FRAMEWORK FOR INSTRUCTION AND ASSESSMENT ON ELEMENTARY INFERENTIAL STATISTICS THINKING[1]

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ABSTRACT

The main objective in this paper is to describe a framework to characterize and assess the learning of elementary statistical inference. The key constructs of the framework are: populations and samples and their relationships; inferential process; sample sizes; sampling types and biases.

To refine and validate this scheme we have taken data from a sample of 49 secondary students sample using a questionnaire with 12 items in three different contexts: concrete, narrative and numeric. Theoretical analysis on the results obtained in this first research phase has permitted us to establish the key constructs described below and determine levels in them. Moreover this has allowed us to determine the students' conceptions about the inference process and their perceptions about sampling possible biases and their sources.

The framework is a theoretical contribution to the knowledge of the inferential statistical thinking domain and for planning teaching in the area.

Keywords: inferential statistics, theoretical framework, secondary level.

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1. Introduction

One of the characteristic features of the current society is the enormous technological development that has been applied for the social and economic improvement of the citizens. In this technological society information and communication play key roles and education should provide the citizens with the necessary elements to develop within. The access to information, the use of data, data analysis and the taking of informed decisions in uncertain situations, the understanding and the capacity of criticism of the information provided by the media, etc., form part of the new formative necessities of citizens in the current world. As an answer to these new social necessities the educational systems have introduced reforms in the curricula that affect statistical education in many countries and at all teaching levels, for example, MEC(1990), Junta de Andalucía (1992; 1994; 1997), NCTM(2000). One of the novelties of the reforms in Spain has been the introduction of statistical inference in the curricula for the compulsory teaching level (ESO, 12-16 years old) and the Bachillerato (16-18 years old). Parallel to this, the introduction of more and new statistical contents, and at more elementary teaching levels each time, outlines a bigger necessity of further research on the learning of these contents and their throughout the student's schooling years. Although we already have some results of research carried out in this respect in the field of data analysis and of probability, this field can be considered mainly, as emergent and developing (Shaughnessy, 1992; Mokros and Russell, 1995; Gal and Garfield, 1997; Batanero and cols., 1994; Jones and cols., 2000). In the field of statistical inference the research works carried out are even more scarce (Watson, 2000; Jacobs, 1996; Moreno and Vallecillos, 2001; Vallecillos, 1998; in print). Jones and cols. (2000) propose a framework to characterize the children's statistical thinking based on the cognitive development model described by Biggs and Collis (1991). In our work we have tried to develop a similar framework for the case of statistical inference thinking, so finally we can have an applicable general framework for elementary, descriptive and inferential statistics. To do that, on a review of previous research works and based on our own researching experience on the topic, we have built an initial theoretical framework to evaluate the learning of statistical inference in secondary education students. Then, we have elaborated a questionnaire that 49 students of this level have completed and we have analyzed the results obtained. Finally, by incorporating the obtained information, we have refined the initial framework and we have elaborated the conclusions of this phase of the study.

This theoretical framework of analysis developed to evaluate the learning of the basic statistical inference has been validated with secondary level students but it can be used to plan teaching of the topic and to evaluate the learning of the students in introductory courses at the university level too.

2. Theoretical Framework

Teachers need a good knowledge about how students understand statistical concepts and how they engage in solving problems. Students exhibit statistical thinking over the different school levels and develop in time. So the framework is situated in a general cognitive development model (Bigg and Collis, 1982; 1991). These authors describe three levels of observed learning outcome:

1. Unistructural responses, those taking in to consideration only one aspect of the concept or task considered;

2. Multistructural responses, those in which several aspects of the concept or task are considered but not all, and

3. Relational responses, those in which all aspects are considered and integrated exhibiting an integrated understanding and a meaningful learning.

Situated in this general cognitive model (Bigg and Collis, 1982; 1991), Jones and cols. (2000) formulate a framework to characterize children's statistical thinking. They define four constructs, describing, organizing, representing and analyzing and interpreting data. Within each one of these constructs they establish four thinking levels representing a continuum from idiosyncratic to analytic reasoning. Results of the study, authors say, confirm that children's statistical thinking can be described according to the framework. Our initial framework for inferential statistical thinking is also situated in the general cognitive model (Bigg and Collis, 1982; 1991) and is like Jones and cols. (2000) framework with four construct and four thinking levels within each one. Nevertheless, we consider two related aspects for determining construct and levels in the framework: the statistical content and the result of the questionnaire filled in by students. In the initial framework we have determined the constructs and level in statistical content based; afterwards we have considered the students' responses to the questionnaire too in order to establish constructs and levels in them in the inferential statistical framework. We have established four constructs, population and samples and their relationships (PS), inferential process (IP), sample sizes (SS) and sampling types and biases (ST), and four thinking levels in each one.

3. Method

3.1. Aims of this research

The objectives of this paper are mainly three: a) to develop an initial framework to characterize and assess the learning of basic statistical inference; b) to elaborate a questionnaire to asses statistical inference learning at secondary level; c) to test framework in order to get the first objective and validate and refine it with the questionnaire results.

3.2. The constructs

We seek to describe and to fix, in the first place, the elements and key concepts of statistical inference for the basic training of the students at introductory teaching levels. To do that we will use the expression "construct" that is used in the field of Psychology to describe complex phenomenon such as the personality, motivation, etc., of difficult definition. For us each "construct" represents a category of concepts all of them under only one epigraph in which they can be described. We believe that the description of the samples, the populations of the ones that have been extracted and their relationships; the questions related with size, the selection methods and the possible sources of biases in the selection of samples are important conceptual nuclei that are in the basis of learning in statistical inference that can be described in these terms. All these elements have already been recognized previously as such by teachers, researchers or curricular documents. Our proposal includes a novelty: we have included as a differentiated construct the one that we have called "Inferential process" because we find that it deserves a special mention. Indeed, the students sometimes do not distinguish well between population and sample and they are not conscious, therefore, that the conclusions obtained in the study of a sample are not those that we need, and it is necessary to make them aware that a generalization under the conditions of the study is carried out and therefore subject to certain limitations and to the possibility of error. Other times the students do not admit the generalization possibility and they only believe in the carrying out of census and so it is necessary to make them reflect about the impossibility of these in certain situations, such as destructive tests or with unbroachable temporary or economic costs. We describe the key constructs below:

A) Populations and samples and their relationships

We try to understand the ideas of the students about the sample and population concepts as well as the relationship between them. These concepts are intuitively used in many environments of daily life, outside the school environment. Concepts such as the variability and sample representativeness have a great incidence in many aspects of social life. Kahneman and cols. (1982) have investigated thoroughly on these aspects and find that people reason using heuristics that lead them to erroneous conclusions most times. Among secondary level students the presence of thinking heuristic has also been detected (Rubin and cols., 1991; Moreno and Vallecillos, 2001). In another order of things, we are also interested in discovering if the scheme 'part-everything' used in the teaching of contents of numerical type such as the fractions and rational numbers, is also used in this context and how it is used.

B) Inferential process

We try to understand how the students conceive the process that allows them to describe the population on the basis of the information obtained from the observation of one of its samples. To do that we have determined the students' conceptions (Artigue, 1990) about the process, such as theoretical models built which supposedly guide the students' answers.

C) Sample sizes

In order to get a good learning relative to the sample concept it is necessary to keep two aspects that are essential in it in mind: the sensitization of the students about the importance of the sample size and the appreciation of the same when judgements are emitted or decisions are made based on samples. The works of Kahneman and their colleagues determined the "law of the small numbers" as a very widely believed among the population, even among people with statistical training. This belief is part of the representativeness heuristic leading people to believe that the samples, even the very small ones, always reproduce the population's characteristics from which they proceed, showing an insensitivity towards the size of the sample (Kahneman and cols., 1982).

D) Sampling types and biases

The sampling based on the randomization of the statistical units provides the representative samples of the populations under study. In this section we consider two aspects basic for the good teaching of the topic: the sensitization of the students about the importance of the randomization in the selection of the samples used as well as about the presence of biases in any other case and of the derived pernicious effects of the use of biased samples.

3.3. The inferential statistical thinking framework

In Table 1 we described it.

Table 1: The inferential statistical thinking framework

Construct	Level 1: Idiosyncratic	Level 2: Transitional	Level 3: Quantitative	Level 4: Analytical
Populations and samples and their relationships (PS)	Usual population concept Usual sample concept Neither identifies population nor sample Confuses population and sample	Statistical population concept Population of discrete type Identifies population or sample in concrete context	Statistical population concept Population of discrete/continuous type Identifies population and sample in certain contexts only	Statistical population concept Sample space concept Identifies and poses in relation to population and sample in all contexts
Inferential process (IP)	Subjective criteria Previous conception	Subjective criteria and/or numeric with errors Deterministic conception	Numeric criteria with informal expression Identity conception	Numeric criteria and formal expr ession Correct conception
Sample sizes (SS)	Sample size characteristic recognizing Sample size insensitivity	Sample size characteristic importance recognizing Recognizes sample size interest in some context	Sample size and estimation relationship Recognizes sample size and/or put in relationship with estimation in numeric contexts	Sample size and estimation relationship in all contexts Sample size sensitivity in all contexts and in relation with characteristic estimated
Sampling types and biases (ST)	Sampling concept Different sampling types possibility recognizing Type sampling insensitivity Biases insensitivity	Sampling methods Randomization Different sampling types recognizing Recognizes the biases possibility	Random sampling types Recognizes different random sampling types Recognizes the biases possibility	Sampling method and characteristics estimation Recognizes the more adequate sampling type Biases sensitivity

3.4. Participants

Participants are 49 secondary students from two Spanish high schools distributed in two different courses. 30 students from 3º de ESO (14-15 years old) without any previous statistics information and 19 from COU (17-18 years old). COU is the last course at secondary level and these students had some statistical knowledge.

3.5. Questionnaire

The questionnaire was made up of two different parts with 12 questions each one about elementary inference concepts. Items are presented in three contexts, concrete, narrative and numeric. We include two different items, one of Part I and one of Part II of the questionnaire for readers illustration. The complete version of the questionnaire may be obtained from authors on request.

Item I.1. We have a bag with 100 balls of the colors red and green. We want to study the number of balls of each color. To do that we take 25 balls from the bag and we observe that 14 of them are red and 11 are green. Write:

a) The set objects we are studying:

b) The sample observed:

Item II.3. The town council is starting a campaign for explain to the citizens what they may do when they need to get rid of old furniture. They want to know if the instructions have been clear and understandable. The population of the city is 300.000 people and so they decide to ask 2000 adult citizens about their opinion. They are asked in small and big quarters, some male and some female, some old and some young people, some who live in flats and who live family houses and so on. They think that they have a varied group of people. They are 73% of these people say that the given instructions are clear and the 27% say not.

¿What can you say to the town council about the percentage of adults in the whole city who think that the given instruction are clear?:

a) 50% because probably half of the people think the instructions are clear and the other half think they are not.

b) 73% because the adults asked gave a general idea about the results as if the whole population were asked.

c) I can't say anything because the result of the inquiry could have been anything.

d) I can't say anything because I can't ask all the adults in the city.

e) Were .... because....................................

3.6. Procedure

Third course ESO's students filled in Part I questionnaire in one 60 minute session and Part II in another 60 minute session. In some questions in Part I of the questionnaire the researcher intervened for concrete material handling required or to explain to students what they are being asked. Then they fill in the questionnaire individually. COU students use only a 60 minute session to individually fill in both parts of the questionnaire.

3.7. Results

A) Populations and samples and their relationships (PS): a lot of students have not identified the sample and population studied correctly, although there are notable differences in correspondent items success percentages in the different contexts. The higher age group (COU) have got better global results than the ESO group and in the numerical context. About two thirds of the ESO students can not identify either population or sample while in COU only a fifth of them cannot do so.

B) Inferential process (IP): we have grouped students' responses under three headings characterizing each one determined conceptions. They are summarised below:

C1) Correct conception: the inference process is a chance ruled process and can not permit the precise population characteristics determine on the basis of the information obtained from one of its samples.

C2) Identity conception: the inference process permits to us describe the population with characteristics identical to the one of its samples.

C3) Previous conception: the population has characteristics described by previous ideas and not for the ones observed in the extracted sample.

C4) Deterministic conception: the population can only be described by doing a census and not by studying samples extracted from it.

In this category we have found very great differences between contexts: not all conceptions appear in each context, e. g., in narrative context the previous conception do not appear and the deterministic conception only appear in the narrative contexts.

C) Sample sizes (SS): in the lower age group (ESO) about 50% of the students do not take in to consideration the sample size and in the COU group the success percentage is a little better but only a quarter of all the students relate the sample size and the population characteristic estimation.

D) Sampling types and biases: most of all the students recognizes the different sampling types and most of the higher age group students, the different kinds of random sampling too, e. g., simple versus stratified sampling.

4. The revised inferential statistical thinking framework

This actual inferential statistical thinking framework needs to be tested in an other field: the instructional field. The students who have participated in the research have been taken in their natural classes and without any special preparation to do that. In order to prove the inferential statistical thinking framework for instruction usefulness we are designing didactic resources to use in secondary classrooms and based on the four constructs described previously. We need to take data about teaching and learning in the classroom functioning of the framework. Selected students will be interviewed afterwards and the inferential statistical thinking framework will be profoundly and globally revised. That will be the second research phase.

5. Conclusions

In this paper we have presented an initial inferential statistical framework for instruction and assessing secondary student learning of the same. We have in synthesis described the four constructs and the four levels within each one that the scheme constitutes. We have tested it with secondary students from two different courses in Spain. With the results obtained from the questionnaire filled in by them we have revised and completed the inferential statistical framework that we have describe before. As a first general conclusion we have experimented several difficulties in two different areas mainly, of a theoretical and of a didactical nature. In the theoretical area, to determine the essential theoretical aspects, concepts or constructs that are basic and essential and so it is necessary to include them it in any general elementary curriculum for statistical education for all citizens in order to make peoples aware and be able to take informed decisions. In the didactic area, once the adequate curriculum content has been determined, how do the students get the bests results?. The inferential statistical framework in our actual personal contribution to these problems. This research is now completing its instructional slope, developing classroom resources for testing it and for a global revision of the inferential statistical framework.

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