ACTIVITY MATHEMATICS

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ABSTRACT

Mathematics is something that *people do*. In the ages before the recent rapid developments in technology, this activity called "doing mathematics" has been restricted to those who happened to be able to master a variety of artificial, mechanical, formal processes. The necessary conditions for one to be a candidate to practise mathematics have included for instance mastery of the mindless symbolic process of manipulation of formulae, and possession of the magician's box of techniques such as symbolic integration.

Now though, technology allows freedom for many more people to benefit from being able to "do mathematics", and for others to benefit from the results of that. Doing mathematics has always been much more than just being able to carry out manipulations on paper. It is now easier to perceive it and to present it to people at large as a broader activity which enables one to gain insight into the world, encompassing a rich combination of communication between reality both internal and external, words, pictures, and numbers, and a formalised language. Thus the idea of thinking logically and analytically in order to make human sense of the world can receive more emphasis than the repetitive practice of mechanical skills. In propagating this wider view, mathematics becomes more obviously a "people" activity.

We present in this paper some of our own recent experience of positively developing courses, for students of mathematics and others, to incorporate developing technology. Packages involved include for example Mathsoft Studyworks, TI-Interactive and Cabri Geometry, but the important issue is not precisely which packages we currently use, but how we have changed what we now perceive as "doing mathematics" now that rapidly changing technology is here to stay.

Keywords: TECHNOLOGY AND MATHEMATICS, CURRICULA INNOVATIONS,
INNOVATIVE TEACHING METHODS

1. Introduction

Before the recent rapid advances in technology, “doing mathematics” was restricted to those who could master a variety of artificial, mechanical, formal processes; what some have called mindless symbolic manipulation of formulae. The mathematician’s magic box of symbolic techniques, such as integration, has been an essential tool for a candidate actually to practise mathematics. Technology now allows freedom for many more people to benefit from being able to "do" mathematics. There has always been much more to doing mathematics than just being able to carry out the manipulations, but it can now be viewed more obviously as a broader activity allowing insight into the world, through its structures for communication between reality, words, pictures, and numbers, and a formalised language. Mathematics thus becomes more obviously a "people" activity.

Here we present some recent examples from our practice of incorporating a variety of technology and software into our teaching of what we now perceive as doing mathematics, now that rapidly changing technology is here to stay.

2. Going for a *SONG*

To many people, “mathematics” is practically synonymous with mental arithmetic and algebraic manipulation. To emphasise our point that these aspects form only a part of the mathematical way of way of looking at the world, we encourage our students to "go for a ***SONG***" (Challis and Gretton 1999 and 1997). We encourage them to approach a mathematical concept from a variety of directions, by combining ***S***ymbolic, ***O***ral, ***N***umeric and ***G***raphic approaches, and thus hope they will acquire a richer understanding. The pervasive presence of mathematical technology is the major factor in allowing, prompting, and perhaps even demanding this broader approach.

‘O’ is for Oral

Let’s consider the collection of symbolic manipulation, mathematicians’ tricks and technical jargon to be a language. This language is useful to us as mathematicians, in that it is precise and concise and often helps us to ‘do the sums’. But it isn’t widely spoken. Most problems are communicated in some other language, and even if we can find a solution to the problem, we usually need to communicate it in the same language as the problem was posed, to those who are not fluent in ‘our’ mathematical language.

A typical process of solving a problem is to specify it in some formal, possibly symbolic way (translate from English, say, to our mathematical language); use the tricks of our language to find a solution; and then justify our solution to the person who originally posed the problem (translate back from the mathematical language into the original language). A mathematician (and therefore a mathematics student) does not only need to be fluent in the mathematical language, but also be able to translate into the original language.

Consider the problem:

> *Think of a number. Multiply by 3. Add 8 more than the original number. Divide by 4. Subtract the original number. Does everyone get the same answer or does it depend on the number you started with?*

Using the TI Interactive™! package we can consider specific examples, say of 4 and 5 as our original number:

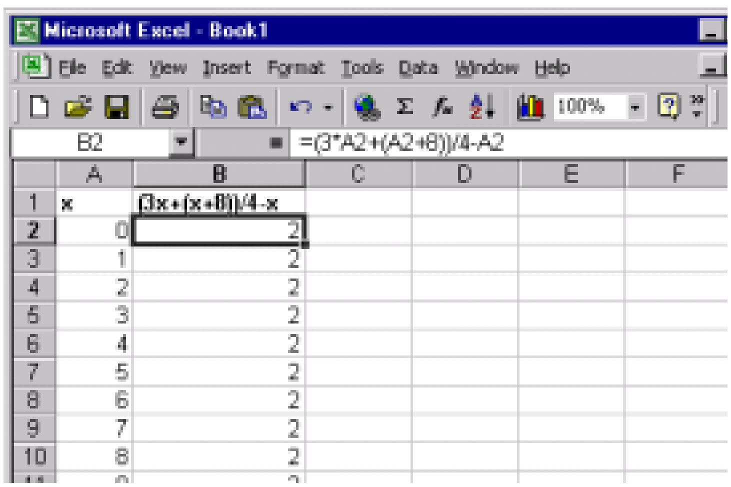
$$\frac{4\cdot 3+(4+8)}{4}-4 \qquad 2 \qquad\qquad \frac{5\cdot 3+(5+8)}{4}-5 \qquad 2$$

This suggests that we will get 2 whatever we start with. Most mathematicians would prefer a generalised approach, ie let our original number be *x*:

$$\frac{(x\cdot 3)+(x+8)}{4}-x \qquad 2$$

Thus we see the process always gives the same answer 2.

This solution is relatively easy for someone with some algebraic experience, but what if it had to be explained to the general user who had never come across algebra? Here we come across a significant problem in doing mathematics: explaining to someone who does not have our expertise. So what do we do? One possibility is to resort to convincing. Enumeration of many possibilities is one way of doing this, even though as mathematicians we know this is not equivalent to a proof (in fact it can be a good opportunity to make that point!) This can be made slightly less exhausting if we use a spreadsheet or a calculator:



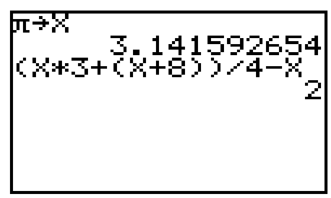


Figure 1

Since we start with words it is interesting to try to find a convincing argument which has *no* algebra in the solution. To try to convince your audience with just words is not easy, but it can be a useful ingredient of convincing a non-mathematician, and a thought-provoking activity for our students *and* us! Of course we may also note that *convincing* someone that something is the case is not the same as *explaining* to them precisely *why* it works - or even dare we say, proving it. It is something of a challenge to do this without symbols!

‘G’ is for graphic

This process of convincing audiences can perhaps be facilitated by pictures. Mathematicians are used to the notion of drawing graphs from some data or function, but graphics can come in many more formats than this. They could be a video of a process, an analogue output from a system, a picture in digital format, and so on. A student can collect, view or even extract their own data from 'pictures' thereby gaining ownership and highly interactive engagement with the problem.

For example, a student with a TI calculator, a CBL™ (Calculator-based Laboratory) and a light measuring device, let loose collecting information on light bulbs, computer screens, calculator, or mobile phones, can provide a plethora of real data for the construction or validation of mathematical models. Figure 2 shows such data from a computer monitor and a room light.

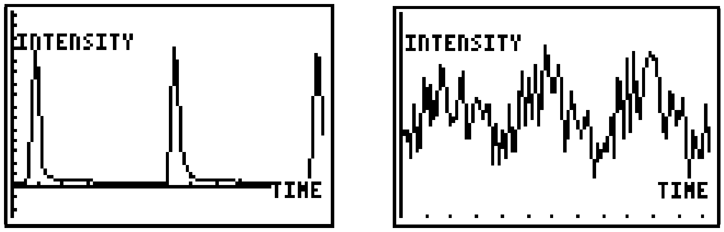


Figure 2 Light intensity data from a computer monitor (left) and room light (right)

Understanding the significance of the pictures can certainly be seen as a mathematical process, regardless of whether we think graphically (what kind of shape is it?), symbolically (what function does it represent?) or numerically (what discrete data does it show?).

Extracting numerical data from graphics is not restricted to graphs. Video evidence (home or otherwise) provides another opportunity for students to use graphics and to extract data from an experiment that they cannot easily reproduce in reality. Consider video of a motorcycle crash (Fig. 3).

Figure 3 (Produced in Multimedia *Motion*, ©Cambridge Science Media 2000)

The student proceeds with the active investigation by collecting personal data from the video of the crash thereby ensuring ownership of the problem.

Motorcycle crash 1

	Dataset 1		Dataset 2	
t/s	x/m	y/m	x/m	y/m
0	0.699	2.007	0.135	2.563
0.03	1.134	2.014	0.556	2.578
0.06	1.532	2.022	0.954	2.593
0.09	1.953	2.029	1.36	2.593
0.12	2.366	2.044	1.773	2.608

Table 1(Produced in Multimedia *Motion*, ©Cambridge Science Media 2000)

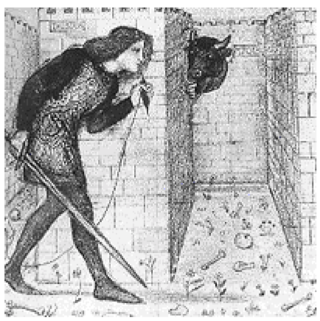
Thus pictures lead to numbers, which leads us to…

‘N’ is for *Numeric*

Increasingly in mathematics (and the rest of the world!) we are dealing with discrete numeric data – whether collected from some experiment or extracted from some video. Even the solutions of classical calculus problems are often sought from numerical methods. (We might eventually ask, if calculus is derived from letting small differences tend to zero, but solutions are sought by approximating over discrete differences, can we miss out the nasty business of infinitesimal changes altogether? But perhaps that is an argument for another time.)

Think of the major developments in mathematics over the past decade: mobile phones, money, chaos, rendering images etc. They all have their roots firmly planted in discrete mathematics. We even view continuous processes in discrete packages - the eye scans at a rate of one per 1/30 of a second.

In fact, Crete’s ancient guest, the Minotaur, is reducible to a collection of numbers defining a bitmap image now (see Figure 4).

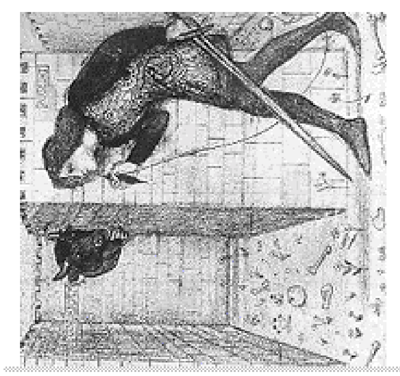


$M =$

	1	2	3	4	5	6	7	8	9	10
1	185	197	194	175	191	196	204	198	203	204
2	202	181	197	210	210	192	191	192	197	211
3	181	219	204	191	186	205	204	200	201	198
4	142	181	174	202	211	206	206	205	203	212
5	119	148	167	162	201	203	208	207	203	202
6	111	183	233	173	173	200	187	200	198	201
7	188	198	218	180	81	106	145	195	191	171
8	181	193	208	204	75	128	203	166	215	187
9	184	168	202	207	188	143	202	202	154	121
10	207	201	198	187	210	185	168	221	18	145

Figure 4 Picture from Island-of-Crete.net (2001)

These pictures (numbers) can be easily "moved" by standard matrix manipulations (see Figure 5). Whilst we learnt about matrix transformation by looking at a moving a unit square on a graph, our students can look at much more interesting problems.



$M^T =$

	1	2	3	4	5	6	7	8	9	10
1	185	202	181	142	119	111	188	181	184	207
2	197	181	219	181	148	183	198	193	168	201
3	194	197	204	174	167	233	218	208	202	198
4	175	210	191	202	162	173	180	204	207	187
5	191	210	186	211	201	173	81	75	188	210
6	196	192	205	206	203	200	106	128	143	185
7	204	191	204	206	208	187	145	203	202	168
8	198	192	200	205	207	200	195	166	202	221
9	203	197	201	203	203	198	191	215	154	18
10	204	211	198	212	202	201	171	187	121	145

Figure 5 Picture from Island-of-Crete.net (2001)

We have argued so far then that mathematical thinking resides in pictures, numbers and words. Finally being ‘proper’ mathematicians we must necessarily turn to symbolic processes.

‘S’ is for Symbolic

Whilst many professional mathematicians might recognise our wide view of mathematics, this is not always reflected in mathematics teaching at universities, where what we teach, and indeed particularly what we assess is often firmly rooted in the symbolic. While we do not wish to underplay the importance of the symbolic, this can contribute to the distorted view amongst the public at large about what mathematicians do.

It is easy for *mathematicians* to use precise, compact, symbolic notation. This specialised language enables ‘simple’ solutions of problems. To those fluent in this language, it is easier and more efficient to write down an equation or some shorthand mathematical expression than to express the idea in words. Unfortunately it is commonplace also to assume or hope that any student (or broader) audience has equal facility with this language, and often to complain when they do not. A series of reports in the UK (London Mathematical Society 1995, Engineering Council 2000) has indeed concentrated on and identified what are perceived as increasing shortcomings in this respect in UK students newly arriving at university.

We do not doubt that students need to understand symbolic mathematics, but there is a difference between recognising the meaning of something and being able to perform *the full range* of repetitive, algorithmic **m**indless **s**ymbolic **m**anipulation. Concentrating on manipulation, which many people view as ‘proper’ mathematics, obscures the richness of the subject, and is unnecessary when computer algebra software (CAS) is so readily and cheaply available. A major task for researchers and developers over the next few years is to find out how much of the “pencil and paper” manipulations a student needs to be fluent in, before being able to use CAS with complete comfort. For example how many integration techniques must a student of engineering be able to use before being able to then trust Maple or Derive to give the answer to one they cannot do? Exactly what constitutes competence in algebra now?

Once the CAS can be used comfortably, benefits accrue. For example on a TI-89 (Texas Instruments 2002(2)) the solution of differential equations becomes simple, leaving time to think and reflect, to justify the process, and to interpret the answers (Figure 6).

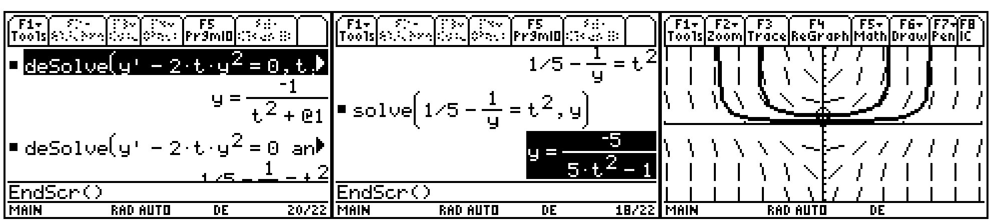


Figure 6

In some ways of using CAS, for example using the Script facility on say a TI-89, the CAS becomes part of the communication strategy. This is illustrated in Figure 6 for the Newton-Raphson process.

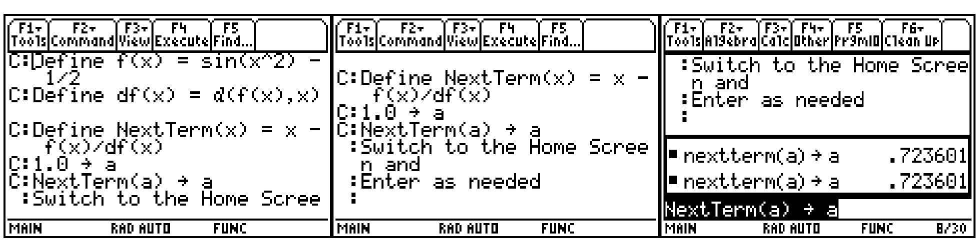
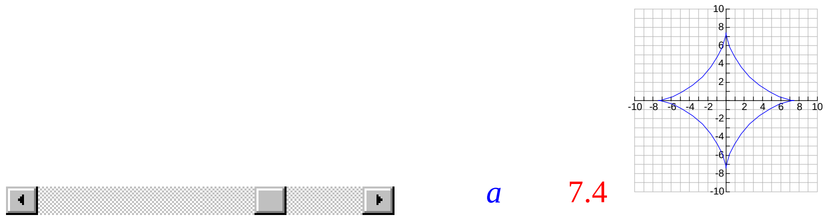


Figure 6

Technology which deals with *SONG*

Integrated software packages now appearing reflect our views here. One example is TI-Interactive™ (Texas Instruments (1)). This PC package provides an integrated environment, which includes CAS, word processor, spreadsheet, graph plotter, and a variety of links, to the internet, hand-held machines, and other students. It thus can be used to address the range of activities described above. One interactive process sheet, in this case the "magnetic bottle" function, is shown in Figure 8.



Cartesian equation: $x^{2/3} + y^{2/3} = a^{2/3}$ Parametric equation: $x = a\cos^3 t, \quad y = a\sin^3 t$

Figure 8 TI-Interactive™ output

The user can make the connection with reality through the package in a variety of ways. Figure 9 shows the wealth of data available for modelling from the associated web site. Alternatively one can collect personal data using a data logger such as CBL™ with the appropriate probe or a CBR™, or gather data from less accessible places using a package such as Motion (Cambridge Science Media 2000).

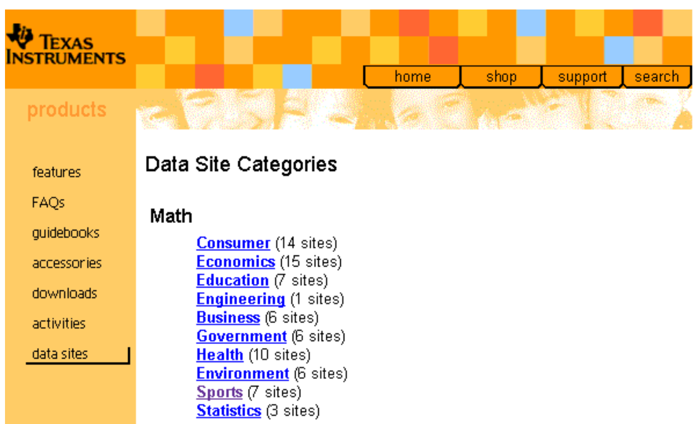


Figure 9

Discussion and conclusion

"We must not train people for our past but for their future" (Jones 2000)

The view of what constitutes 'mathematics' varies according to whom you ask. To mathematically uninformed people it is almost exclusively symbolic manipulation; a little like equating geography with English because one studies one using the other. Mathematics students might add that it includes learning what some people long dead did a long time ago, and being able to imitate them. Professional mathematicians might choose to define it as being about logical and analytical thinking processes, the role of rigorous proof, and so on. Some would choose to label what we have described here as science rather than mathematics, despite the fact that the kind of activities we have described are a core part of many (especially, perhaps, applied) mathematicians' work.

Before the advent of widespread technological tools for doing mathematics, it could be argued that anyone wishing to 'do' mathematics needed many years of training in mathematical language and history before being able to be let loose on the exciting tasks of formulating problems, interpreting results and relating mathematics to the real world. Hence the widespread experience that you really only started *doing* mathematics if you stuck at it until doctoral level. In this paper we have reported examples from our experience of how using technological tools can help in making mathematics a subject which all students can *do* for themselves. In a way, technology is allowing students to engage in mathematical processes, within the constraints of their knowledge, in a similar way to professional mathematicians. The issue of how fluent one really has to be in what range of symbolic processes, or how much one can depend in that respect on technology, is a critical one.

In the end, mathematics is wider than almost any one-sentence definition one could give. We certainly recognise the importance of our mathematical history and the role of analytical processes and rigorous proof, but mathematical ideas and concepts arise and are generated from our experience of the world. We believe that the kind of activities we describe in this paper fall within the wide range of what constitutes mathematical activity, and is a valid and useful part of a student's mathematical education. Technology gives us an opportunity to re-balance mathematical education to include these broader aspects, although the extent of that balancing must be the subject of much debate yet.

The educational processes described here are proving useful at various levels of mathematics. They enhance the development of concepts, and motivate students to engage in the subject. Interest in the world around them, perhaps through practical examples from sport, music, nature or the environment, can be used as an engaging vehicle. The process works both ways: using real problems enhances the development of mathematical concepts, *and* the mathematical ideas and language can be used to help make sense of reality.

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