There are different computer languages for the representation of Mathematics formulae, like *TeX* and *MathML*. The *TeX* language is broadly used in writing Mathematics books and papers, whereas *MathML*, (MathML), was conceived for the representation of Mathematical texts in web pages and for their future interactive treatment. The web is progressively becoming an appropriate instrument for the spreading of mathematical texts. However, W3C's *MathML* has not yet become extensively used in web browsers, and in fact only a few browsers, e.g. (Amaya), support *MathML*, though there are also examples of plug-in software (TechExplorer) to display both *TeX* and *MathML* documents in general purpose browsers. Nowadays formulae are usually represented in the web by using image files, nevertheless, *MathML* is being used as a common language to represent formulae in several computer systems, since interoperability and reusability are guaranteed from its use.

On the other hand, there are systems such as *Mathematica*, (Wolfram 1999), which support symbolic calculations (so are *Mapple*, *Matlab*, etc). In this system, a specific language is used to represent formulae, though the user writes formulae in the usual way (e.g. by entering "x+y"). Moreover, it contains a module for the transformation of *Mathematica*-based formulae to other languages such as *TeX* or *MathML*.

Graphical WYSIWYG equation editors are also widely available. For instance, *Mathematica* incorporates a built-in one that allows the user to select from palettes the corresponding operators. Other common tool is the Microsoft equation editor, used to generate formulae included in Microsoft Word documents. Another example is *WebEQ*, (WebEQ), a Java equation editor that can be used as a program included in web pages (applet). However, all these editors are ad-hoc elements that cannot be used in a project for Mathematics learning such as the one we propose in this paper. Specially, their extensibility is the main problem since they are not open source tools.

As far as networking, web browsers are the common means for distance learning. In particular, web pages can contain formulae represented as image files, or can have Java applets inside. Networking capabilities of the Java language allow communications between web browser users (students), and either a server or other students. The *Mathematica* software, apart from being able to be used as a standalone application, can communicate with a Java program by means of *MathLink*. Therefore, it is possible to build a distributed computer system containing modules such as a *Mathematica* evaluator, and one or several Java programs (or applets). Recently, the *WebMathematica* module has been released, (*WebMathematica*), which basically enables the access to the *Mathematica* standalone application by using the web as its front-end; *WebMathematica* actually relies on *JLink* for its networking processes. Nevertheless, by using *WebMathematica* it is difficult to achieve a high level of interactivity due both to the limitations of form-based web pages obtained and the usability of the language required for creating these pages (*Mathematica Server Pages*).

Finally, it is worth mentioning the existence of a very limited amount of computer systems that can be considered authoring tools for the creation of interactive sets of Mathematics problems, comparable to *ConsMath*. *PAT*, (Koedinger 1998) and MathEdu (Díaz 2001) are the most remarkable ones. Both systems achieve a higher degree of interactivity than *ConsMath* does, since in particular *MathEdu* includes dialogs between the student and the system, and also the possibility to define subproblems that must be solved during the solution of a problem, but *PAT* is very limited about the fields where it can be applied, that is restricted to Linear Algebra, and *MathEdu* has many more limitations than *ConsMath* from the point of view of the knowledge it requires from the teachers for its use, since they have to know a non trivial amount of programming in *Mathematica*.

Extending a system such as the previous ones to fulfil the goals proposed in this paper is a hard issue. The *Leibniz* system, (Leibniz), a tool built on top of *Mathematica* for the creation of mathematical documents by carrying out evaluations or by applying operators to formulae appearing previously in the document, is an example of this kind. However, the level of interactivity the system allows is somewhat simple. Moreover, a higher level of extensibility and reusability is needed, since these conditions are important in the generation of digital interactive learning resources, (Roschelle & Pea & Digieano & Kaput 1999).

3. The *ConsMath* Approach

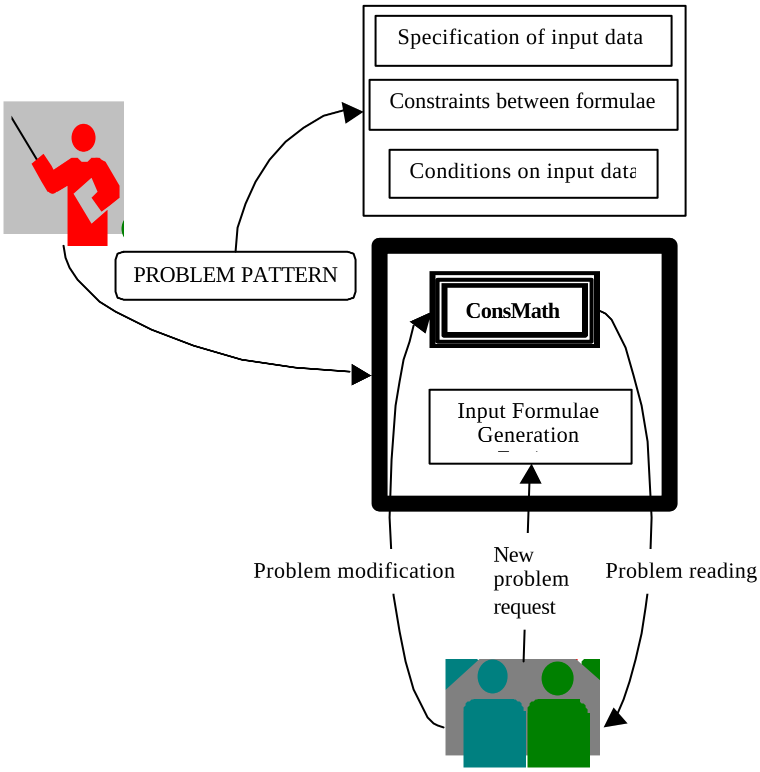
In this section, the approach used in the *ConsMath* computer tool shall be described. We shall present the system from three different perspectives: 1) a user-centred description in terms of the possible scenarios of use; 2) the decisions undertaken in relation to the integration of technologies needed, and the architecture of the system being developed; 3) a first assessment about the use of the system by teachers, and considerations about the benefits observed of using a system like *ConsMath*.

3.1. A User-centred Description

The *ConsMath* system has two types of users: the teacher and the student. As for the Mathematics teacher, he or she must have a certain familiarity with a graphical Mathematics editor, such as the Microsoft Equation Editor available in Microsoft Word. An important point is the possibility for the teacher to start with a static document, such as the description of the solution of differential equations by separation of variables. By using the graphical equation editor, he or she writes the equations (steps) involved in the method for a specific example. In a spreadsheet-like manner, when a formula depends on others, the teacher can define a constraint that links two formulae. For example, if a formula (or a portion of it) is calculated as a certain function of a previous one (e.g. on integration), this relation is defined by a constraint. The teacher can also designate the formulae that the student can modify (in our case the f function, for the differential equation y'= f y), namely input formulae, as well as demand certain conditions on those formulae. For instance, in our case the condition would be something like "dependsOnlyOn(x,f)"; as another example, in a problem where the teacher is explaining the method of rational function integration the condition to be held is that the integrand (input formula) must be the quotient of two polynomic expressions.

Students can use *ConsMath* in three different scenarios. Firstly, the student uses the material prepared by the teacher as a reading material, whether in a web browser, or in the *ConsMath* main window. Secondly, the student can interact with the teaching material containing solved problems. Specifically, the student can modify input formulae to generate alternative solved problems. In such a case, the system takes care of allowing those modifications (after checking that the conditions on input formulae defined by the teacher are satisfied), and it responds accordingly, by updating the formulae that are related to them by means of constraints. And thirdly, the student can ask the system to generate a similar problem about a solving method. The system responds by presenting a new problem where the input formulae have changed, while holding the conditions required for that solving method.

Each document created by a teacher that defines a solving method constitutes a problem pattern. The possible uses of *ConsMath* are depicted in the following figure:



3.2. Integration of Technologies, Architecture

The *ConsMath* system is implemented in Java. The reasons for this choice are the networking capabilities available in Java, along with the possibility of use in web browsers. As for the language for representing formulae, we chose *MathML*, particularly using content mark-ups, which give us a semantic representation of formulae, instead of presentation mark-ups suitable only for graphical renderings. Another reason for the choice of *MathML* was the number of efforts being undertaken to consider it as a standard, as well as that many systems, like *Mathematica*, provide support for conversion to *MathML*.

As far as the graphical equation editor concerns, we have our own Java program to graphically build equations, which are represented internally in *MathML*. We could have tried to use other software such as the *Mathematica* front end, *WebEQ*, or even the Microsoft Word equation editor. However, it was impossible to us to use them as specific libraries to include in our Java programs, so we decided to build our own editor. The first prototype supports a limited amount of operators, namely the ones that have been necessary for the tests that have been performed (though we are augmenting the available ones in our operator palette), but it has allowed us to do the tests described in this paper.

On the other hand, we rely on the *Mathematica* system for symbolic processing or evaluation of formulae. *Mathematica* has also a powerful pattern matching capability for Mathematics formulae that allows teachers to establish conditions on them. Moreover, when a student requests a new problem, the system generates random formulae that satisfy the conditions specified by the teacher. This part has been implemented in *Mathematica*, and it benefits from powerful capabilities of this system. Another interesting point is the modularity achieved in *ConsMath* in relation to the evaluation system used, as it can be substituted by a different one without any major changes in the tool.

3.3. Assessment

A complete assessment of *ConsMath* includes two different levels: on one hand, the tool must be tested by teachers in order to build their collections of problems on some specific subjects. On the other hand, the use by students of the materials developed by the teachers is also essential. *ConsMath* has been tested by Mathematics teachers in order to build practising materials on Calculus and Ordinary Differential Equations. The solution of a set of integration problems from (Spivak, 1989), and another set of problems on Ordinary Differential Equations from (Simmons 1981) have been implemented using *ConsMath*. The main difficulties found arose from the need to extend the set of operators included in our equation editor, something due to the fact that it was an unfinished prototype. The teachers who have used *ConsMath* consider that it is a highly suitable tool for teaching purposes. The corresponding sets of problems have started to be used by students recently, and no results are still available from this experience. However, we can already claim that the system is suitable for its use according to the initial goals, although a more detailed evaluation of its possibilities is still needed.

The tests that have been accomplished with teachers point out three important features of the *ConsMath* approach:

1) The spreadsheet-like fashion of problem patterns allows teachers with a certain familiarity to spreadsheets to emulate the constraint-based approach by generating documents that include mathematical formulae and constraints between them. These documents can also be used for simulation processes where the user observes the consequences of the modification of certain input data.
2) *ConsMath* allows teachers to generate documents in an environment that is similar to the one used by students. Moreover, teachers can seamlessly switch to the student role to verify the suitability of the document being generated.
3) With *ConsMath,* interactive documents can be generated from static ones, by specifying which are the input formulae and the constraints to be held between different formulae. Hence, available static documents containing *MathML* formulae are potentially suitable for its use with the *ConsMath* tool. As more an more static documents of this kind are available, the possibilities of use of *ConsMath* and the simplicity of using it will be bigger.

4. Conclusions and Future Work

In this paper, we have described our experience in the design of the *ConsMath* system. This system is an authoring tool for the creation by the teacher of interactive Mathematics documents that describe a given solving method. Teachers can establish spreadsheet-like relations between parts of documents and require certain conditions to be held when used by the student, who can benefit from the relations established by the teacher in a distance learning environment. In relation to the *ConsMath* approach, some advantages have been pointed out concerning to the ease of use by the teacher and the positive aspects of its usability, namely the spreadsheet-like manner of creating documents, the use of the same environments by the teacher and the student, and the possibility of creating interactive documents from static ones.

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