BLENDING TECHNOLOGY AND PURE MATHEMATICS:
Is the hard work worthwhile?

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ABSTRACT

Incorporating a computing component into an undergraduate pure mathematics course is well-established practice. Reasons given for introducing technology include freeing students from the grind of hand calculations so that they can tackle more realistic problems, exposing students to the possibility of exploratory work, and allowing graphical as well as numerical representations of the mathematics. Although a small number of courses have abandoned lectures and are taught entirely in the laboratory, most still retain the traditional format and present the computing component as a supplement.

Integrating the computing work with standard lectures and pen and paper exercises requires a clear understanding of the aims of each type of learning activity. Questions to be considered include: what is an appropriate balance between teaching the students about the software and teaching them mathematics, what do students believe they are learning from computer-based sessions, and are students' perceptions of the purpose of this type of activity markedly different from that of the teacher? Designing a new Matlab-based computer laboratory program for an undergraduate linear algebra course with an enrollment of 850 students presented both a technical challenge and an opportunity to investigate these important questions. Student reaction, both critical and favourable, is discussed.

1 Background and purpose

The practice of integrating computing components into undergraduate pure mathematics courses (usually calculus and linear algebra courses) goes back at least twenty years and with ongoing improvements in software is becoming increasingly common. The primary motivation is usually to improve the learning outcomes for students. One hopes that the technology will free students from the grind of hand calculations so that they can tackle more realistic problems, expose them to the possibility of exploratory work, allow graphical as well as numerical representations of the mathematics, and provide more variety in the students' learning experiences.

The commitment in time, energy and resources to run a computing component is substantial, and so it's important to know if the aims are being met. Alexander (1999) reported results of a survey of 104 teaching development projects involving technology (90% of which had the stated aim to improve student learning) in university courses over a broad range of disciplines, which revealed that only a third could report an improvement in quality of learning outcomes because only a third actually tested for this. The remainder restricted their evaluation to a basic student feedback questionnaire of the type that focused on student reaction to the innovation. Alexander suggests a range of fourteen different methods of evaluation of student learning outcomes, including comparative studies, pre- and post-testing, focus groups, expert reviews, observations of student use, and student questionnaires testing experiences and perceptions as well as reaction.

Anecdotal evidence and evidence based on student surveys suggest that a sizeable proportion of students are lukewarm on the use of computers in maths courses. Coupland (2000) reports that asking students for an overall view of their experiences with *Mathematica* in first year courses produced positive, neutral and negative responses in the ratio 25 : 27 : 47. In the study by Galbraith et al (1999), the open-ended question "How do you feel about using computers to learn mathematics?" elicited 15 positive responses, 14 negative responses and 5 containing both positive and negative comments. In a University of Sydney linear algebra course held during 2000 using in-house software, students were asked if the lab sessions had helped them understand the course. There were 110 positive, 79 neutral and 63 negative responses.

The question of appropriate evaluation became relevant when a new Matlab-based computer laboratory program was introduced in 2001 into a large second year linear algebra course at the University of Sydney. Although a computing component had been part of the course for many years, there were several reasons for replacing it with a new program. Firstly, the Engineering departments had moved to Matlab and wanted their students to use the same system in mathematics. Secondly, it was felt that all students would benefit from an introduction to a commercial program widely used in industry. Thirdly, the previous program had no graphics capability and was somewhat dated; the increasing experience and sophistication of students as computer-users meant that attention had to be paid to visual as well as numerical aspects of the program. The new lab program had two aims: to familiarize students with basic Matlab commands and to improve their understanding of the linear algebra concepts.

Prosser (2000) commends the usefulness of open-ended questions in order to accurately reflect student beliefs and perceptions, especially in the evaluation of new technologies. He notes that "level of agreement" questions produce judgements by students on issues determined by academics as important, which may not coincide with issues students consider important. The purpose of this study is to attempt to discover and analyze students' perceptions and experiences of the lab program using both student-focused questions and questions measuring student reaction, as a first step towards evaluating whether the aims of the programs have been met. Other evaluation methods such as those mentioned by Alexander are expected to be used at a later stage.

2 Method

At the end of the course in which the new lab program ran for the first time (semester 1, 2001), 362 students (218 engineering and 144 science students) volunteered to complete a pen and paper questionnaire. Students were asked to indicate if they were enrolled in engineering, but no other personal data were recorded. The questionnaire contained 19 statements, of which 12 related specifically to the computer laboratory sessions. Students indicated their level of agreement with each statement. The responses were scored 0,1,2,3 or 4, a score of 0 corresponding to strong disagreement and a score of 4 to strong agreement. In addition, three open-ended questions invited students to say what they liked most and disliked most about the lab sessions, and to suggest improvements.

In the following running of the course (summer session 2002), a further questionnaire containing open-ended questions on students' experience of the lab program was completed by a much smaller number (n=28). Seven of these were repeat students, while twenty one were new to the course.

2.1 The students and the course

Students in the course are drawn mainly from Engineering (55%) and Science (42%) degrees. This course is compulsory for engineers, roughly 50% of whom had prior Matlab experience. There are two lectures, one pen and paper tutorial and one computer laboratory session per week, for one semester. The lectures cover standard material: elementary vector space theory, linear transformations, diagonalisation and applications of the theory to the solution of recurrence relations, systems of linear differential equations and quadratic forms. Over the years, many students have said that they find this material abstract and somewhat difficult to understand. Labs contribute 10% to the overall course assessment, the balance coming from quizzes, tutorial participation, written assignments and final examination.

2.2 The lab program

The new program uses Matlab with a graphical user interface to provide a step-by-step path through each problem, giving immediate feedback to students on the correctness of their data entry and allowing for automatic registration of completion of questions and recording/marking of their answers to specific assessment tasks. These features were incorporated to manage the large enrollment, and permitted the laboratory sessions to run (after the first month) without tutorial staff. Around 50 problems (numerical, graphical and experimental) and two special animations were devised, tested and incorporated into the program. Students complete four or five problems each week, either at a scheduled time or at any other time when lab space is available. At present, students can access the program only on campus.

3 Results

3.1 Results of the first questionnaire

Statements asking for level of agreement

Statements concerning the computer labs which scored the highest and lowest averages are given below. There were no significant differences in the responses of engineers versus non-engineers to any question except that which asked about previous Matlab experience.

Highest averages, indicating agreement (average score over n=362, standard deviation):
I would prefer to be able to do the lab work from home via the web (2.92, 1.20)
I now feel reasonably familiar with the basic Matlab commands (2.76, 0.89)
The mix of 2 lectures, 1 tutorial and 1 lab session per week was just right (2.69, 0.91)
I appreciated the structured nature of the lab problems (2.62, 0.88)

Lowest averages, indicating disagreement (average score over n=362, standard deviation):
I was an experienced Matlab user before the course started (1.35, 1.43)
The lab questions are too difficult to understand (1.45, 0.88)

Some of the remaining statements, with averages closer to the "neutral" score of 2, were statements that related in important ways to the pedagogical success of the lab program from the students' point of view. The statement
"The lab sessions helped me to understand the course" (2.20, 1.03)
included 169 positive, 100 neutral and 93 negative responses. The statement
"The lab work was interesting" (2.10, 1.00)
included 139 positive, 128 neutral and 95 negative responses. The statement
"The graphics in the lab sessions helped me to understand the maths" (2.20, 1.03)
included 157 positive, 107 neutral and 98 negative responses.

Answers to the open-ended questions

From the open-ended questions, a total of 303 responses to the question "What did you like most about the lab sessions?" and 304 responses to "What did you dislike most about the lab sessions" were recorded. There were 161 suggestions for improvement to the lab sessions. The students' comments were classified under the following general headings. The numbers in brackets indicate the number of times the response was written. The spread of responses from engineers appeared not to differ markedly from the non-engineers and so the numbers recorded are combined. Students usually wrote at most one comment for each question.

Liked most about the lab sessions:
Easy-to-use system, questions were quick and easy to do (94)

Interesting questions that helped understanding of concepts (68)
Ability to work at own pace and at flexible times (44)
Step by step structure of the questions (43)
The graphical questions and animations (22)
Learning Matlab (14)
Ability to use computer and maths together (8)
Ability to experiment and solve realistic problems (6)
Labs contributed to the assessment (4)

Disliked most about the lab sessions:
Old hardware, lab ambience, lab location, occasional bugs (72)
Step by step structure of questions (49)
Lack of tutorial assistance after first month (47)
Questions sometimes boring or too easy (46)
Problems sometimes too hard (24)
Lack of feedback on whether answers were right or wrong (19)
Can't do labs off campus (17)
Method of assessment of labs (11)
Having to use pen and paper as well as computer (11)
Labs not relevant to lectures (8)

Suggestions for improvement to the lab sessions:
Employ tutors for the whole semester (35)
Arrange access to lab program from home (24)
Buy better computers for the labs (20)
Have more challenging questions (18)
Change the way lab work is assessed to provide better feedback (17)
Abolish the lab program (16)
Replace the existing GUI (8)
Provide more graphical questions (7)
Provide a hard copy of the help manual/question bank (7)
More problems on applications (6)
Make lab work more relevant to lectures and tutorials (3)

Some of the replies to the open-ended questions (with both positive and negative views of the program) were very thoughtfully constructed, others were very brief. It is also possible that these responses were influenced somewhat by the content of the previously-answered written statements, which reminded them specifically about particular issues concerning their lab work. For this reason the second questionnaire, conducted during the next running of the course, attempted to gauge students' opinions of the wider issues relating to the lab sessions, free of the influence of a structured survey.

3.2 Results of the second questionnaire

Twenty eight students volunteered to provide responses to the following three open-ended questions. Students' comments were again classified under general headings, to indicate the range of their replies. Students usually wrote one comment at most. The brackets indicate the number of times that answer was mentioned.

What do you think is the purpose of the computer lab component?
(Comments arranged from emphasis on understanding of mathematics to emphasis on learning Matlab)
To help students understand the course (7)
To help students understand the theory by using a computer to eliminate errors (3)
To revise lecture material by doing complicated questions that cannot be done by hand (2)
To solve practical problems related to the theory in the course (2)
To assist visualization (2)
To expose students to advanced software and lift awareness and understanding of how technology can come into mathematics (2)
To be able to answer questions faster and more efficiently (2)
To gain experience with a computational package that will be used in the real world (8)

Your course contains lectures, tutorials and computer laboratories. What relationships should there be between these components?
Temporal/content relationship: lecture first to present a topic, then pen and paper tutorial with exercises to reinforce the same ideas, followed by lab for practical applications (11)
Content relationship: all components should reinforce each other to widen understanding (6)
Balance is wrong: there should be 1 lecture, 2 tutorials and 1 lab (1)
Matlab should be referred to in lectures to show ways in which it can be used (1)
Lab problems are appropriately easy mathematically, because new computing skills are being learned simultaneously with the mathematics (1)

What do you believe you are learning from the computer lab sessions?
Increasing knowledge of Matlab and seeing how it's used to solve real life problems (19)
Increasing understanding of concepts presented in lectures (8)
Observing patterns, seeing how matrices work, predicting (1)
Nothing much (1)

4 Discussion and Conclusions

In the first questionnaire, approximately one quarter of students had a negative response to the statements "The lab sessions helped me to understand the course", "The lab work was interesting", and "The graphics in the lab sessions helped me to understand the maths". It seems that the computer laboratory program has failed to engage a significant number of students, confirming similar data mentioned in the introduction. Galbraith et al (1999), in a study on attitudes to computer use and to mathematics, present correlations which suggest that positive attitudes to computers are more influential than positive attitudes to mathematics in determining active involvement in the use of computers to learn mathematics. It would be interesting to investigate this further to determine if other factors are also involved.

The most appreciated feature of the lab sessions (94 mentions) was that the work could be completed relatively quickly and easily. Linking this with the 44 favourable mentions of the ability to work at their own pace at flexible times, and the quite strong agreement with the statement "I would prefer to be able to do the lab work from home via the web" suggests that students value highly a reasonable workload that can be managed in their own way at their preferred time. The 68 favourable mentions of questions that helped understanding of concepts, together with the 22 favourable mentions of graphical questions and animations (and other related comments with smaller frequency), suggest that about a quarter of students value the "mathematics plus computer" experience. Many of the features that were disliked were echoed in suggestions for improvement. The most disliked feature, the hardware and the lab environment (72 unfavourable comments), was reinforced by 20 suggestions for improved quality of computers. This type of complaint should become less frequent with progressive upgrading of the equipment. There appear to be roughly the same number of students holding opposite extreme views about the lab program (16 abolitionists and 18 who want more challenging questions), and roughly the same number who were pleased by the structure provided by the GUI as were irritated by it.

Though the sample was much smaller for the second questionnaire, some interesting features can be observed. In answer to the question about perceptions of the purpose of the computer lab component, the two most frequent comments identified each of the two aims of the program, but students did not perceive the possibility of a dual purpose. For the question on the relationships between the different components of the course, most responses focused on the temporal/content relationships. This suggests a preference for a course structure in which topics are well defined by lectures and tutorials, and the role of the labs is to demonstrate their applications. Only 6 of the 28 students mentioned the lab program as a source of understanding of the mathematics. In the question asking what they believed they were learning from the lab sessions, responses focus predominantly on the use of Matlab itself and its capabilities in solving practical problems rather than Matlab as an aid to understanding the mathematics.

Do the responses from the two questionnaires help to determine whether the aims of the lab program have been met? A "level of agreement" statement in the first survey shows that even allowing for the subset of engineers who already had prior Matlab experience, the students as a whole claimed familiarity with the basic Matlab commands by the end of their course, and this is reinforced by the 19 responses in the second questionnaire claiming to be learning and using Matlab. This suggests that the first aim of the lab program has been met. However, disappointingly few students perceive the computer lab component as helpful in learning linear algebra. Further testing along the lines proposed by Alexander (1999) will be required to determine whether this indicates an actual lack of learning or simply a lack of perception.

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