

INTRODUCING EXPERIMENTS INTO A FIRST COURSE IN CALCULUS

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ABSTRACT

It has become widely accepted that mathematical software can contribute significantly to the learning and understanding of Mathematics. In particular, the visualization capabilities of software packages and Computer Algebra Systems, students can explore function behaviour and phenomena that would be impossible without the use of computers. A Mathematics instructor has a wide choice of software tools to consider for use in undergraduate courses. Yet, the problem remains how to construct interesting problems that would challenge the student and where the technology is an important tool assisting in the exploration, yet allowing one to reflect, analyse, modify one's thinking until the appropriate conclusion is reached.

In this paper, I give examples of challenging problems within the conceptual reach and understanding of Calculus students. These problems were given in the fall 2001 to students taking a first course in Calculus. A characteristic of these examples is that without technology it may be difficult for students to do the analysis and to obtain the answer, yet the technology and its visualization capabilities provide the student with a mechanism for experimentation and testing, allowing them to modify their hypothesis and their thinking to lead them to a solution. Second, in these problem tasks, there is not one correct answer and the answer can be given to different degrees of generalization allowing students to go as far as they can in their analysis. In these assignments, a written component was added so that the students can reflect on their own thinking. They were required to do a write up showing the steps used in their analysis and an explanation of why the conclusion they arrived at is a valid one. Technologies used in these experiments are the TI-89/92 calculator and the dynamic software "*Autograph*".

Keywords: Calculus Reform, Technology, Visualization

Introduction

It has become widely accepted that mathematical software can contribute significantly to the learning and understanding of Mathematics. In particular, the visualization capabilities of software packages and Computer Algebra Systems allow students to explore function behaviour and phenomena that would be impossible without the use of computers. A Mathematics instructor has a wide choice of software tools to consider for use in undergraduate courses. Yet, the problem remains how to construct interesting problems that would challenge the student and where the technology is an important tool assisting in the exploration, allowing one to reflect, analyse, modify one's thinking until an appropriate conclusion is reached.

In this paper, I present examples of the use of technology that was done with students taking a first course in Calculus in the fall 2001. A characteristic of these examples is that without technology it may be difficult for students to do the analysis and to obtain the answer, yet the technology and its visualization capabilities provide the student with a mechanism for experimentation and testing, allowing them to modify their hypothesis and their thinking to lead them to a solution. In these assignments, a written component was added so that the students can reflect on their own thinking. They were required to do a write up showing the steps used in their analysis and an explanation of why the conclusion they arrived at is a valid one. The technologies that were utilized were the TI-89/92 calculator and the dynamic software "*Autograph*". The reason these were used was the ease with which the calculators could be brought into the classroom in addition that some students already had them. The "*Autograph*" was used because it was simple to learn and provided powerful animation and a numerical component.

Background of this experience

The students in this class came from quite a varied background. Some of the students come from Lebanese schools but who either did not pass the official exams in the math and science sections or some who may have been in a literary and humanities sections and therefore did not do enough mathematics to enter at the sophomore level to major in Engineering or in Computer Science. A large number of students come from Arab countries and may have studied in Arabic but generally have a poor background in Mathematics. There are two sections of the course and one section was set up to be an experimental section where the teaching would be non traditional with a strong emphasis on the visual and numerical aspects. The textbook used for the course was "*Calculus, from Graphical, Numerical and symbolic Points of View*" by Ostebee and Zorn [4].

The rationale for this experimental section was that we were dissatisfied with what students were learning after two years of Calculus, as have been experienced by many math educators everywhere. Students come out not having any conceptual understanding of the concepts in Calculus and after two years have almost forgotten everything as I witness this in my Numerical Analysis course. Even though in our teaching, we tried to emphasize the visual as well as the conceptual, and we assigned projects using mathematical software, the students seem not to place the same importance to the topics done in class and would be satisfied if they were able to do the routine problems specified in the textbook. The large number of routine exercises as well as what they were used to in high school seemed to define to them what "mathematics" was important. Even though we added "*Mathematica*" projects, these were done in a mechanical way and little or no benefit was gained. Thus we were coming to the conclusion that you cannot change student's perception of what is the mathematics that is important, and therefore their attitudes to the subject

as long as we use a traditional textbook. As Porzio [4] concludes: “in revising the curriculum it is not sufficient to tag on technology on the topics covered, but to emphasize equally the various representations in the study of various concepts and to design problems that will reflect this philosophy, and that will engage the student in moving between the various representations in their solution.” At the same time, we were reluctant to make changes for all of the courses without first having an experience of what it would entail and how students and the departments for whom Calculus is a service course would react to this change.

A large number of handouts were given to the students and a number of projects were assigned to be done in groups of two, where a written component was required. The other section was taught in a traditional way. I will not go into the details of how the course was conducted but will focus on the experiments that were done using the TI 89/92 calculators and “*Autograph*”.

The Experiments:

1. Experiment 1: Studying the graph of a polynomial of degree 4

The graphs for this experiment were generated using “*Mathematica*”, however the students were expected to do this using the TI 89/92 calculator or any other graphic calculator available to them. Students could also borrow graphic calculators from me. The graph of a polynomial of degree 4 having four distinct roots and opening upwards was given along with its equation. The defining equation was $f(x) = 3(x-2)(x-1)(2x+1)(x-3)$ and they were given that it expands to $6x^4 - 33x^3 + 48x^2 - 3x - 18$. The graph as given is shown in the following diagram, and no scale was indicated in the diagram:

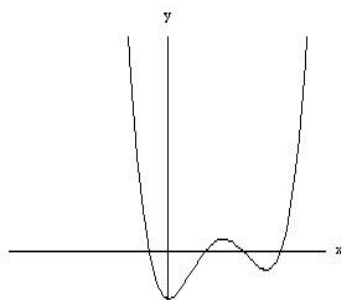


Figure 1

Following that they were a number of graphs of 4th degree polynomials and they were asked to use experimentation with the graphic calculator in order to obtain an equation yielding a graph which has similar properties as the ones indicated in the given graphs. They were also asked to describe the process by means of which they were able to obtain their answer, including those that did not yield the needed graph. So, here it was important for the students to realize that experimentation is valid and that it was important for them to reflect on their thought processes.

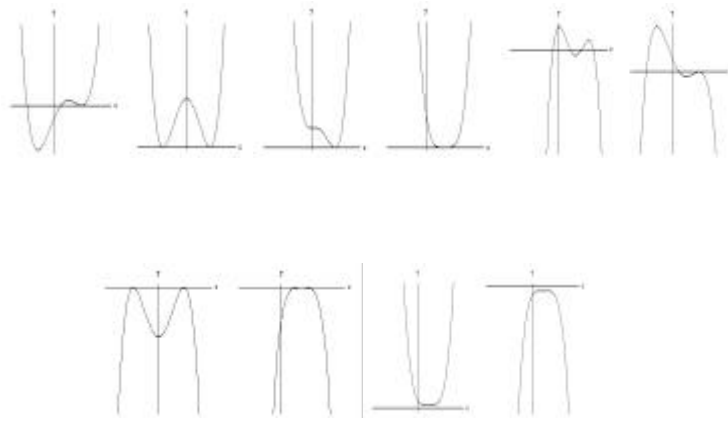
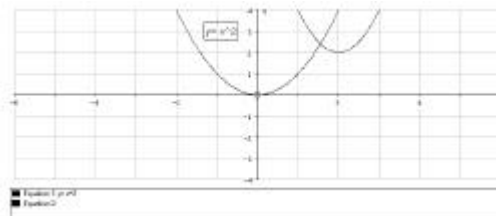


Figure 2

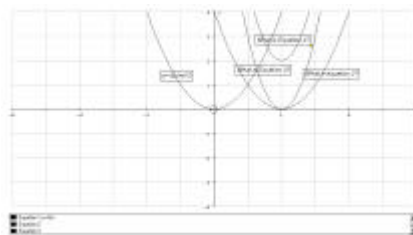
At first the equation of the given polynomial was not given in the factored form, and the students had difficulty in trying to figure out how to go about experimenting with graphing different equations. What also made it more difficult for them was to manage the window defining the part of the graph needed. When the hint was given, that the equation had four distinct roots and could be written in its factored form, they were then able to determine equations whose graphs were given. They chose a scale on the x-axis, found the zeros of the polynomial and thus were able to define the polynomial. They had to discover that if the tangent line was horizontal, then it meant that the root was a multiple root. Finally, where the required graph did not intersect the x-axis, they were able to take a similar graph, and take its defining equation and add an increment to shift it up or down as was needed.

2- Experiment 2- Transformations

This experiment involves the study of how transformations affect the graph of a function, and “Autograph” was used for that purpose. In class, we studied the effects of linear transformations on functions, and several handouts and homework exercises were given for that purpose. The students were given a function and its graph as well as the graphs of other functions obtained by applying a transformation on that function. The objective of the exercise was to determine the transformations and hence the equations defining these graphs. Initially, the graph of $y=x^2$ and that of equation 4 were given in class (figure 3 (i)) as an exercise, but students were unable to find the answer, so the intermediary graphs (figure 3 (ii)) were given to help the students how to determine the equation of the graph.



(i)



(ii)

Figure 3

This exercise was instructive in that it showed the students that in case one cannot find the answer from what is given, one can introduce intermediary elements that will help bridging the gap from the given to reach the required result.

3. Experiment 3- The Derivative

The purpose of this experiment was to help the students understand the concept of the derivative as the limit of the difference quotient of the function as well as the tangent line to a function at a given point. They were asked to use “Autograph” in order to plot the graph of $y = x^2$ and to consider what happens when $x=1$. They were asked to zoom in at the point (1,1), and to

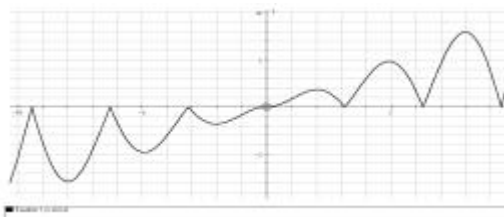


Figure 4

make their observations. The idea that the function is differentiable at a point if it is locally straight is an important idea (see Tall [6]) that is much easier for students to grasp than the concept of the limit. In fact they can see that if they zoom at any point, the graph will be locally straight. However, when they are asked to plot the graph of the function $y = x |\sin x|$, they will be able to see that at the point (0,0), the graph is locally straight, whereas at $(\pi, 0)$, the function is not locally straight and hence not differentiable at that point.

4. Experiment 4- The Derivative as a limit

The next experiment was designed to help the students learn the concept of the derivative at a point as the limit of the difference quotient. Here they were asked to draw the graph of a function for example $y = x^2$, and select two points (1,1) and another point close by, and to draw the gradient by selecting the two points, then select to the gradient as is shown in figure 5

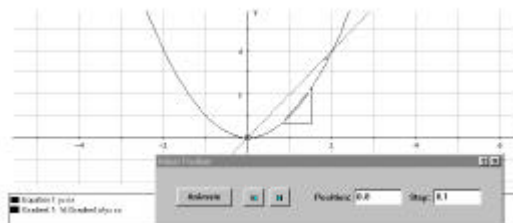


Figure 5

The picture can be animated and at the bottom of the graph is shown the value of x , y , the gradient, the change in x and y as well as the equation of the secant line. Students were asked

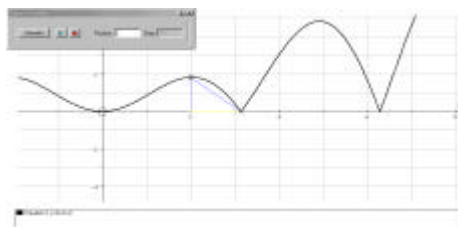


Figure 6

to make a table of these values and to write their observations. This exercise is repeated but with another graph namely that of $y= x |\sin x|$, and are asked to repeat this points at which the function is differentiable and points where the function is not differentiable.

4. Experiment 5- A family of functions

In this project they were supposed to study the family of functions $x^2 + a x$, the derivative function as well as the tangent line at $x=0$. “Autograph” allows a user to enter a family of functions and initially the parameter is set to 1. The project is provided in the appendix, and students are expected to see how the graph changes as a varies. They can either do manual animation or an automatic animation or have simultaneously the graphs drawn at the same time. They were asked to draw the graph of the tangent line at $(0,0)$ a common point to the family of graphs. Here, they had to figure out what the equation of the tangent line is and enter its equation. In another part they had to draw the graph of the derivative of the function and to draw the family. Following is the output:

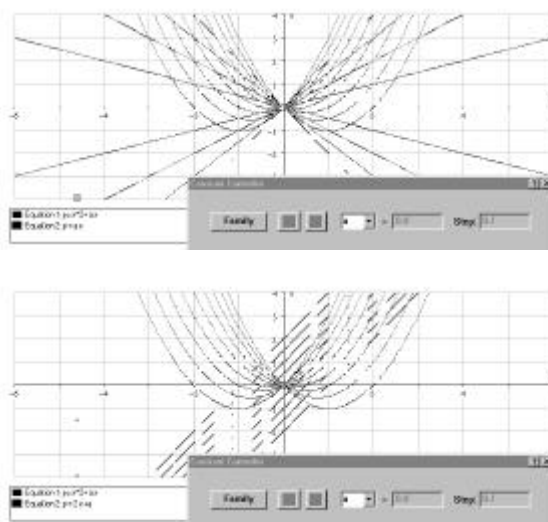


Figure 7

Further elaborations on this exercise are indicated in the appendix. It is clear that students who did these exercises developed a strong geometrical meaning to the concept of the derivative.

Conclusion

From this experience, it is clear to me that the students got engaged in their projects done with the TI 89/92 calculators and with “Autograph” and gained a better and deeper understanding of concepts studied in the course. This is contrary to previous experiences I had in introducing technology in other math courses such as Calculus III. I believe that the reasons are: first that the entire course emphasized the multiple representations. This was done during class time, homework assignments as well as questions on tests. The algebraic part of Calculus, namely obtaining the formulae for the derivatives was only done in the last three weeks of the semester. The second reason was that the calculators were often used in class and many of them also used them at home. So they were familiar with them. On the other hand, “Autograph” was easy to learn and had transparent syntax. I believe that this experiment was successful but it required a lot of work and also determination as the students were unhappy with the approach at the beginning and wanted to do mathematics the way they were used to in high school.

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**Appendix
Sample
Calculus I
Using Autograph
Plotting a family of functions**

This handout explains how to use the *Autograph* software to plot a family of functions. This project is similar to the one demonstrated in class. This project will be done in groups of 2 using the “Autograph” software, which is available in the Computer Center.

Consider the family of quadratics whose equation is given by $y = x^2 + ax$, where a is a parameter. Define a function f given by $f(x) = x^2 + ax$. We would like to study their behaviour as a varies by using Autograph software.

Questions:

Q1 What do you notice about the graph? How many roots are there? What is the axis of symmetry of the graph?

Q2 Now zoom in around the point (0,0)! Continue zooming around that point? What do you notice about the graph around the point (0,0)? Can you read the slope of the tangent line around that point? What is its equation?

Step 3- We will now show what happens to the graph as the value of a is varied. Go to the **Constant controller** button, which is on the item before last on the second tool bar. Open that and you will see that you can vary the value of a . Choose the step to be 0.5. Now go with the forward arrow to allow a to take values 1.5, 2, 2.5, 3, 3.5. Now go backwards to allow a to decrease until $a = -3$

Q3 Now describe what you have observed?

Step 5- Now go back to the **Constant Controller** dialog and choose **Family** and then **Family Plot**. Now the graphs will be plotted simultaneously.

Q4 Can you identify which function belongs to which curve? You may want to print the graphs! You can do that by going to file and then press the Print button

Step 5- Now delete all equations by going to the equation button and select the Delete all equations, so that we can start with a fresh page. Now we enter equation1 to be $f(x)$ and we want to enter another equation namely that of its derivative. Enter for equation the derivative of $x^2 + ax$. Now we will go over step 3, 4 and 5 to see how the function and its derivative at the same time.

Q6 Can you find where the derivative is equal to zero and hence where the function has a minimum value. Can you read the minimum value. Make a table of the values of a , the function $f(x)$, the value of x where the derivative is 0 and the minimum values of the function.

Step 5- Now we want to draw the tangent line at a given point we start with (0,0). Now go to the **equation** button and select the **Delete all** button to get a fresh start. Enter equation1 to be $y = f(x)$ and for equation2, the equation of the tangent line at (0,0). What is the equation of the tangent line? The derivative at $x=0$ is a , therefore the equation of the tangent line $y = ax$. Repeat steps3, 4 and 5 and describe what is happening with the tangent line at the point (0,0)?

What if we want the tangent line when $x = 1$? What is the value of y ? What is the derivative at $x=1$? What is the equation of the tangent line at $x=1$?