

LANGUAGE AS A COMMUNICATIVE AND INTERPRETIVE TOOL IN MATHEMATICAL PROBLEM SOLVING

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ABSTRACT

Discourse analysis of transcribed protocols is a very interesting field of educational research. In mathematics education, cooperative problem solving has proved to be an effective way to promote mathematical discourse. Using linguistic analysis techniques and symbolic interactionism as our theoretical framework we analysed the language that students used while they worked in pairs to solve a geometrical problem. Our analysis revealed how language was used as a communicative and interpretive tool by the participants. The use of everyday or quasi-mathematical language didn't seem to affect the establishment of shared meanings. Each student adopted a specific role during the interaction and used language to maintain it. The whole process was governed by social and sociomathematical norms which were constituted before or during the interaction.

KEYWORDS: Discourse analysis, language, problem solving, symbolic interactionism, interaction, social norms, sociomathematical norms.

1. Introduction

Verbalisation is a major component of the thinking process. Within this view, great effort is made by all educators to engage their classes in some sort of discussion. Depending on the teacher's scope and the practical limitations involved, different kinds of discussion are practised: whole class discussion, teacher-student discussion, discussion between groups or discussion within a group. Discussion is a component of mathematical discourse which is defined as "the ways of representing, thinking, talking, agreeing and disagreeing" (NCTM, 1991). One of the most effective types of discourse is the one that is produced in small groups of students engaged in what Pirie and Schwarzenberger (1988) define as mathematical discussion: "It is purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction". This kind of discussion has proved effective in all levels of education, from primary school (Pirie and Schwarzenberger, 1988) to university (Yackel, 2001). The term effective means that it either promotes students' learning and socio-cognitive development (César, 1999) or that it contributes to gaining equal-status interaction or positive intergroup relations (Cohen, 1994).

2. Theoretical framework

Symbolic interactionism is a social psychological theory developed from the work of Cooley and Mead in the early part of the twentieth century. The actual name of the theory comes from Blumer, one of Mead's students. According to Blumer (1969), this theory is based on three core principles: **meaning**, **language** and **thought**. These principles lead to conclusions about the creation of a person's self and the socialisation into a larger community (Griffin, 1997). The basic assumptions of the theory according to Mead (1934), Blumer (1969) and Yackel (2001) are:

- a) People act toward things on the basis of the meanings that the things have for them.
- b) Meanings are not intrinsic in things, they have to be defined before they have human reality.
- c) Everything that people act upon or that has an impact upon them must go through the process of subjective meaning.
- d) The meaning of a thing for a person grows out of the ways in which other persons act toward the person with regard to the thing.
- e) Meaning is not merely individual and subjective, but it derives from and arises out of the social interaction.
- f) Meanings are handled and modified through the interpretive process used by the person in dealing with the things he encounters.
- g) Human action is created in the process of interpreting meanings.

Language gives humans a means by which to negotiate meaning through symbols. Human communication is made possible through the use of symbols (symbolic interaction). Although symbols seem to be a fixed entity, people use them in a shifting, flexible and creative manner. That process of adjustment and change involves individual interactions and larger scale features such as norms and order. Finally, thought modifies each individual's interpretation of symbols; thought based on language is a mental conversation that requires role taking or adopting different points of view.

Another notion which we found very useful in our research is "**face**" as described by Goffman (1967). "Face" is the image of the self presented; it is what the others see or consider having been expressed by the "actor". Both the actions of the "actor", and the perception and view of others, establish whether or not "face" is maintained. We analysed the kind of language that students used to maintain "face" and tried to identify some characteristics of that "face".

Discourse analysis in our research was based on the notion of the cooperative principle for the exchange of information as suggested by Grice (1989) and on the notion of context as something which is not just given as such in interaction, but it is made available in the course of it.

According to Slembrouck (2001) the cooperative principle is based on the assumption that language users tacitly agree to cooperate by making their contributions to the talk as it is required by its current stage or the direction into which it develops. Adherence to this principle entails that talkers simultaneously observe four maxims:

- a) quality, i.e. make your contribution truthful and sincere,
- b) quantity, i.e. provide sufficient information,
- c) manner, i.e. make your contribution brief, present it in an orderly fashion and avoid ambiguities,
- d) relation, i.e. make your contribution a relevant one.

There are various conditions under which these maxims may be violated or infringed upon. Our analysis did not intend to convey if these maxims were enforced or not; our aim was to observe how language was transformed by the students in order to be consistent with these maxims and if the attempt was successful or not.

The view of the context was very useful in our attempt to create a dynamic analysis, in which we sometimes had to “go back in time” in the text in order to justify our contentions.

3. Methodology

The aim of our qualitative research was “subjectively and empathetically to know the perspectives of the participants” (Jacob, 1988). The subjective character of qualitative analyses is stressed by many researchers. Lemke (1998) states quite lucidly that “It is not always possible to say what a particular choice or move means, but you can say what it might mean... Even the participants in a discourse may disagree about the rhetorical meanings of particular features, or change their minds in retrospect or with additional information.”

The exploratory process of data reduction was mainly based on the assumption that everything the subjects said made sense in some ways. We intended to look for anything and everything of interest (Mitchell 2001) using different types of analyses; this is not unusual in qualitative research and an interesting example of that practice is the multiple analysis approach as described by Dekker, Wood and Elshout-Mohr (2001).

The subjects of our research were twenty undergraduate students from the Department of Primary Education of the University of Ioannina. Their ages varied from 18 to 21 years and they were asked to choose the person that they would like to work with. Thus, ten pairs were formed, seven “girl-girl” pairs and three “girl-boy” pairs. The type of task that we would assign to the students was a challenge for us. We noticed that the vast majority of research in linguistic area involves algebra problems because they allow many interpretations and solving strategies, and because they can be easily altered to produce variations of the original problem. Thus, we thought that an Euclidean geometry problem would be a challenging alternative to that tradition. The only instruction given to the students was that they should work together to solve the problem and that they should verbalise every thought they make.

The first observation we made was that the students used everyday or quasi-mathematical language in many cases. We categorised these cases according to what the purpose of that language seemed to be and then we studied the effect of it on the common understanding that the students were supposed to create.

In addition, we noticed that the students tried to use mathematical justification in many cases, although they were not clearly asked to. This was attributed to the establishment of sociomathematical norms. According to Yackel (2001) the norm is a collective notion which describes the expectations and obligations that are constituted in the classroom. Thus, our concern was to identify these norms in every episode, and then compare the findings of all the protocols.

The third and most interesting observation was that there was a clear distinction between the “roles” that were acted by each one of the two students in every couple. We studied how language was used to achieve that.

These three observations helped us to shape our research questions:

- a) Does the use of everyday or quasi-mathematical language affect the common understanding of the participants?
- b) Were there any observable social and sociomathematical norms constituted in the interaction?
- c) How is language used by the participants to reveal their roles in the interaction?

4. A sample analysis

The following text is made of excerpts from the actual solution process followed by a pair of female students when they were given the problem: “Given a right triangle ABC and D be a point on the side BC. Let DE, DZ perpendicular to AB, AC retrospectively. Draw the line segment EZ and locate the position of D so as the length of EZ to be minimal”. The two students are marked as “A” and “B” and the researcher as “K”. Researcher’s notes are in brackets¹.

Once the two students had read the problem, they made figure 1 (see Appendix). Then the following discussion took place:

10. B: What are we going to do now?

11. A: Now, I guess, we have to find the position of D on BC, so that we can find the minimum possible distance of ZE, of EZ. The straight line is the shortest way, isn’t it? What if we draw a vertical line?

12. B: Vertical to which?

13. A: To ZE from D?

14. A: Shall I try putting D in a lower position?

15. B: On another figure?

16. A: On the same one.

(A draws D'E', D'Z' and compares ZE with Z'E').

17. B: Is it shorter now?

18. A: It’s nearly the same though.

19. A: If we put it in a higher position, would it be larger?

20. B: How?

(A suggests a point near C).

21. A: What if we put D here? It would be larger.

22. B: No.

The two students continue measuring ZE’s length for various positions of D.

(A draws KP, KL).

31. A: It’s larger now.

32. A: What if we draw a vertical line from D to ZE?

¹ The original text of the problem and of the transcriptions of students’ spoken interactions is in Greek.

(B reads the last sentence of the problem).

33. B: Maybe it's somewhere here, in the middle, because this one and the other distance that we found earlier is greater compared to that one.

34. B: If we put D in the middle of BC, wouldn't that distance be smaller?

35. A: What if we draw something like a square? And put D at its center where its diagonals intersect? Do you understand what I'm saying?

36. B: Yes.

(A draws ABIC, but figure 1 has become too complicated, so the researcher suggests that they should draw a new figure. So A draws figure 2).

39. A: Shall I draw a square again?

40. B: Yes.

(B points at D in figure 2)

41. B: This has the minimum distance.

42. A: But we don't know it.

43. B: And those we had made before, below and over the middle, had the same distance, isn't that so?

44. A: We have now made a small square and one of its diagonals which is EZ is the minimum distance.

45. A: We took the center of the triangle's hypotenuse. Why did we take the center?

46. A: What if we put numbers at the sides?

47. B: This must be it.

The students felt that they were at a dead end, so the researcher decided to intervene. He explained to the students that the given triangle is not isosceles and that, in case it was isosceles, the middle of its hypotenuse would be the point that they were looking for. The students then drew figure 3 and compared the length of EZ for various positions of point D.

89. A: What if we use a formula?

90. B: Which formula?

91. K: Which formulas do you know?

92. A: We don't remember any.

93. K: You don't have to use any formula.

94. B: Do we have to prove it too? Can it be proved?

According to our research questions, the analysis of the protocol consists of three parts:

a) The use of ordinary and quasi-mathematical language is obvious throughout the whole excerpt. Examples of that use are the words "way" (11), "lower" (14), "higher" (19), and the expressions "like a square" (35), "small square" (44), "center of the triangle's hypotenuse" (45), "put numbers at the sides" (46).

The word "way" is included in the expression "the straight line is the shortest way" which is very common in Greek education although it is not considered formal mathematical language. (This is an example of contextual analysis, where the word is analysed with respect to its surrounding context).

The words "lower", "higher" and the expressions "small square", "put numbers at the sides" are examples of the enforcement of Grice's maxims for the exchange of information. A mathematician would replace the word "lower" with the expression "towards point B" or "near point B" (see figure 1). But the enforcement of the maxim of manner made student A use the least words possible without violating the maxim of quality: everybody present at the interaction understood what A was talking about. The same is the case with the other words and expressions.

The expression “like a square” reveals a shared meaning that is constructed in the course of the interaction. The students had drawn an isosceles triangle, although this was not the case, and that fact is revealed in 35. One might argue that figure 1 gave enough evidence for that; but once the word “square” is uttered, the notion of the isosceles triangle is constituted. In 39 student A says “Shall I draw a square again?” and the word “again” is what Gumperz (1999) calls a contextualisation cue, that is a word the role of which is to connect the sign (in our case the square) with its context.

b) An interesting feature of our research was to find out if this setting will constitute any social or sociomathematical norms. Previous research in this field was primarily conducted in classroom settings, over a long period with continuous assistance and guidance by the teacher. Our protocols provided some stimulating examples of social and sociomathematical norms. In this particular episode we can find two such cases. The first is in 35 where student A, after having expressed her proposal, asks student B: “Do you understand what I’m saying?”. We can say that she accepts that she has to explain, if necessary, her proposal to student B, which constitutes a social norm. The second case is in 94 where student B asks the researcher: “Do we have to prove it too? Can it be proved?”. Here, the norm of what is mathematically efficient doesn’t seem so stable; the student wonders when she asks if it can be proved, but on the other hand her question means that she is aware of what mathematical proof is about.

c) Finally, we face the most interesting phenomenon in our research: the role playing. In all the protocols that we analysed, each student played a distinct role which was evident during the whole discussion. In the particular episode, student A played the role of the initiator – the one who proposes tasks or procedures – and the way she did it was through questions. Consider these: “Shall I try putting D in a lower position?” (14), “If we put it in a higher position, would it be larger?” (19), “What if we put D here?” (21), “What if we draw a vertical line from D to ZE?” (32), “What if we draw something like a square? And put D at its center where its diagonals intersect?” (35), “What if we put numbers at the sides?” (46), “What if we use a formula?” (89). Using a symbolic interactionist perspective, student A interpreted the original instruction “to verbalise every thought they make” by taking the role of the initiator. Student B on the other hand, interpreted her fellow student’s questions as proposed solution strategies, but her reactions were different in each case. She either chose to accept the proposal (see the next lines of 14, 19, 35), reject it (see the next line of 21) or ignore it (see the next lines of 32, 46, 89).

The cross-examination of the protocols confirmed that the use of ordinary and quasi-mathematical language exceeded the use of formal mathematical language. The students were reluctant to use sophisticated mathematical expressions, but this did not prevent them from creating and handling the shared meanings that were necessary to deal with the problem. The cross-examination also helped us to clarify the social and sociomathematical norms that were constituted. Examples of these norms are that each student had to justify her thinking, listen to her partner and try to make sense of her thinking, and realise the need for mathematical justification whenever it was possible. Finally, we noticed that there was little difference between the roles that were acted by the students in all episodes. In every single pair, one of the participants proposed new ideas and the other one evaluated them. Each student, using language, tried to maintain her “face” throughout the interaction. In very few circumstances this role playing was reversed.

5. Concluding remarks

The linguistic analysis of the protocols has shown that despite the fact that students used

everyday or quasi-mathematical language, they had no difficulty in creating shared meanings and understandings. This creation was not uncontrolled; social and sociomathematical norms defined the behavior – namely the language – of the participants. These norms were the same as the ones which were observed in other studies, but the interesting part was that in our case there was no previous preparation or assistance during the process. The analysis was also based on the symbolic interactionist perspective using elements from interactional sociolinguistics. From this point of view, we noticed that each participant interpreted differently the norms that were established before or during the interaction; this resulted in a role playing which proved to be extremely important for the flow of the interaction because it helped the participants to handle the discourse and cooperate in it.

Multiple analysis approach proved to be very helpful in our attempt to clarify some aspects of the interaction that takes place in cooperative problem solving. But it also raised some new questions: Is there any way for the teacher to establish all the necessary norms from the beginning of the interaction? What is the evolution of these norms in time? What makes one student adopt a specific role? Beyond these questions we can see the most important implication of our analysis: the mathematics educators must be very careful in handling the language that they and their students use in problem solving. Making the right move at the right moment can help the teacher achieve the expected cooperation and make the interaction effective.

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APPENDIX

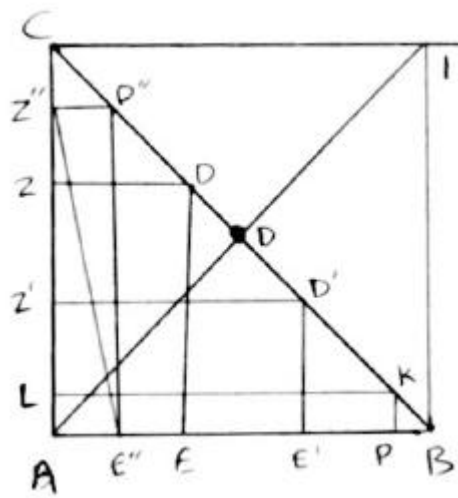


Figure 1

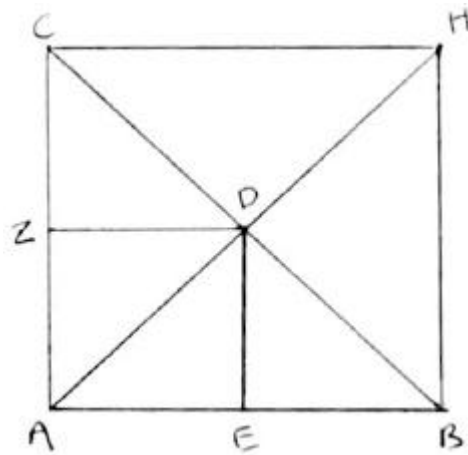


Figure 2

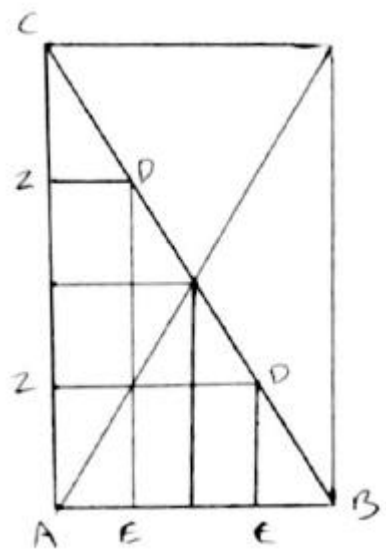


Figure 3