SERVICE WITH A SMILE

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ABSTRACT

In this paper I will discuss some aspects of *specialised service teaching*, by which I mean the teaching of mathematics to an identifiable group of students with a shared primary interest which is not mathematics. I will first argue for the vital importance of service teaching in general, not because of its budgetary implications for mathematics departments, but because of its role in ensuring the overall health of mathematics as a discipline. I will then examine two key issues concerning the teaching of specialised service courses, namely whether mathematicians should teach such courses, and if they do, how they should approach this task.

1. Introduction

In this paper I will be mainly concerned with what I will call *specialised service teaching*, by which I mean the teaching of mathematics to an identifiable group of students with a shared primary interest, which is not mathematics. A course in complex variable for engineering students would qualify, as would an introductory course in calculus for biology majors, but not a general introductory course in calculus for students including mathematics majors, or for all students other than mathematics majors. First, though, I want to say something about service teaching in its broadest sense, that is, the teaching of mathematics to students whose primary interest is not mathematics.

The issues I want to discuss fall under three headings:

- Why is service teaching important?
- Who should teach specialised service courses?
- How should mathematicians approach the teaching of specialised service courses?

2. Why is Service Teaching Important?

> *"Of all the resources which the human spirit possesses ... none is so momentous and so inseparable from our inner nature as the concept of number. ... Every thinking person ... is a number-person, an arithmetician".*
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> J.W.R. Dedekind, undated manuscript

Some years ago I took part in a study of the mathematical needs of school-leavers in New Zealand. The particular aspect that I was involved in was the investigation of the mathematical needs of everyday life. The detailed conclusions that we reached are not relevant here. But the study brought home forcibly to me the fact that virtually everybody does mathematics frequently in the course of their daily lives, in many different contexts: shopping, completing tax returns, working out household budgets, calculating quantities for home decorating, playing games ... — the list is endless. The fact that everyone *does* mathematics makes it almost unique among academic disciplines; people may take an interest in history or geography, but they do not *do* it inescapably in their daily lives.

There is nothing new here, of course, we all know this. But the point I want to emphasize is that this is why society regards mathematics as deserving of a special place in the school curriculum — not because of the aspects of mathematics that we mathematicians regard as important. By most ordinary standards of importance, it is arithmetic and elementary geometry that are the most important parts of mathematics, not functional analysis or group theory.

Much the same can be said at the level of tertiary education. What gives mathematics a special place in tertiary education is the fact that it is needed by scientists, engineers, economists, sociologists ... — and again the list is endless. If it were not so, mathematics departments would be small groups teaching small classes of a few devotees.

Once again there is nothing new here. We all know that large service classes are a budgetary necessity for most mathematics departments, so of course service teaching is important! But that is not the point I want to make. If, as I argue, almost everyone does mathematics at least some of the time, then service teaching is important simply because it is the way almost all of those who do mathematics learn the subject. It is vital for the health of the discipline that it should be done well. If most of the people who do mathematics do so unwillingly, inexpertly and with feelings of dislike if not actual nausea, then mathematics is in a bad way. If on the other hand they do mathematics with

a sense of enjoyment and view it as a friend rather than a foe, then we as teachers have done well and our subject will flourish at all levels.

The teaching of mathematics majors is of course essential for the continuation of the subject, but we do not need encouragement to be attentive to that aspect of our educational task. Service teaching, on the other hand, often risks being neglected because it is seen as a tiresome necessity, a digression from our main task of educating mathematicians. I believe that for service teaching to receive the attention it deserves, it needs to be seen for what it is — one of the most important things that we as teachers of mathematics do.

3. Who should teach specialized service courses?

It may not be so everywhere, but certainly in the university systems that I have worked in, the question of who should teach specialized service courses is a perpetual source of tension. Because of its budgetary implications, the question is all too often seen as a purely political one, but here I want to focus on the academic question. Who are the best people to teach such courses — the mathematics subject specialists or the specialists in the students' primary interest subject? We might like to say that mathematicians are the best qualified people to teach such courses, but what reasons can we advance to justify this?

The most obvious reason is that mathematicians are the experts in mathematics, and university students should be taught by experts. When it comes to teaching mathematics majors, this argument is conclusive. In the case of service courses, it remains valid, but the acknowledged expertise of mathematicians does bring disadvantages as well. As mathematicians, we see the subject from a particular viewpoint, which is not the same as the viewpoint of students in service courses. For example, a mathematician would probably see Fourier series as a special case of the general phenomenon of the representability of elements of a Hilbert space in terms of orthonormal bases. But if the students are electrical engineering students, they will see the subject in terms of signal processing and spectral analysis. Unless their mathematician teacher takes this into account, the students may feel (perhaps rightly) that they are being taught by someone who does not understand their needs. Again, mathematicians tend to be excited by singular cases and exceptions, which help to sharpen our understanding of the conditions under which various results hold good. But students in other disciplines care much less about such things since they seldom or never arise in practice. We need to keep a sense of proportion when teaching service courses and not get too carried away by "interesting" special cases, which are really of interest only to ourselves.

The second reason that might be advanced is that we are the experts on the teaching of mathematics. Here again it can safely be said that we are the experts on teaching mathematics to budding mathematicians (though even so we are not always conspicuously successful). We tend to take it for granted that this expertise will easily transfer to service courses, and are unimpressed by the doubts sometimes expressed by our colleagues from other disciplines. But when teaching service courses we are not teaching people like ourselves (or even people with ambitions to be like us). We need to keep reminding ourselves that while we may be teaching mathematics, we are not teaching mathematicians. Making our teaching acceptable to students who do not necessarily share our interest in mathematics is not easy. It may require us to take an interest in things non-mathematical, rather than assuming that the students have an interest in things mathematical.

The contrary case for leaving the teaching of specialized service courses to specialists in the discipline being served is of course made by turning the negative features of teaching by mathematicians into positive arguments for the contrary. The positive features of teaching by

mathematicians will naturally then become arguments against the contrary! But the case against teaching by mathematicians is not without strengths and we certainly cannot simply dismiss it as ill-conceived.

In short, I do not think we can or should expect others to take it for granted that we are the people best fitted to teach service courses in mathematics. To prove our case we need to take such teaching very seriously and put in the effort required to overcome some of the handicaps that I have mentioned. This brings me to my last section.

4. How should mathematicians approach the teaching of specialized service courses?

I think one of the biggest problems facing mathematicians teaching specialized service courses is that the students tend to see both the teacher and the subject as alien. Advanced students in, say, engineering or ecology usually form a coherent group, attending many classes and laboratories as a group and getting to know the teachers in their chosen fields very well. By contrast the mathematician appears for a few hours each week and may well seem like a being from another world, particularly if the mathematics is obviously being taught from the point of view of a mathematician rather than an engineer or a biologist. Terminology and notation that is different from what the student sees in other subjects can increase the feeling that mathematics is an alien subject. To take a very simple example in connection with the teaching of engineering students: mathematicians (and textbooks on engineering mathematics written by mathematicians) invariably denote the solutions of $x^2 + 1 = 0$ by $\pm i$, while engineers (and textbooks on engineering mathematics written by engineers) denote them by $\pm j$. So students are immediately conscious of a distinction between the two worlds, yet there is really no reason why a mathematician teaching engineers should not adopt their notation.

You can probably guess what my proposed solution is: as far as possible, mathematicians teaching specialized service courses should try to see the subject from the point of view of the discipline being served. Now you may say: "But I am a mathematician, not an engineer or ecologist or whatever. How can I not see the subject from the point of view of a mathematician?" — and of course there is some truth in that. But as professional mathematicians we are often confronted by problems brought to us by people outside mathematics, and in order to help them we have to understand their points of view and interpret our mathematical solutions in their terms. On the whole, I think we are pretty good at this, and their is no reason why we cannot do the same in our teaching. It does require some extra effort though: it is important to talk to practitioners of the other discipline and to read the textbooks that the students will use in their other subjects. Just using some of the terminology and notation that these textbooks use can make a big difference. And perhaps most important of all, the teacher should have or be willing to develop a genuine, even if only amateur interest in the other discipline. A service mathematics teacher who really has no interest in the discipline being served is not likely to be successful.

Let me give a few examples of what I mean:

(i) Textbooks on calculus for economics generally define concepts such as marginal cost, elasticity of demand and so on in terms of derivatives, give a brief explanation of their significance and then plunge into examples and exercises involving the calculation of these quantities for specific, often quite arbitrary, functions. This has its place, of course, but textbooks on introductory microeconomics do very little of this. The focus is much more on qualitative questions involving the interpretation of these quantities and effects of changes in them. Often the material in the

mathematics text is of little direct help in understanding these matters. Yet it would not be difficult to incorporate such ideas into the mathematics course and thereby make it much more relevant to the students' real needs.

(ii) Functions of a complex variable are very important for engineers in connection with control theory. Textbooks on mathematics for engineers typically focus on residue theory, leading towards applications such as the evaluation of certain definite integrals. This is a mathematically beautiful theory, but it is of only marginal relevance to control engineers. Certainly they need to know about poles, but their interest is in the location of poles in connection with the stability and behaviour of control systems. There is plenty of interesting mathematics here, but it needs to be dug out of texts on control theory, not mathematics texts, and it tends to use its own specialized language. Time spend on finding out these things and incorporating them into a service course is well rewarded by having a much more motivated class.

(iii) Mathematicians may find themselves teaching a course to ecology students on the mathematical modeling of populations using differential equations. It is very easy to get carried away by the mathematical tidiness of the models involved and forget that real populations do not always behave in the tidy way predicted by our models. A look at texts and journal articles on ecology will provide plenty of material for a more critical look at the relevance and applicability of our models, surely just as important as training our students in the mathematical techniques, and probably more interesting for most of the students, since only a few will go on to become specialists in mathematical modeling.

To sum up: I have argued that service teaching is of the highest importance for the health of mathematics. I believe that we as mathematicians are the best people to do it provided we are prepared to make the effort to meet the students halfway. My experience is that if we do this, service courses can be immensely satisfying and enriching for both teacher and students.