NAME, ARTS, MATHEMATICS, AND TECHNOLOGY

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ABSTRACT

For some years the author has been experimenting with an activity based on designs formed by her students' names. In order to form the design, we transform the student's name into points on the coordinate plane, then we connect points to form a closed polygon, then we subject this basic polygon to three 90-degree rotations. The result is an individualized, frequently interesting and complex polygonal design. Students can then, by coloring in regions, obtain interesting and often quite beautiful designs. One can then ask mathematical questions about these figures, how many pairs of parallel or perpendicular lines there are, and what are the areas of the various regions, and so on. Students seem to enjoy this activity, and the fact that the activity automatically yields individualized projects seems to enhance the students' interest.

When the lines involved happen to pass through points of the graph paper grid, it is comparatively easy to find their slopes and thus determine properties such as being perpendicular. Areas of squares and other quadrilaterals can be computed readily if their vertices lie on grid points. There will be some quadrilaterals whose vertices do not lie on grid points and we must learn how to solve pairs of linear equations to find these vertices. There may be many quadrilateral figures in a given design, and so we encourage the use of calculators to keep the computational labor from being excessive.

We find this project has been helpful to students about to enter calculus, because it affords an amusing and motivated review of the important pre-calculus notions. It also is good for prospective teachers because it gives a way to vertically integrate parts of the curriculum.

The author has been working with this pedagogical device for several years and new ideas still seem to be coming up. In this talk we illustrate in detail how our activity works with the example "ICTM".

Keywords: Art; Calculator; Spiral Curriculum; Problem Solving; Problem Posing

1. Background and Literature Review

The National Council of Teachers of Mathematics (NCTM, 1989, 2000) suggests that the emphasis of the mathematics curriculum should move away from rote memorization of facts and procedures to the development of mathematical concepts, and that students should investigate through problem solving not only to make connections among various representations of those concepts, but also to make these concepts meaningful to themselves.

It is believed that the use of real world representations helps students develop understanding of abstract mathematics (Fennema & Franks, 1992). Real-world problems are commonly used as vehicles to introduce or deepen students' understanding of mathematical concepts and relationships. To be successful problem solvers, however, students must develop inquiring habits of mind. They not only need to seek what are the solutions to problems and to determine why the solutions work, but also to pose questions to answer. Of course, teachers also play an essential role in developing inquiring minds. In particular, teachers must themselves be models of inquiry and must establish classroom context in which questioning and proving are the norm. They should pose questions about the problem situations and challenge students to defend their problem-solving strategies.

House (2001) concludes that a good problem is a problem just keeps giving and giving. What are the characteristics of investigations that can lead to good mathematics problems for students? Good problems (adapted from Russell, Magdalene, & Rubin, 1989; Wheatley, 1991: Clements, 2000):

1) Are meaningful to the students;
2) Stimulate curiosity about a mathematical or non-mathematical domain, not just an answer;
3) Engage knowledge that students already have, about mathematics or about the world but challenge them to think harder or differently about what they know;
4) Encourage students to devise solutions;
5) Invite students to make decisions;
6) Lead to mathematical theories about a) how the real world works or b) how mathematical relationships work;
7) Open discussion to multiple ideas and participants; there is not a single correct response or only one thing to say;
8) Are amenable to continuing investigation, and generation of new problems and questions.

Miller (2001) reports that he uses an interdisciplinary project to teach the mathematics concepts of transformations of periodic functions. Through this project the author has increased the students' understanding of the mathematics concepts and helped them to see mathematics in art. This use of interdisciplinary units in art can satisfy requirements found in NCTM Principles and Standards for School Mathematics (NCTM, 2000).

Mustafa (2001) finds a new method to determine the end point of a segment. In his article the author presents a new method for determining the coordinates of the endpoint of a segment, given its slope, point of origin, and length. For example, given segment AB=12.5, point A (2,2) and slope m=$\frac{4}{3}$, find endpoint B. His method consists of five steps:

Step 1: Z=(2+3, 2+4)=(5,6)

Step 2: $d_1=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(2-5)^2+(2-6)^2}=5$

Step 3: $k=\frac{12.5}{5}=2.5$

Step 4: $\frac{\Delta y_2}{\Delta x_2}=\frac{4\times 2.5}{3\times 2.5}=\frac{10}{7.5}$

Step 5: B=(2+7.5, 2+10)=(9.5, 12)

B=(2-7.5, 2-10)=(-5.5, -8)

This seems an easy-to-follow set of steps for finding the endpoint of a given segment AB, but my question is why do we need to find the endpoint? We want students to understand why things such as Mustafa's method are worth knowing. Mathematicians all over the world are trying to find a better way of teaching mathematics and for students to learn mathematics. However, students are always wondering; why do we need to study mathematics, how is it related to us? With the help of technology, plugging into a formula to compute the right answer is not the main issue, what is important is that our students understand mathematics and know the relevance of mathematics to every one of us.

2. The Questions

Question 1: When we ask teachers to "integrate technology into their classrooms" we are asking for the biggest change in educational practice in the last 200 years. This task is so difficult, so painful, so challenging and so directionless. How to help our teachers to try and to try very hard to "integrate technology into their classrooms" in order to improve their teaching and improve their students' learning, will be a main focus of this presentation.

Question 2: Mathematics has been taught based on the chapters of the textbooks. Usually there are few direct connections among chapters of books or among branches of mathematics. In collaborative, open-ended designed problems, how do the teacher and students build and maintain a common understanding of the task? From the teacher's perspective, how can she or he guide and assist them as they invent and design an artefact? From the students' standpoint, how do they know where to start? How do they know what to do and whether they have all the information to complete the task? How are they able to figure out which knowledge is useful or which is not? How do they decide on a goal?

We do not have answers for the above questions. However, we do propose an activity that can be used to tie in some of the mathematics curriculum which can be used as review for calculus students or a spiral curriculum for combining algebra and geometry, and which also motivates student to learn how to use technology.

3. Designed Activity: The Shape of "ICTM"

In the following activity, we combine interactive teaching and collaborative learning. Students become active participants in the learning process. In this activity, this pedagogy, proved by our experience to be helpful, is used to cover topics in pre-calculus, linear equations and inequalities, algebra, geometry and analytical geometry. The goal of this activity is to let students gain competence in mathematics, and be prepared to go on to successful completion of the calculus sequence. We believe that this activity meets most of the characteristics of NCTM standards. It represents a "big idea", uses processes that are appropriate to the discipline, is thought provoking, fosters persistence, develops thinking in a variety of ways, and has multiple avenues of approach, making it accessible to all students.

For general mathematics classes, we use this activity to get the students interested in learning some basic mathematics. For calculus classes, we use this activity as concept review and prior knowledge checking. For prospective teachers, we use this activity in their mathematics methods course to demonstrate how to integrate material into a spiral curriculum.

We now describe our activity in detail, in the following five steps

Step 1:

1) First, we create a rule for relating alphabets to sets of numbers. We use a simple, order-preserving rule for assigning numbers to letters of the alphabet, given by the following Table 1.

Table 1: Chart showing correspondence between alphabets and numbers

1	2	3	4	5	6	7	8	9	10
A	b	c	D	E	f	g	h	i	j
K	l	m	N	O	p	q	r	s	t
U	v	w	X	Y	z				

2) Next, we find a point set corresponding to ICTM (the acronym of "International Conference on Teaching of Mathematics"). The letters I, C, T, M, according to Table 1, correspond to numbers 9, 3, 10, 3. Let these be the x-coordinates of our points. Start from the second number to form the y-coordinates (No specific reasons, just to have the y- coordinates. You can use any method to get y coordinates as long as it is consistent). In order to make the first point's coordinates and the last point's coordinates the same, we will have to add the first number at the last. Therefore the points of (9,3), (3, 10), (10,3), (3,9), (9,3) will form a closed basic shape of our name, ICTM.

Step 2:

Connecting the five ordered pairs (x, y) on the following coordinate plane, we construct the basic figure of our graph (See Figure 1). We can identify some mathematics concepts here. We hope our students will able to pose some questions. For example, what are the lengths of these four lines? What are the slopes of these four lines? What are the equations of these four lines? Do we have parallel lines? Do we have perpendicular lines? What is the intersection point of any two lines? What kinds of geometric shapes do we have? Do we have isosceles triangles? Do we have trapezoids?

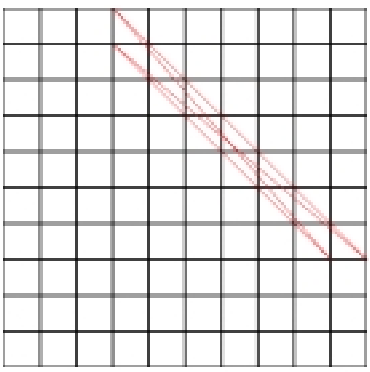

Fig 1: Basic Shape

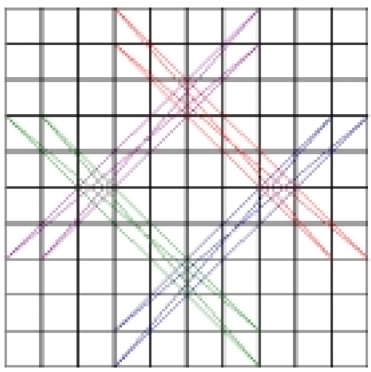

Fig 2: Transformed Shape

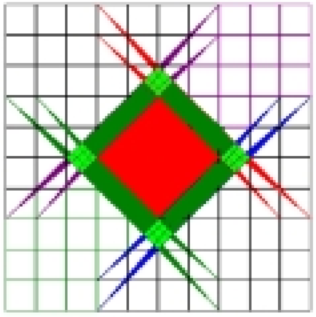

Fig 3: Squared Table

Step 3:

In order to make the shape of ICTM more complicated and interesting, we form the following Table 2, adding on three extra correspondences of ICTM and the numbers. Now we connect all the other transformed ordered pairs of (y, 10-x), (10-x,10-y), (10-y, x) to form Figure 2. What do you see? We can ask the same questions as for the basic shape and ask further questions. For example, are there any squares in this transformed shape? We can color the graph to make some beautiful pictures (See Appendix). What is the name of this picture? Does this picture fit the theme of our "International Conference on the Teaching of Mathematics"?

Table 2: ICTM and the transformed ordered pairs

X	y	(x, y)	(y, 10-x)	(10-x, 10-y)	(10-y, x)
I	c	(9,3)	(3,1)	(1,7)	(7,9)
C	t	(3,10)	(10,7)	(7,0)	(0,3)
T	m	(10,3)	(3,0)	(0,7)	(7,10)
M	i	(3,9)	(9,7)	(7,1)	(1,3)
I	c	(9,3)	(3,1)	(1,7)	(7,9)

In the above Figure 3, the author sees the design of our shape of ICTM as Squared Tables. The squared tables at which mathematicians all over the world are sitting here to share ideas, to talk about teaching strategies, to learn more about our students, to assess new technologies, to make a better environment for teaching and learning mathematics.

Step 4: Exploring the transformed shape and posing mathematics questions

We ask the students to explore their name shapes and find out what kind of mathematical concepts show up. Can the students pose their own mathematical questions, such as are there equal line segments, deciding if there exist parallel or perpendicular line segments, or are there any parallelograms and so on. In the ICTM case, we have the following mathematics concepts show up: 1) plotting points; 2) connecting two points; 3) finding distance between two points; 4) finding slopes of line segments; 5) finding equations of line segments; 6) looking for perpendicular or parallel line segments; 7) solving systems of linear equations; 8) area of geometrical figures; 9) angle of two segments; 10) different geometric figures; and more.

Step 5: Doing mathematics

We pose a problem that uses most of the above mathematic concepts. With the help of a calculator, the students can plug in the formula to get the answers quickly. Rather than giving credits for the answers, we ask students to observe the relationships between the answers. Or we can, for instance, ask them to predict whether two given segments AB and CD have the same length, and use the formula to find out if their prediction is correct and similar questions of making and checking predictions. We can generate more mathematics by, for example, asking students to decide whether the triangle formed by vertices $A(x_1, y_1)$, B (x_2, y_2) and $C(x_3, y_3)$ is an isosceles triangle or an equilateral triangle. Different students might have different figures to work on.

The question: In our ICTM case, to prove that the triangle formed by vertices A (3,1), B (9,7) and $C(x_1, y_1)$ is an isosceles triangle, where $C(x_1, y_1)$ is the intersection point of line segments formed by points (3,1) and (10,7) and the line formed by points (3,0) and (9,7), respectively.

In order to prove it is an isosceles triangle, we need first to find the third vertex which is the intersection point of line segments formed by points (3,1) and (10,7) and by points (3,0) and (9,7), respectively. In order to do that, we need to solve a system of linear equations. Second, we need to find the equations of these two line segments. In order to find the equations of line segments, we need to find the slopes of line segments. Reverse the operation processes, we find the slopes first and proceed until we find the intersection point.

(1) Find the slopes: $m_1 = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-7}{3-10} = \dfrac{6}{7}$ $m_2 = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{0-7}{3-9} = \dfrac{7}{6}$

(2) Find the line equations:

Line 1: $y = mx + b \Rightarrow y - 1 = \dfrac{6}{7}(x-3) \Rightarrow 6x - 7y = 11$

Line 2: $y = mx + b \Rightarrow y - 0 = \dfrac{7}{6}(x-3) \Rightarrow 7x - 6y = 21$

(3) Solve a system of linear equations:

$$\begin{cases} 6x - 7y = 11 \\ 7x - 6y = 21 \end{cases}$$

With the help of a calculator, we can find the intersection point is ($^{81}/_{13}$, $^{49}/_{13}$). Our next question is whether the triangle formed by (3,1), (9,7) and ($^{81}/_{13}$, $^{49}/_{13}$) is isosceles? How do we start? Again with the help of a calculator, we can find that the lengths of the segments formed by (3,1) and ($^{81}/_{13}$, $^{49}/_{13}$) and by (9,7) and ($^{81}/_{13}$, $^{49}/_{13}$), respectively, are equal. We can continue to pose questions; for example, what is the area of this isosceles triangle?

The area of this isosceles triangle= $\dfrac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 7 & 1 \\ \dfrac{81}{13} & \dfrac{49}{13} & 1 \end{vmatrix} = \dfrac{18}{13}$

Our last question for the moment will be, what is the acute angle of this isosceles triangle?

Angle Formula: $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

From the previous calculation we know a=b=4.2552, and c=8.4853. Thus

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{4.2552^2 + 4.2552^2 - 8.4853^2}{2 \times 4.2552 \times 4.2552}$$

Again the calculator helps us to find the angle C that is 2.98934 radians or 171.1968866 degrees. And we can ask the students to explore whether the other two angels are equals and as reinforce checking for the concept of equal sides of triangle have the equal angles. Notice how this question about the isosceles triangle requires students to use much of their prior knowledge.

4. Discussion

In this activity, we ask our students to explore lines that have the same slope, and be able to conclude that they are parallel lines. Or we can ask students to find the slope of two apparently perpendicular lines and explore the relationship between the two numbers and find out what pairs of perpendicular lines have in common algebraically.

We would like our students to find out for themselves why we need to solve systems of linear equations. When the intersection points are on the grids of coordinate system, it is easy to identify the coordinates. However, if points are not on the grid of the coordinates what can we do? These kinds of questions lead to solving systems of linear equations. Students now see a purpose for systems of linear equations, and they will be willing to work on them. If a few problems are not enough to make the students familiar with the mathematics concepts, we can always find further questions or problems. For example, we could ask them to do more, to observe more, to conjecture more and to learn more, there is always more to learn.

In the ICTM case, if we rotate the basic shape 90 degrees, 180 degrees and 270 degrees, then we get the transformed shape. The shape is symmetric horizontally, vertically and diagonally with respect to x=5, y=5 and point (5,5). We can explore different ways to transform names into shapes. For instance, in Step 1, we could choose the y-coordinates to be some other cyclic permutation of the x-coordinates. We could modify Table 1 to make the alphabet letters correspond to numbers 0 through 9, for instance. It would be interesting to see how such changes in the correspondence affect the shape of the names. Also, instead of permuting the x-coordinates to get the y-coordinates, we can let y be a pre-determined function of x. For example, if we take y=x, we have one straight line instead of a closed polygon, and with the transformations we get a figure consisting of two perpendicular lines intersecting at (5,5). If, as another instance, we let y=x/2 and perform the transformations we get four lines enclosing a square. Another interesting function to use is $\left[\frac{x^2}{10}\right]$ (integer part).

5.Conclusion

To be successful problem solvers, students need to develop inquiring habits of mind. Many educators believe students learn better when they have a personal interest in the assigned projects. In our background and literature review we have presented some main features of a good problem according to some educators.

The challenge to teachers is to come up with good problems and activities. For some years the author has been experimenting with an activity based on designs formed by the students' names. We have had enough success to believe this fits Clements' criteria for a good problem. Due to the individualization of this activity, we hope your name produce some interesting mathematics that might be totally different from the example we presented here.

In this paper we have illustrated our name-design activity by focusing on one example; the acronym of our organization. We have seen how a number of interesting questions come up, and we have explored a few of them. Answering all these questions by hand calculations would be too tedious, and students would lose momentum: so we are naturally led to the appropriate use of calculators.

Not mentioned above in our paper, but a facet that makes this activity appeal to students, is that very beautiful designs frequently appear when regions of the name-graphs are colored in various ways. In an appendix, we show several designs based on "ICTM".

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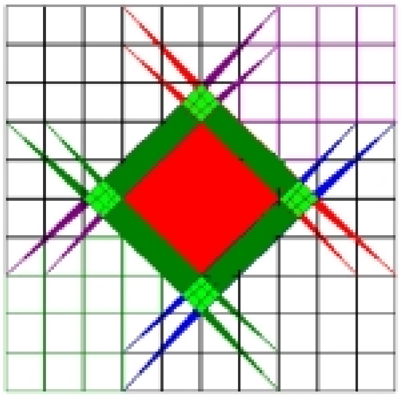
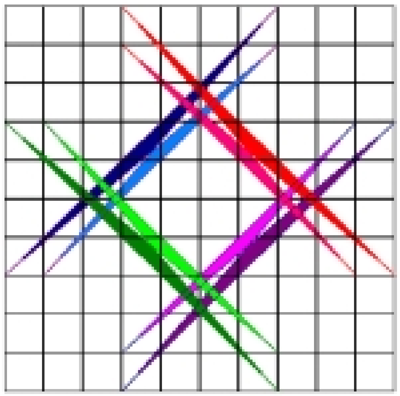
Appendix

The designs of ICTM

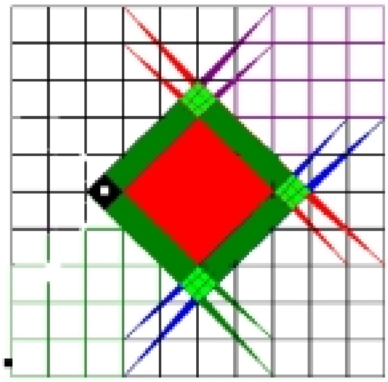
The Squared Tables



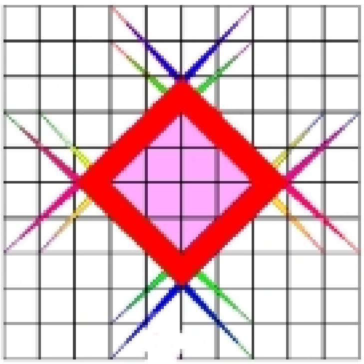
The Isosceles Triangles



The Tropical Fish (Side View)



The Tropical Fish (Front View)



The Trapezoids