USE OF THE COMPUTER IN MATHEMATIC TEACHING FOR ENGINEERS: A POWERFUL CALCULATOR?

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ABSTRACT

Since a few years ago, the use of the computer as a tool to support teaching has been extended in University, especially in practical subjects. However, we should think about the following question: are computers under-used in mathematical teaching?

Nowadays, computers are being used in university teaching as great powerful calculators, but not as tools which help to carry out a substantial change in Mathematics teaching. Their use as a tool which helps to encourage the mathematical creativity of the students has not been extended yet. In most cases they are used as calculators, since they are used as a tool for calculation with numbers, although they are also used for algebraic manipulations, representation of curves, etc. All these uses are good to complement or simplify the traditional method of teaching, but they do not constitute an important improvement within Mathematics teaching.

This is partly due to the fact that we −the teachers− have been educated within an education system in which we have never received any training on this matter. Therefore we have not taken on the computer culture necessary to be able to prepare activities in order that our students receive the adequate mathematical backgrounds for their professional future that is ahead of them.

In short, the challenge we must face up in the future is overcoming this situation in order to use computers as tools for increasing mathematical creativity. As this is not an easy task, it is advisable that, at least, exercises with computer should be included in every subject related to Mathematics, and especially in courses for undergraduate students in Engineering.

In this paper, we present the experience carried out in courses for undergraduate students in Technical Telecommunication Engineering, using some kind of innovative exercises with computer. More concretely, we focus on the exercises of integration in several variables developed within the subject *Vectorial Analysis and Differential Equations*. We will end with the obtained conclusions and with the corresponding references.

1. Introduction

Since a few years ago, the use of the computer as a tool to support teaching has been extended in the University, especially in practical subjects. However, we should think about the following question: are computers under-used in mathematical teaching?

The answer is clearly affirmative. In most cases they are used as calculators, since they are used as tools for calculation with numbers, although they are also used for algebraic manipulations, representation of curves, etc.

All these uses are good to complement or simplify the traditional method of teaching, but they do not constitute an important improvement within Mathematics teaching.

This is partly due to the fact that we –the teachers– have been educated within an education system in which we have never received any training on this matter. Therefore we have not taken on the computer culture necessary to be able to prepare activities in order that our students receive the adequate mathematical backgrounds for their professional future that is ahead of them.

Nevertheless, an important improvement has been achieved: up to a few years ago, pupils of secondary education were taught, for instance, how to use log tables. Now, this is not explained to them anymore; instead, they learn how to operate with a computer or a calculator. Without any doubt, the present procedure is faster, but we must not make the mistake of thinking that we have improved the way of explaining the meaning and use of logarithms. The improvement we referred to above lies in the fact that this use is already considered as normal and it is not seen as an extraordinary thing or a new experience. That is, mathematicians have contributed with their work to the creation of tools, which are being used by non-mathematician professionals, up to the point of being fundamental in their work.

In short, the challenge we must face up in the future is overcoming this situation in order to use computers as tools for increasing mathematical creativity.

As this is not an easy task, it is advisable that, at least, exercises with computer should be included in every subject related to Mathematics, and especially in courses for undergraduate students in Engineering. Thus, it will be possible that future professionals be certainly capable of elaborating programs, which change substantially the way of teaching Mathematics.

The fact that the student could carry out this kind of exercises with any mathematical software will benefit him not only for the Mathematics subject in question, but he will also be able to use it in other subjects which need to make calculations of a certain complexity. Moreover, he will be ready to face up the resolution of problems that may occur in his professional future.

In this paper, we present the experience carried out in courses for undergraduate students in Technical Telecommunication Engineering, using some kind of innovative exercises with computer. More concretely, we focus on the exercises of integration in several variables developed within the subject *Vectorial Analysis and Differential Equations*.

2. Software and work setting

The choice of the program to be used is one of the most important matters of the entire process. Among the great amount of mathematical software now available on the market, we have chosen the program DERIVE® for several reasons:

1. First of all, this software is, from our point of view, easier to use than other mathematical programs which are “more powerful” as it operates with a very simple syntax.

2. Due to what we have set out in the previous point, the student is capable of starting to solve problems by using the program in a short period of time, since basic functions and operations are available in several menus.
3. It needs few requirements, with regard either to memory and physical space, when it comes to installing it.

In the following we will use the term *practical* for those exercises developed in a computer laboratory using DERIVE®.

The practicals are performed in a laboratory fitted with 30 units, with a maximum of two students per computer, and they are carried out in every Mathematics subject of the degree course.

The distribution of the practicals is as follows:

1. A first practical, which is two hours and a half long, to provide the student with the basic notions about how to use the program. This practical is carried out during the first weeks of the first four-month period, and it is aimed at students of the first year.
2. A specific practical for each subject where typical problems of such subjects are cleared out. This practical, which is two hours long, is carried out during the last days of the corresponding academic year in order to cover as much syllabus as possible. Another practical with particular contents can be fixed in the middle of the course, if the teacher thinks it would be convenient. Thus, for instance, it is usual, in the subject *Vectorial Analysis and Differential Equations*, to fix a practical on vectorial analysis and another one on differential equations, whenever the availability of time permits it.

Each specific practical consists of three different parts:

1. In the first part, theoretical-practical aspects which are to be developed in the practical are pointed out and DERIVE® own functions or macros to be created to solve eventual future problems are indicated.
2. The second part consists of examples of application of the concepts referred above, which will be solved during the course of the practical. Within these examples, and depending on the subject, some of the macros needed for the resolution of problems are elaborated.
3. In the third part, the students can solve a list of proposed problems in order that they reinforce the knowledge acquired during the practical.

The two hours of each specific practical are distributed as follows: one hour and a half for the first two parts and half an hour for the third part.

These practicals are carried out in a guided way, that is, by means of the teacher’s explanations, so that, in order to obtain a better assimilation of the introduced contents, the teacher can make the appropriate comments. It is important to point out that the development of the practicals is not reduced to the mere execution of the application examples, but that each example is useful to remind the student the theoretical-practical aspects seen in the conventional lectures. Thus, these practicals serve also as a review of the subject.

3. Innovative aspect of the practicals

All that has been commented before would fit in the development of classical practicals for Mathematics subjects. We now go on by presenting our contribution, which consists of the elaboration of innovative practicals insofar as the student participates actively in their creation. We emphasize that, in these practicals, apart from solving typical problems of the subject in question, the students elaborate macros in order to solve such problems. This elaboration of macros constitutes the principal innovative aspect of the practicals and requires the student to have an exhaustive knowledge of the subject.

Thus, for instance, for the elaboration of a macro which proves if a differential is an exact one, the student will need to know which one is the condition for this differential to be exact. Whereas for the elaboration of macros to calculate triple integrals he will have to take into account the following elements: the function to be integrated, the system of coordinates and the three variables of integration with their corresponding limits of integration. Besides, as the order of integration is important, he will have to take it into account when it comes to elaborating such macros. Obviously, the fact that the student himself is the one who elaborates the macros has a very positive influence when it comes to applying the macros in order to solve concrete examples.

So, with this kind of practicals the student does not just solve problems but he creates the macros to solve them as well. Our aim is that the use of computers will not further be reduced only to its most classical and usual application (that is, making calculations as if it were a powerful calculator), but that the computer is also used as a tool that encourages mathematical creativity.

4. Development of the practical about vectorial analysis

By way of example, now we develop the practical carried out as part of the subject *Vectorial Analysis and Differential Equations* about vectorial analysis. Note here that the original language has been conserved in the names of the macros.

Practical with DERIVE
Vectorial Analysis and Differential Equations
Technical Telecommunication Engineering

First of all, for the correct developing of the practical, it is necessary to load the file ANALVEC.MTH (use the option **Load – Utility file**). This file contents the definition of some commands to solve the exercises.

Gamma and Beta functions. Scalar and vector fields.

- *Gamma function*
  - Syntax: GAMMA(value)
  - Example: gamma(7/2) to calculate $\Gamma\left(\frac{7}{2}\right)$
- *Beta function*
  - Syntax: BETA(value1,value2)
  - Example: beta(3/2,5) to calculate $\beta\left(\frac{3}{2}, 5\right)$
- *Gradient*
  - Syntax: GRADIENTE(scalar field)
  - Example: gradiente(x^2+y^2+z^2) to calculate the gradient of the scalar field $x^2 + y^2 + z^2$
- *Divergence*
  - Syntax: DIVERGENCIA(comp1,comp2,comp3)
  - Example: divergencia(x^3y,2xzy,z^2) to calculate the divergence of the vector field $(x^3y, 2xzy, z^2)$
- *Curl*
  - Syntax: ROTACIONAL(comp1,comp2,comp3)

- Example: rotacional(x^3y,2xzy,z^2) to calculate the curl of the vector field $(x^3y,2xzy,z^2)$

- *Laplacian*
  - Syntax: LAPLACIANO(scalar field)
  - Example: laplaciano(x^2+y^2+z^2) to calculate the laplacian of the scalar field $x^2+y^2+z^2$

Line integrals

- *Exact differential in R^2*
  - Syntax: DIFERENCIALEXACTA2(comp1,comp2)
  - Example: diferencialexacta2(y^2,2xy) to check if $y^2\,dx + 2xy\,dy$ is an exact differential
- *Exact differential in R^3*
  - Syntax: DIFERENCIALEXACTA3(comp1,comp2,comp3)
  - Example: diferencialexacta3(x+z,-(y+z),x-y) to check if $(x+z)\,dx - (y+z)\,dy + (x-y)\,dz$ is an exact differential
- *Potential function in R^2*
  - Syntax: POTENCIAL2(comp1,comp2)
  - Example: potencial2(y^2,2xy) to calculate the potential function of $y^2\,dx+2xy\,dy$. If the differential is not an exact one, the macro answers "this is not an exact differential"
- *Potential function in R^3*
  - Syntax: POTENCIAL3(comp1,comp2,comp3)
  - Example: potencial3(x+z,-(y+z),x-y) to calculate the potential function of $(x+z)\,dx - (y+z)\,dy + (x-y)\,dz$. If the differential is not an exact one, the macro answers "this is not an exact differential"
- *Line integral of non-exact differentials in R^2*
  - Syntax: LINEAPARAMETRICA2(comp1,comp2,cur1,cur2,a,b)
  - Example: lineaparametrica2(xy^4,x^2y^3,t^3,t,0,1) to calculate the line integral of (xy^4,x^2y^3) along the curve $y^3 = x$, from $(0,0)$ to $(1,1)$
- *Line integral of non-exact differentials in R^3*
  - Syntax: LINEAPARAMETRICA3(comp1,comp2,comp3,cur1,cur2,cur3,a,b)
  - Example: lineaparametrica3(3x^2+6y,-14yz,20xz^2,t,sqrt(t),t^(1/3),0,1) to calculate the line integral of $(3x^2+6y,-14yz,20xz^2)$ along the curve $x = t$, $y^2 = t$, $z^3 = t$, from $(0,0,0)$ to $(1,1,1)$

Double and triple integrals

- *Double integration in cartesian coordinates*
  - Syntax: DOBLE(function,var1,lim1,lim2,var2,lim3,lim4)
  - Example: doble(xy,y,0,x+1,x,0,2) to integrate the function $f(x,y) = xy$ in the region bounded by $x = 2$, $y = x+1$, $y = 0$ and $x = 0$
- *Double integration in polar coordinates*
  - Syntax: DOBLEPOLAR(function,r,r1,r2,theta,theta1,theta2)
  - Example: doblepolar(x^2y^2/(x^2+y^2),r,0,1,theta,0,pi) to integrate the function $f(x,y) = \dfrac{x^2y^2}{x^2+y^2}$ in the region bounded by $x^2+y^2 = 1$ with $y \geq 0$
- *Triple integration in cartesian coordinates*
  - Syntax: TRIPLE(function,var1,lim1,lim2,var2,lim3,lim4,var3,lim5,lim6)
  - Example: triple(x+yz,z,0,5-x-y,y,0,5-x,x,0,5) to integrate the function $f(x,y,z) = x+yz$ in the solid bounded by $x+y+z = 5$, $x = 0$, $y = 0$ and $z = 0$

- *Triple integration in cylindrical coordinates*
  - Syntax: TRIPLECILINDRICA(function,z,z1,z2,r,r1,r2,theta,theta1,theta2)
  - Example: triplecilindrica(sqrt(x^2+y^2),z,r,1,r,0,1,theta,0,2pi) to integrate the function $f(x,y,z) = \sqrt{x^2+y^2}$ in the solid bounded by $z = 0$, $z = 1$ and $z^2 \geq x^2+y^2$
- *Triple integration in spherical coordinates*
  - Syntax: TRIPLEESFERICA(function,r,r1,r2,theta,theta1,theta2,alpha,alpha1,alpha2)
  - Example: tripleesferica(x+y+z,r,0,2,theta,0,2pi,alpha,0,pi/2) to integrate the function $f(x,y,z) = x+y+z$ in the solid bounded by $x^2 + y^2 + z^2 = 4$ with $z \geq 0$

Surface integrals. Gauss' theorem

- *Surface area in polar coordinates*
  - Syntax: AREASUPERFICIERXYPOLAR(z surface,r,r1,r2,theta,theta1,theta2)
  - Example: areasuperficierxypolar(sqrt(x^2+y^2),r,0,1,theta,0,2pi) to calculate the area of the part of the surface $z^2 = x^2 + y^2$ inside of $z = 2 - x^2 - y^2$
- *Unit normal vector to an explicit surface*
  - Syntax: COSENO1(explicit surface)
  - Example: coseno1(x^2+y^2) to find an unit normal vector to $z = x^2 + y^2$
- *Unit normal vector to an implicit surface*
  - Syntax: COSENO2(implicit surface)
  - Example: coseno2(x^2+y^2+z^2-4) to find an unit normal vector to $x^2 + y^2 + z^2 = 4$
- *Flux of a vector field. Double integral in polar coordinates*
  - Syntax: FLUJORXYPOLAR(comp1,comp2,comp3,z surface,r,r1,r2,theta,theta1,theta2)
  - Example: flujorxypolar(x,2y,x+z,x^2+y^2,r,0,4,theta,0,2pi) to calculate the flux of $(x,2y,x+z)$ over the part of the surface of the paraboloid $z = x^2 + y^2$ for which $0 \leq z \leq 16$
- *Gauss' theorem. Triple integral in cylindrical coordinates*
  - Syntax: FLUJOGAUSSCILINDRICA(com1,com2,com3,z,z1,z2,r,r1,r2,theta,theta1,theta2)
  - Example: flujogausscilindrica(x,2y,3z,z,r,2,r,0,2,theta,0,2pi) to calculate the flux of $(x,2y,3z)$ over the closed surface bounded by the cone $z^2 = x^2 + y^2$ and the planes $z = 0,\ z = 2$

Examples

1. Build the macro BETA.
2. Evaluate $\Gamma\left(\frac{9}{2}\right)$ and $\beta\left(\frac{7}{2}, 9\right)$.
3. Build the macros GRADIENTE and LAPLACIANO.
4. Given the scalar field $f = 2x^2y-xz^3$, calculate its gradient and laplacian.
5. Given the vector field $F = (xz,-y^2,2x^2y)$, calculate its divergence and curl.
6. Build the macros DIFERENCIALEXACTA2 and POTENCIAL2.
7. Find, when possible, the potential function of:
   a. $(xy^2 + x + 1)\,\mathrm{d}x + (x^2y - 2)\,\mathrm{d}y$
   b. $(yz+y+z)\,\mathrm{d}x + (xz+x+z)\,\mathrm{d}y + (xy+x+y+2z)\,\mathrm{d}z$
8. Calculate the line integral of (xy^2+x+1, x^2y-2) along any path from $(1,2)$ to $(-2,5)$.
9. Calculate the line integral of $(yz+y+z,xz+x+z,xy+x+y+2z)$ along the segment that joins $(1,2,3)$ with $(-2,7,3)$.
10. Build the macro LINEAPARAMETRICA2.
11. Calculate the line integral of $(xy,2x)$ along the ellipse $x^2 + \frac{y^2}{4} = 1$.
12. Build the macro DOBLE.

13. Integrate the function $f(x,y) = x^2 + y^5$ within the rectangle with vertices $(0,0)$, $(2,0)$, $(2,1)$ and $(0,1)$.
14. Calculate the area of the circumference $x^2 + y^2 = 4$.
15. Build the macros TRIPLE and TRIPLECILINDRICA.
16. Integrate the function $f(x,y,z) = x + yz$ in the solid bounded by $x+y+z = 5$, $x = 0$, $y = 0$ and $z = 0$.
17. Integrate the function $\mathrm{f(x,y,z)} = \sqrt{\mathrm{x}^2+\mathrm{y}^2}$ in the solid bounded by $z = 0$, $z = 1$ and $z^2 \geq x^2+y^2$.
18. Calculate the volume of the sphere $x^2 + y^2 + z^2 = a^2$.
19. Build the macro AREASUPERFICIERXYPOLAR.
20. Calculate the area of the portion of the sphere $x^2 + y^2 + z^2 = 16$ outside of the paraboloid $x^2 + y^2 + z = 16$.
21. Build the macro COSENO1.
22. Calculate an unit normal vector of the following surfaces:
    a. $z = x^2 + y^2$.
    b. $x^2 + y^2 + z^2 = 4$.
23. Build the macro FLUJORXYPOLAR.
24. Calculate, using two different methods, the flux of the vector field (x,y,z) over the closed surface bounded by $x^2+y^2 = 4z$ and $z = 4$.

Exercises

1. Calculate $\Gamma\left(\frac{7}{2}\right)$, $\Gamma(6)$, $\beta(5,6)$ and $\beta\left(\frac{5}{2},\frac{7}{2}\right)$.
2. Given the scalar field $f = 2x^2y^3 - xy^2z^5$, calculate its gradient and laplacian.
3. Build the macros DIVERGENCIA and ROTACIONAL.
4. Given the vector field $F = (xyz,-y^2z,2x^2y+z)$, calculate its divergence and curl.
5. Build the macros DIFERENCIALEXACTA3 and POTENCIAL3.
6. Find, when possible, the potential function of:
    a. $(x+y+z)\,\mathrm{d}x - (y+z)\,\mathrm{d}y + (x-y)\,\mathrm{d}z$
    b. $ye^{xy+z}\,\mathrm{d}x + xe^{xy+z}\,\mathrm{d}y + e^{xy+z}\,\mathrm{d}z$
    c. $(ye^{xy}+x)\,\mathrm{d}x + (xe^{xy}+3y)\,\mathrm{d}y$
    d. $(2xy+y^2)\,\mathrm{d}x + (x^2+y^3)\,\mathrm{d}y$
7. Calculate the line integral of $(ye^{xy+z},xe^{xy+z},e^{xy+z})$ along any path from $(1,2,-3)$ to $(-2,5,11)$.
8. Calculate the line integral $(ye^{xy}+x,xe^{xy}+3y)$ along the segment that joins $(1,2)$ with $(-2,9)$.
9. Build the macro LINEAPARAMETRICA3.
10. Let $A = (3x^2+6y,-14yz,20xz^2)$. Evaluate $\int_C A\,\mathrm{d}r$ where C is the path from $(0,0,0)$ to $(1,1,1)$ given by:
    a. The curve $x = t$, $y^2 = t$, $z^3 = t$.
    b. The segment that joins both points.
11. Calculate the line integral of $(xy,x+3)$ along the ellipse $x^2 + \frac{y^2}{4} = 1$.
12. Integrate the function $f(x,y) = x^2+y^5$ in the region bounded by $y = x^2$ and $y = 2- x^2$.
13. Build the macro DOBLEPOLAR.
14. Calculate the area of the region bounded by $x^2+y^2 = 2ax$, $y = x$ and $y = 0$.

15. Integrate the function $f(x,y,z) = x+yz$ within the solid bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 1$, $z = 1$ and $z = 3$.
16. Build the macro TRIPLEESFERICA.
17. Calculate the volume of the solid bounded below by the cone $z = +\sqrt{x^2+y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$ in both cylindrical and spherical coordinates.
18. Calculate the volume of the solid bounded by the cylinder $x^2 + y^2 -bx = 0$ and the cone $x^2 - z^2 = -y^2$.
19. Calculate the area of the part of sphere $z^2 = x^2 + y^2$ inside the paraboloid $z = 2 - x^2 - y^2$ with $z \geq 0$.
20. Build the macro COSENO2.
21. Calculate an unit normal vector of the following surfaces:
    a. $z = 2 - x^2 - y^2$.
    b. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{z^2}{c^2}$.
22. Build the macro FLUJOGAUSSCILINDRICA.
23. Calculate, using two different methods, the flux of the vector field $(2z,x,y^2)$ over the closed surface bounded by $z = 4 - x^2 - y^2$ and $z = 0$.

5. Conclusions

After having carried out this kind of practicals for the last four years, we have been able to verify advantageous results either for students and teachers. Among other aspects, we can emphasize the following:

1. The student is provided with a powerful tool for the resolution of problems that, besides, can be used to verify the results obtained in the exercises he does.
2. The student feels more motivated towards the subject because of the chance of working with the computer to solve problems that occur to him.
3. This motivation leads to a better preparation of the subject by the student, what, at the same time, entails that classes can be given more easily since the student is more prepared and receptive.
4. Before carrying out the specific practicals of each subject, the student is reminded that in such practicals some examples and exercises concerning the whole content of the subject will be solved, so that he must get conveniently prepared prior to them. This, together with the fact that during the practical itself the most important theoretical-practical concepts of the subject are reminded, makes the student be better prepared when it comes to facing up the final exam.
5. As the attendance at the practicals is voluntary and students must register previously on a list, the students who attend these lectures are those who feel really motivated towards the subject and towards the carrying out of these practicals, what has a very positive influence on their progress.

Finally we would like to insist on the innovative aspect of this kind of practicals that gives rise to a double consideration. On the one hand, these practicals require something more than the mere fact of using the computer as a tool for calculation, because, if the student wants to solve a problem with the macros that have been created, he has to set it out previously. On the other hand, the elaboration of the macros by the student requires him to have some knowledge of programming and some mathematical reasoning. Therefore, with this kind of practicals, we show that computers are more than powerful calculators.

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