THE FRONTAL COMPETITIVE APPROACH TO TEACHING COMPUTATIONAL MATHEMATICS

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ABSTRACT

The paper develops a general methodological framework for teaching Mathematics so that some minimum learning outcomes are achieved for all students and at the same time student-dependent learning outcomes due to individual creativity and effort are also possible.

Although the theoretical ideas and pivotal concepts behind the paper, such as motivation, feedback, reinforcement and others, are well known, specific implementation of these ideas may present a real challenge to practical teachers. The paper contains a comprehensive detailed view of one aspect of differentiated teaching: behavior modification. This aspect is considered as a prime necessity in circumstances when Mathematics is being taught to a large student audience. The methodological conditions and technological practices necessary to implement such an approach are discussed. Examples in the context of teaching numerical methods of Linear Algebra and other related courses are given.

Key words: Mathematics Education in Universities, Creativity in Mathematics Education, Mathematics Competitions in Mathematics Education.

1 Introduction

As Russian mathematician Yakov Tsypkin wrote jokingly in the preface to one of his works (Tzypkin 1970), reading mathematical books results in the three levels of knowledge. The first level means that a reader has understood the author's argumentation. The second level means that the reader has become capable of reproducing the author's arguments. And the third level means that the reader has acquired a capacity to refute the author's argumentation.

This joke reflects the fact that mathematics as a subject has a specific feature that it can not simply be put in memory. It means that a mathematics student can not stop at the first level of understanding. He needs to reach at least the second level. To do this the student has to pass all the information through his mind by solving a large number of tasks independently, thus as we say "adjust his head and hand". But even this is not enough, because as Hungarian mathematician Alfred Renyi said (Renyi 1967), "who learns the solution without understanding the matter can not use it properly". Independence, critical approach and creativeness – these are the third level features and only such knowledge has real value when learning mathematics.

Of late years, Russian mathematics community has been really feeling the need for novel teaching methods to stir the students to greater activity. One would expect a wealth of methods to choose from and apply. But upon examining methods that are practically being used, one finds a lack of appealing and interesting approaches that would create and hold students' interest and make them continue to study.

Here it is pertinent to note that mathematics has another feature. Mathematics may be defined as "chamber" science by the nature and mathematicians are often told to be "piece-goods". However many universities traditionally practice teaching to a large student audience. For example, large audience-oriented teaching has become the trait of Russian universities to the extent that the large audience have been assigned a special term,"a stream".

It is almost evident that the great size of the stream stands on the way, – it erects obstacles to awaken and hold students' interest in learning mathematics. It is a handicap to independent thinking, as many of students get used to "flow over the stream" and prefer to be as ordinary as their classmates. How can teacher transform this obstacle into advantage? How can he encourage students' independence? How can we help them to understand their outstanding abilities? And finally, how can we prove that mathematics is a live, beautiful science but not a collection of incontestable proofs, unquestionable facts and irrefutable arguments?

Fortunately, mathematics itself often prompts us how to achieve these goals. One of such methods termed the Frontal Competitive Approach (FCA) is discussed below. We consider it mainly in circumstances when the subject is being taught to a large student audience where students' behavior modification is a prime necessity.

2 Main idea of FCA

It may be explained by the definition itself. "Frontal" means general, involving all students to meet one common goal. "Competitive" means opportunity for a success due to individual's creative and non-standard solutions or actions. To implement FCA we need to do the following: (1) Organize creative environment. (2) Encourage students' creative potential. (3) Give start to students' instinct of competition. (4) Ensure transparency of assessment.

Let us examine these components in detail in the context of teaching Computational Mathematics.

2.1 Creative environment

Most of the existing educational materials on Computational Mathematics provide main theoretical data and sometimes theoretical instructions on how to program a numerical method or algorithm. However, it seems to be inadequate to the end. We believe that the true understanding of a numerical method may be achieved if: (a) a student completes assignments related to a challenging programming project; (b) each project results in practical use of that particular method assigned for the student; (c) the student conducts a set of extensive computational experiments with the program he developed independently; and finally (d) frontal rating of the projects is carried out by the teacher together with the students.

Programming in itself is beneficial for student due to a number of reasons. First, it provides an opportunity to understand and learn a numerical method "from inside". This is quite different from utilizing ready-made software and significant for any creative professional. Second, it improves student's computer proficiency, as it requires keen programming. And finally, it develops general analytic and solution seeking performance and implants practical skill to attack and solve computationally oriented problems.

To organize creative environment while teaching numerical methods to a large audience requires to make programming assignments as varied as possible in terms of the methods' algorithmic significance rather than their initial data. However, the number of variations on every method is usually limited. In these conditions, finding as many as possible versions of every numerical method becomes a matter of great methodological importance for each teacher.

Organizing creative environment means, also, that we should evaluate any laboratory programming project as a single study objective which possesses all the features of a completed software product. Among them are modular structure, convenient interface, efficient utilizing computer resources (memory and time), and possibility to implement a wide plan of computational experiments. This differs definitely from a widely used technique when the students work on one and the same ready-made software when they only enter their initial data and wait passively for a result. The approach we apply makes them perform valuable creative operations, stimulates each student's competitiveness, prevents cheating and helps to improve overall class performance.

A classic example of how to find as many as possible variant forms of a numerical method is the topic "Elimination and Matrix Inversion Methods". First of all, the teacher should systematize a set of Gauss and Gauss-Jordan elimination specific characteristics. They are: (1) direction of elimination of unknowns, (2) mode of access to the matrix entries, (3) mode of updating the active sub-matrix, (4) pivoting strategy etc. (Ortega 1988). Then independence of these characteristics will result in a significant number of different variants of assignments on the same topic being studied.

Over the course on many years, our work is focused on the possible ways of applying FCA to teaching numerical methods in Linear Algebra, Least Squares, Optimal

Filtering, Optimal Control, Linear Programming and Nonlinear Optimization. As a result, we recommend that teachers use textbooks, that offer a good choice of various project assignments. The first one contains: Topic 1 - Elimination and Matrix Inversion Methods 26 assignments in total, Topic 2 - Sparse System Solution 48 assignments in total, Topic 3 - Cholesky Decomposition 40 assignments in total, Topic 4 - Orthogonal Transformations 28 assignments in total, Topic 5 - Simultaneous Least Squares 28 assignments in total, Topic 6 - Sequential Least Squares (Semoushin & Kulikov 2000), and Optimal Filtering 25 assignments in total. The second one contains: Topic 7 - Simplex Method 70 assignments in total, and Topic 8 - Nonlinear Optimization 30 assignments in total(Semoushin 1999).

2.2 Student's creative potential

Lectures usually prove one variant of a numerical method in a certain topic. For example, we prove: LU-factorization theorem, LU-factorization theorem with the choice of pivots, LU-factorization algorithm replacing the original matrix by factors L and U, and so on. However, practically always for each proven theorem or algorithm there exists a dual variant. In our case the dual variants are those with $UL-$factorization (LDU and UDL factorizations are also possible). Therefore it will be expedient if each assignment contains formulation and proof of the theorem/algorithm for the assigned variant, which has to be made independently. Thus students are trained to understand subject of mathematics in a wider sense, their creative abilities and potential become more active and may be well evaluated especially for the gifted students.

2.3 Instinct of competition

As a rule, students of mathematical departments are very eager to gain and demonstrate professional knowledge of computers and modern programming technology. They are not interested in "hanging about" the initial level of computer proficiency. More to it, they express definitely their wish to show their skills for creating "outstanding" software. FCA ideally supports this instinct of competition. Indeed, multiplicity of assignment variants fortunately comprises two features of the variants: their resemblance and difference. Due to the resemblance between variants students' projects can be compared, and due to the difference they are of individual nature. Teachers applying FCA note surprising cases when a student who has got already a credit on the project, for example in sparse system solution, continues upgrading the software by changing access mode to matrix entries in order to achieve faster operation. Sometimes students arrange a kind of competition between them, in whose software has better interface or faster operation. Sometimes in holidays we hold a presentation of the best programs developed by the students. At first we required that a software should be written in Pascal but now we accept usage of various tools: Visual Basic, Delphi, Builder C++ etc.

2.4 Transparency of assessment

Teacher's role in FCA application is very important. Besides all above mentioned, a teacher should put forward a precise and definite system of requirements and evaluation criteria for students' projects. A student should know definitely what mark and for what work quantity and quality he will get. Starting his work student chooses by himself the level of assessment he initially pretends for. System of assignments should be designed in such a way that allows each student to move independently from one assessment level to another according to his own work. For example, the system of assignments on simplex method (Semoushin 1999) contains 70 different variants divided into three groups dependent on their complexity: basic level (20 variants), advanced level (30 variants), and higher level (20 variants). The marks are given accordingly: sufficient, good, and excellent. This transparency of assessment have a notable effect on the students' activity.

3 Technological summary of FCA

Some general tools indicative of the FCA are the following.

1. *Creative environment.* A broad assortment of assignments and tasks is offered to students together with the clearly differentiated scenarios of their accomplishment.

2. *Goal setting.* A clear formulation of both short-term and long-term goals is offered: (1) marked improvement of programming skills together with the deeper understanding a particular numerical method, and (2) profound understanding the subject of Computational Mathematics and ability to attack computationally oriented problems.

3. *Challenge.* The environment including non-trivial assignments and tasks with increasing levels of difficulty, challenges the student to keep self-independent working.

4. *Student-controlled navigation.* Putting the locus of control in the hands of the student has a great psychological effect: even the weak students put in a claim for higher grades and try to move to the upper level of difficulty while choosing the assignment.

5. *Competition.* Competition by "playing" against others appears to be naturally embedded in the teaching process because the above tools are in excellent agreement with this human instinct.

6. *Rewards.* Rewards, such as higher grades or "automatic" credit for the course, may be offered as students show an obvious increment in skill and success.

4 Some empirical data

Since 1988 the standard approach (SA) in teaching Computational Mathematics and Optimization Methods was used at two Ulyanovsk universities. The SA meant that, while studying Computational Linear Algebra, students were offered to fulfill only one laboratory programming project per semester in order to be allowed to take examinations, and this project was on Elimination and Matrix Inversion Methods without essential differences between separate project assignments. The courses of Numerical Methods and Optimization Methods included some problems solution using the Dialog Computing System (DCS) developed in Mathematics Institute of Byelorussia Academy of Sciences for the purpose of teaching and learning Applied Mathematics (Fourunzhiev, 1988). The DCS contained a set of subroutines. The student had to send his/her commands to the system and wait for the system requests and messages thus organizing a dialog during the work. In 1993 we switched to the FCA at Lomonosov Moscow State

Table 1: Percent student distribution under the two teaching approaches conditions

Grading system		NoW under SA conditions		NoW under FCA conditions	
Grade	NoW[1]	attempted	fulfilled	attempted	fulfilled
Excellent	3	12	4	40	24
Good	2	36	12	44	48
Satisfactory	1	42	56	16	24
Failure	0	0	28	0	4

[1]Number of works.

University Branch in Ulyanovsk (transformed into Ulyanovsk State University in 1996) and in 1995 at Ulyanovsk State University of Technology.

Over the four years of the SA usage and then the nine years of the FCA application we collected our observations, so this time can be considered as the length of our teaching experiment intended to analyze performance differences between the two teaching approaches, SA and FCA.

Usually the researcher has a number of techniques to help gather and make sense out of the data collected. However in education practice, there is a lack of definite, standard metrics for teaching performance assessment. All the data should be treated as subjective in some way or another as human participation is inherent in several stages of the education experiment. Nevertheless, subjective data from human observations and judgements can be considered objective and valid as performance measures if the observations are verified and judgements are derived from what is purported to be measured.

To obtain the "big-picture" of what is good and bad and why in the two approaches, we used the following student performance measures: (M1) Amount of work attempted at the beginning of semester, (M2) Amount of work fulfilled by the end of semester, (M3) The week when a student begins to work at full power, and (M4) The week when a student defends the first work fulfilled. A large amount of data from teacher observations and student questionnaire responses were collected, summarized and averaged for two core requirement (compulsory) courses: Computational Linear Algebra (Topics 1 through 4) and Optimization Methods (Topics 7 through 8), and also for two major requirement (elective) courses: Recursive Least Squares and Optimal Filtering Algorithms (Topics 5 through 6), see Section 2.1 for topic numbering. For Computational Linear Algebra, Table 1 shows the averaged percent of students who attempted and fulfilled three, two, one or zero laboratory work assignments under the SA and FCA approaches, and Table 2 shows the above mentioned "averaged" student performance measures.

These results indicate convincingly that FCA is superior in effectiveness to SA as better stimulating students' interest in Mathematics. This is also reflected in the fact that the students who choose to be enrolled for Recursive Least Squares and Optimal Filtering Algorithms have increased in number after completion the course of CLA. Analogously, having completed the Linear Programming case study project under FCA conditions, the majority of students express their desire to fulfill the Nonlinear Optimization case study project independently, i.e. only on the basis of going into the

Table 2: “Averaged” student performance measures (Topics 1 to 4)

Measures	M1	M2	M3	M4
SA	1.50	0.92	10	16
FCA	2.24	1.92	3	10

recommended literature. We explain this by the increased students' self-reliance and confidence in their ability to make sense of new material without assistance.

5 Conclusion

Designing efficient education process in a large audience is complicated and time consuming. This paper has touched on a few basic teaching tools to exploit when motivating students to learn Mathematics. Called FCA and applied in different Russian universities, this approach has proved that students have a generally positive response to it. Individual, self-dependent work within a large audience encourages students' sound competition, desire for creative solutions and better performance indices.

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