WHAT CAN WE KNOW ABOUT PRE-SERVICE TEACHERS' MATHEMATICAL CONTENT KNOWLEDGE THROUGH THEIR E-MAIL DISCUSSIONS WITH 6TH GRADE STUDENTS?

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ABSTRACT

In this study, my aim was to understand whether pre-service teachers (from a nontraditional mathematics classroom) have developed good understanding of fraction concepts and are able to do algebraic thinking. Furthermore, I sought to determine whether they could use their knowledge of fractions and algebra clearly and effectively in their communications with 6th grade students (also from a nontraditional mathematics classroom.)

The approach in this college mathematics class, "Learning Mathematics via Problem Solving"(Masingila, Lester, & Raymond, 2002), is very different from a traditional approach. In this class, students construct their knowledge through active involvement with challenging mathematic problems while the instructor facilitates, guides, and helps students share their own knowledge. During a semester in 2001, pre-service teachers at our university studied mathematics in groups of four, shared their mathematical ideas and thinking with the entire class, kept daily math journals, and participated in a math communication project in which they discussed mathematics problems about fractions and algebra with 6th-grade middle school students via e-mail. The instructor and I wanted to determine whether a "learning via problem solving" approach enabled our students to understand fraction concepts and engage in algebraic thinking.

In this research, I analyzed the e-mail messages/discussions written to the 6th graders by the pre- service teachers in order to understand the pre-service teachers' content knowledge. This analysis enabled me to relate the type of pre service teachers' content knowledge to how they responded to middle school students' e-mails on fractions and algebra. The findings included here are the result of a preliminary analysis of the data, other possible categories or depth of the analysis will be included in further publications.

Keywords

Pre-service Teachers, Prospective Teachers, Content Knowledge, Subject Matter Knowledge, Mathematical Communication, Mathematical Writing, Group Work, Problem Solving, Children's Thinking.

1. Introduction

Many researchers have investigated the relationship of mathematical content knowledge of teachers and their teaching. It is generally accepted that teachers have to have strong mathematical knowledge for effective teaching. As Liping Ma (1999) stated, "Teachers with profound understanding of fundamental mathematics are able to reveal and represent ideas and connections in terms of mathematics teaching and learning." The teachers who do not have the understanding of mathematical concepts cannot engage students in productive discussions or cannot recognize student understanding when it occurs (Fennema, Romberg, 1999). When teachers have a strong understanding of math, this understanding will help them choose and implement tasks that have the valuable mathematic content and the potential to motivate students (Hiebert et all, 1997). The literature about teacher's mathematical content knowledge shows parallelism with Martin A. Simon's "Teaching Cycle." According to Simon, teachers' knowledge of mathematics affects their learning goals, the plan for learning activities, the hypothesis of students' learning process, and their assessment of students' knowledge (Simon, 1995).

For pre-service teachers, as well, the mathematical content knowledge should affect their instruction by how they choose tasks, how they lead mathematical discussions or how they asses their students' knowledge. Higher education has considerable impact on pre-service teachers' mathematical content knowledge before they enter the teaching profession (NCTM, 2000). Generally, Schools of Education and Mathematics Departments at universities are responsible for the mathematical content courses in which pre-service teachers learn the mathematics they need to teach. After the content courses, pre-service teachers take method courses, in which they learn how to teach mathematics and concentrate on children's mathematical thinking.

In this study, Dr. Beatriz D'Ambrosio, my mentor and the instructor of the course, and I wanted to know more about pre-service teachers' mathematical content knowledge as they took their last content course before entering the teacher education program. The pre-service teachers of this mathematics course had a project in which they discussed and wrote about mathematical problems to their 6th-grade partners through e-mail. In this research, I investigated the e-mail messages/discussions of pre-service teachers to understand their level/type of mathematical content knowledge.

2. Context for the Study

For my graduate internship, I assisted Dr. D'Ambrosio with her semester-long college-level mathematics course. Students in this course expected to be admitted to the elementary education program after finishing this content course. In this class, they learned mathematics quite differently from traditional classes.

In this class, the "Learning via Problem Solving" method was used (Masingila, Lester, & Raymond, 2002) and group work was central to the "Learning via Problem Solving" method. By the help of our instruction and group work, it became important for pre-service teachers to "understand" other people's solutions and to rely more on their own mathematical abilities, as well as to know the importance of building new mathematical knowledge through their own efforts (e.g. asking questions for understanding in group work).

While pre-service teachers were getting used to reflecting on their thinking in their small group discussions, the instructor and I required daily mathematics journals. In these journals, we wanted them to reflect deeply on the main problems, which were discussed in the class session. Our aim was to build habits of reflective thinking while improving their mathematical content knowledge.

The assessment in this class was very comprehensive. In addition to math journals, we assigned homework problems related to the mathematical content studied during the week. The pre-service teachers had tests, which also emphasized group work. They did group projects and presentations at the end of the semester. They also wrote a paper about children's thinking based on the experiences in an e-mail discussion project with 6th-grade students as a requirement of the course.

In the e-mailing project, every pre-service teacher was paired with at least one 6th-grade student who sent e-mails about one problem they solved in their mathematics classroom every week. Our pre-service teachers were familiar with the problems because they had solved them by themselves as an assignment before they responded to the 6th graders. These responses were about the childrens'mathematical approaches and the pre-service teachers' own approaches to the same problems.

The e-mailing project was designed for two related considerations:

The first consideration was the mission of higher education in pre-service teacher's training (NCTM, 2000). The course was designed to give strong mathematics knowledge to future teachers before they started to teach preK-12 mathematics and to prepare them to understand children's mathematical thinking in school based learning communities. The course was not a methods course but this project helped pre-service teachers to experience children's' ways of thinking in a content course. We wanted pre-service teachers to come to their own realization of the "need" for strong mathematical knowledge, through the interaction they had with 6th-grade students in the project. Pre-service teachers had more real situations about what kind of students and mathematical thinking to expect in their future classrooms rather than the examples in the written resources.

The other consideration was the curiosity of the 6th-grade classroom teacher. She wanted her students to have a real audience for their mathematical writing, thinking this would help them to improve their writing. She suggested a partnership project of pre-service teachers and her 6th graders in order "to make a more meaningful, real-life mathematical communication opportunity" for her students (Schoen Strabala, 2000).

At the end of the semester, while grading the pre-service teachers' papers about children's thinking according to the professor's guideline, I wanted to see the actual e-mail discussions they had with the 6th-grade students. I thought that writing back and forth was a real situation for those future teachers, in which they were using their mathematical content knowledge. Pre service teachers were using mathematical knowledge in more realistic situations compared to our classroom assessments, such as, when they were reading 6th-grade students' e-mails, when trying to make sense of 6th graders' mathematical explanations, when struggling with the mathematical ideas, when asking questions for clarification, and when giving feedback on the 6th graders' solutions and explaining their own approaches.

I was thinking that pre-service teachers were doing those tasks (the process of replying back and forth to 6th-grade partners) depending on what they knew about mathematics and how they knew it. This experience was my starting motivation for why I did this study: "What can we know about pre-service teachers' mathematical content knowledge through their e-mail discussions with 6th-grade students?"

3. Assumptions

In this study, we assumed that pre-service teachers were learning "communication" while working in groups of four people and while reflecting on their ideas in daily math journals. The group work gave the students an opportunity to ask appropriate questions in group discussions to clarify their thinking about the solutions to the same problem. The math journals helped individuals to realize how they were using mathematics. Writing demanded more effort for them than just discussing verbally; they had to think about what they wrote and why they found the written pieces meaningful and mathematical. The instructor and I were commenting on their journals in that way. Therefore, we expected them to use those communication skills during the e-mail discussion they had with 6th-grade students.

Because we knew that the 6th-grade teacher was using similar instructions to ours, we assumed the 6th-grade students were used to thinking deeply about the problems, develop strategies and plan for the solutions. Based on this assumption, we also thought that the 6th-graders would solve problems and write about them independently of their teacher, which would make rich discussion opportunities for our pre-service teachers.

4. Data and Analysis

Pre-service teachers, who took Problem Solving in Context of Teaching Mathematics (6-hour credit), posted their first message in which they introduced themselves to their assigned 6th-grade partner early in the semester. In return, the 6th-grade students also introduced themselves via e-mail before a face-to-face meeting. During this project, pre-service teachers visited the middle school two times, when we started (to say "hello") and when we finished at the end of the semester (to say "bye") and in each occasion, they played a mathematical game. Between these meetings, pre-service teachers and 6th-grade students used a technological support service for classroom instructions for communicating via e-mailing.

There were 28 pre-service teachers. Each of them sent, between 7 to 24 reply e-mails to their 6th-grade partners regarding four mathematical problems related to fractions and algebra. In this paper, the e-mail responses related to "the Fair Share Problem," one of the problems discussed in the project, will be presented as a sample analysis of the study.

Content analysis of the e-mails was used to develop categories for understanding of pre-service teachers' content knowledge. According to Fraenkel and Wallen (2000):

> Content analysis is a technique that enables researchers to study human behavior in an indirect way, through an analysis of their communications. A person's or group's conscious and unconscious beliefs, attitudes, values, and ideas often are revealed in their communications. Analysis of such communications (newspaper editorials, graffiti, musical compositions, magazine articles, advertisements, films, etc.) can tell us great deal about how human beings live (p.469).

Analyzing the e-mailing task and learning a great deal about pre service teachers' content knowledge is similar to Fraenkel and Wallen's (2000) "indirect way" of studying human behavior.

5. Categories and Findings

Preliminary findings of the analysis, and sample email messages are discussed below. Ma's (1999) and Hibert's (1986) categorization of "conceptual knowledge" and "procedural knowledge" helped me in my categorization of pre-service teachers' mathematical content knowledge.

Based on the e-mail messages, the pre-service teachers fell into three categories. The categories are conceptual knowledge, non-conceptual and "others."

The pre-service teachers who demonstrated conceptual knowledge asked good questions for understanding or making the child's explanations clear. They were able to discuss or introduce different solutions to a problem, or create related examples in their response pushing the child to think further and thus demonstrating that they themselves had thought more deeply about the problem.

The pre-service teachers who had no conceptual knowledge just accepted the child's thinking and did not try to understand child-constructed mathematics deeply. They sometimes did not recognize the child's explanation or solution as a legitimate solution since the child was not using procedures that the pre-service teachers were used to. At that times, they asked procedural questions, since they were lost in child's explanation, or they were not sure with their results after comparing them with the child's results.

In addition to the previously-defined two categories, there was a third category of pre-service teachers who totally avoided writing mathematically or even showing mathematical procedures for the solutions. This category is called "others." There were 6 out of the 28 pre-service teachers who showed a pattern of not responding to any of the children's emails related to the four mathematical problems. Since, these pre-service teachers wanted to be teachers, we thought they cared about children and childrens' thinking; they chose to be in this profession. Therefore, the explanation for not responding appears to us that they do not have enough knowledge, interest, and confidence in mathematics to show their work in this e-mailing project, which was done with 6th grade children.

The categorization in the "Fair Share Problem" is used as a sample to show how the analysis was done in this study (you can find the problem and more examples of analysis in Appendix):

Category One (Conceptual Knowledge):

The pre-service teachers who demonstrated conceptual mathematical knowledge, made appropriate connections of mathematical concepts in the emails, asked clear questions, followed student's mathematical constructions, and explained their own thinking clearly in different ways. There were 4 out of 28 pre-service teachers who showed conceptual understanding in different levels of their categorization.

Email-1: (6th grader's)

Posted: 2/27/2001

Now let's get down to business with the Fair Share problem. I 1st set the treasure # at 1 because it would give me the fractions. I subtracted 1/3 because that is what the 1st guy took. Next, I took away 1/6 for the 2nd guy, and I don't know why now, and that gave me 3/6 or 1/2 for the last guy.

I am confident that my fist answer is correct, but my 2nd and 3rd aren't. I know that a third of a third is a nineth. That would make 1/3 of 2/3, 2/9. You multiply because that is what of means. 2/3-2/9=4/9. Those are the only parts I had touble with. Either way the 3rd guy comes out ahead. Ask if you have any questions.

Email-1: (pre-service teacher's response)

Posted: 3/5/2001

You ended up with all of the shares being divided ok and yes the last guy ended up with more than the other two. Can you think why after the first guy took 1/3 leaving 2/3 you would say the second guy took 1/6? There were 2 shares remaining equal to 2/3 of the total. The second guy took only 1/3 of the 2/3. It might help if you look at the whole in fractions that are divisible by 3, 6 and 9.
Let me know if this is too confusing or if it helped.

Posted: 3/9/2001 7:16:08 PM

Okay lets finish 'Fair Share' first. I didn't mean to confuse you with the 3, 6, 9. Here's my thinking on the problem. We have a whole that guy 1 divides into three and then takes 1/3. Guy 2 splits the 2/3's remaining into thirds and takes 1/3 of it. How can we divide the 2/3 equally into 3/3? If I split the 2/3 into sixths there would only be 4 to split up because 2 of those sixths were taken by the first guy. I can't split 4/6 into 3 groups equally. Now I decide to split the 2/3 into ninths. Since I know the first guy took 1/3 of the ninths that is equal to 3/9. There are 6 ninths remaining to split into 3 groups (2 per group) of 2/9 each. Guy 2 took 1/3 equal to 2/9, leaving guy 3 with 4/9.
This sounds more complicated than it should. I am often not very good at explaining things so don't worry about telling me you don't understand what I'm saying!!

The child was struggling with the idea that there was something wrong with her thinking on the shares of the second and third treasurer. The pre-service teacher was asking a simple but powerful question to open it, "Can you think why after the first guy took 1/3 leaving 2/3 you would say the second guy took 1/6?" she was not restricting the child's thinking into procedure (e.g., how did you get 1/6. multiply or divide?) by her question; her question was very open.

This pre-service teacher had also an interesting thinking on the Fair Share Problem. She was different from the other thinkers on how she thought to have thirds, sixths, and ninths. Her explanation to decide what to use for dividing the whole (into 3, 6 or 9) was related to the amount of the treasure that was left for the second and third treasurer, and their shares.

In her explanation, she had some misuses related to the whole or the language linked to the fraction parts. She wrote "how can we divide the 2/3 into 3/3?" (She was considering how we could divide 2/3 into thirds, because second treasurer needed to take 1/3 of the remaining treasure-2/3 of the treasure) or "If I split the 2/3 into sixths there would only be 4 to split up because 2 of those sixths were taken by the first guy"(she thought splitting the whole treasure, not the 2/3 of the treasure, into sixths; which would give her 4 split for 2/3 of the 6 splits). However, when you follow her explanation, it is easy to understand what she meant in those sentences.

Category Two (Non-Conceptual Knowledge):

The pre-service teacher in this category could not demonstrate conceptual knowledge. They did not use mathematics in depth; generally, pre-service teachers gave their own results or compared them to the results of the 6^{th}-grade partners. When pre-service teachers asked questions in their response e-mails, the questions showed that either they were unable to follow the child's solution or they were only able to ask non-conceptual questions based on their knowledge. According to the categorization of this study, there were 12 out of 28 pre-service teachers who didn't reveal conceptual knowledge.

Email-2: (6^{th} grader's)

Posted: 2/27/2001

On the Fair Share Problem I got that the 3rd guy had the most treasure because if there were 9 the first guy took 1/3, which is 3 there would be 6 left and 1/3 of 6 is 2, so there were 4 left and the last guy took the rest. At first I didn't know that the last guy only took 1/3 of the 4, so I thought the first guy had the most, but I was wrong.

Email-2: (pre-service teacher's response)
Posted: 3/5/2001
The Fair Share Problem: What was the correct answer and how did you solve the problem?

The pre-service teacher was not able to follow the child's work. Child was already explaining how he solved the Fair Share Problem and what he got. However, since the pre-service teacher was concentrating on the results and might be on the procedural solutions; she didn't see the child's obvious solution as a satisfactory and mathematical solution. To be able to understand child's thinking, one needs to have algebraic thinking to ask further question. The child's reasoning should be questioned when he chose the number "9" to represent the treasure in his model for the solution.

Category Three (Other):

The pre-service teachers who were in this category avoided discussing mathematics. They did not demonstrate either conceptual or non-conceptual mathematical knowledge. The y did not apply the mathematics communication experiences from their own mathematics classroom to the e-mail discussions. There were 12 out of 28 pre-service teachers who fell into this categorization for this problem. 5 of them didn't respond in continuou sly for other 3-mathematics problem and 6 of them skipped this problem either they didn't have any idea about this problem and related mathematics or they were in a hurry in that particular time of the semester, while 1 of those 12 pre-service teachers was writing about everything but not about mathematics in this problem.

7. Conclusion

E-mail discussions of pre-service teachers can be used as an additional assessment tool in pre-service teachers' content courses. The pre-service teachers demonstrated different types of mathematical understanding in their e-mail messages/discussions with 6^{th}-grade students. Approximately 4 out of the 28 pre-service teachers showed conceptual understanding by exchanging and exploring mathematical knowledge deeply with their partners. Most of them (around 18 pre-service teachers out of 28) showed non-conceptual understanding of mathematics by just giving the results or asked simple questions that showed they were unable to follow the children's mathematical thinking or just agreeing on the solution to make their 6^{th} grade partners feel better or confident in mathematics. Approximately 6 of the 28 pre-service teachers did not discuss mathematics at all. In addition to tests, the real communication pieces of pre-service teachers can be used for building a hypothesis about their knowledge and improving the mathematical experiences of these students.

Based on the previous research [like Ma (1999), Ball (1988), since researchers used direct interview], we did not expect to find e-mail responses like those placed in the "others" category-avoided mathematics. These pre-service teachers' attitudes and knowledge about mathematics can be analyzed further with different research techniques.

This research raised other questions about pre-service teachers' knowledge and their future teaching and training. For example: What experiences must the teacher education program provide in order to help pre-service teachers in each category to grow? Can the pre-service teachers who avoided writing mathematically become good teachers? What experiences do non-conceptual thinkers need in order to become more open to children's thinking?

It is hoped this study will be an opening for understanding and/or assessing the mathematical content knowledge of elementary pre-service teachers with different methods. In this case, we used their real communication pieces written to children throughout the semester in a content course.

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Appendix (Additional Analysis)

The Fair Share Problem:

Three brave, but not very bright, treasure hunters recovered a small box of Spanish doubloons aboard a sunken ship. They took the coins back to their campsite. Since it was late, they decided to go to sleep and divide the treasure the next day.

One of the treasure hunters, fearing the others didn't understand mathematics well enough to give out fair shares, took 1/3 of the coins in the middle of the night and fled into the darkness.

Later that night, another treasure hunter awoke and saw that some of the coins were missing. The treasure hunter took 1/3 of the remaining coins and fled into the darkness.

The third treasure hunter awoke and was surprised to see the others gone and many of the coins were missing. Trusting that the others left a fair share, the third treasure hunter took the remaining coins and walked away whistling happily.

Which of the treasure hunters ended up with the greatest share of doubloons?

Category One (Conceptual Knowledge):

Email-4: (6^{th} grader's)

Posted: 2/27/2001

The third man would have gotten the most coins. Say there were 33 coins toi begin with. If the first man took 1/3, there would be 22 coins left. The first man got 11 coins. If from that 22 coins, the second man took 1/3, he would have taken around seven. There would be 15 coins left. The third man took all 15, making him have the most coins.

Email-4: (pre-service teacher's response)

Posted: 2/27/2001

I do believe that your answer to the fair share problem is correct. I did the problem last night and I figured out that the last guy would get the most coins also. No matter how many coins they started out with, the last person would always get the most. Do you know why that is?

This pre-service teacher was looking at the problem by considering that the amount (the number of the coins) in the whole was not matter as long as the fraction parts were same for each case (e.g., the first treasurer always took the 1/3 of the treasure or the second treasurer always took the 1/3 of the remaining treasure). She had the ability to generalize, in which case she didn't depend on the certain number of coins. This shows different understanding; she went beyond the procedures that were valid for just one occasion (e.g., 33 coins in the treasure) and she was generalizing it conceptually.

Category Two (Conceptual Knowl edge):

Email-5: (6^{th} grader's)

Posted: 2/27/2001

I noticed that the second guy was the stupid one, because if the dubloons were to be even, #2 would have taken 1/2. Instead, he took 1/3, accidentally leaving 2/3 left for 2 more people, in which there was only one person, so getting this from the original number, the 1st guy got 1/3, the stupid one got 1/6, and the lucky guy got 1/2.

Email-5: (pre-service teacher's response)

Posted: 3/13/2001

I think your reasoning on the fair share problem wa s awesome, although I really had to think about the problem. When I read your reply, I was like, "Oh, yeah, that makes sense." That second one really wasn't very smart.

The pre-service teacher was thinking that the child's way was "awesome"; so what was awesome in this solution for this pre-service teacher? How could one be sure about the child's thinking without asking, "the second guy took 1/6 of what? And how can you compare his share to the first one's share and the third one's share?" The child said "the stupid [*the second treasurer*] one got 1/6, and the lucky guy got ½" and the pre-service teacher didn't asked the child about how the child got 1/6 for the second treasurer or ½ for the third treasurer.

Email-6: (6^{th} grader's)

Posted: 2/26/2001 4:39:34 PM

The paragraph says the firs person takes 1/3 of the treasure. The second person got one sixth. I know this because he took 1/3 of what was left. There was 2/3 left 1/3 of 2/3 is 1/6. The last person got the most because he got 1/2. I know he got 1/2 because I converted 1/3 into 6ths so I could add 1/6 and 2/6 which equals 3/6. 6/6 - 3/6= 3/6. Which when reduced equals 1/2.

Email-6: (pre-service teacher's response)

Posted: 3/5/2001 10:13:09 PM

When you said you took 1/3 of 2/3 and got 1/6, were you multiplying or dividing? How do you know this? I was just curious why you thought that way. We are also going over fractions in class right now, and it has been a long time since I have worked with fractions, and by reading your steps to solving problems I have started to remember them again,

This pre-service teacher asked a simple question to follow how the child got 1/6 when he took 1/3 of 2/3; but thinking procedurally, she was asking procedural question "were you multiplying or dividing?" Her question showed that she was not thinking algebraically because either way when one multiplies or divides 1/3 with 2/3, the result can't be make 1/6. Pre-service teacher's questions might show that she realized that the child got 1/6 of the treasure for the second person, so the child got wrong for the third person depending on what he found for the second person. However, her intent to know about used operations were not related to analyze the child's thinking, as well as the meaning of operations that the child used while getting 1/6 as his answer.