# TEXT STRUCTURES IN TEACHING MATHEMATICS

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## ABSTRACT

Any good learning material must try to anticipate the learner's problems. The author should take into account that the reader is not with him and understanding his good intentions.

Any given text can be understood as an alphanumeric string that is a rather annoying structure. We can distinguish three dimensions of the text: line, column, and the block one. There are usually many internal relationships between parts of the string.

The transformation between linear and structured text can be explained as two opposite processes: aggregation and decomposition. Natural destruction of the text linearity can be applied to implications, classifications and parallel formulations. The modern word editors offer a large amount of possibilities for structuring texts.

For several years, the Department of Mathematics of the TU in Liberec tries to observe the influence of a mathematical text written in structures on an acceptation of lectures and textbooks. The research started in 1999 and continued in 2000 and 2001 with the goal to verify what type of a mathematical text is better for students – classical linear or structured. The results of the student's polls are presented and discussed which were passed through in exercises of Mathematics. Hundreds of students of five faculties at the TU in Liberec participated in them. The last polls of our research show a shift in the direction of the structured form. According to the student's answers the structured versions of the text are appreciated. We could also read many remarkable, wonderful answers in the student's questionnaires. It could be very interesting for psychologists and pedagogues.

**Keywords:** Text structures, aggregation, decomposition, students' poll.

# 1. Introduction

*Motto*
**One structure is better than a thousand of words.**
(Paraphrasing Confucius 552-479 B.C.)

In the last ten years of the twenties century, the possibility (sometimes necessity) of the lifelong learning has started to leak in the awareness of the Czech public. The distance learning became the modern form of knowledge acquisition. Hand in hand with it, the progress of the undergraduate mass education or self-learning in mathematics took part in the Czech education system. Transforming the usual full-time studies to the distance learning the requirements to the intelligibility and suitability of learning materials are increasing rapidly. It is clear that many specific features of open learning material could be used for the full-time studies too. From the psychological view, it is evident that a book-like text can attract readers by its design, size (numbers of pages), a graphical form etc. – shortly by its presentation.

Deciding between two textbooks with almost the same contents, a student/reader will choose inadvertently that book which is written in the more readable and understandable form. (Evidently, there is a difference between expert and student decision-maker.) The famous Confucius (552-479 B.C.) saying, "One picture is better than thousand words", stresses the importance of usually neglected attribute of information (esp. textbook), i.e. of its structure and graphical presentation. Everybody knows that the first-quality textbooks and all learning material, the strong basic literature, the brief and effective textbook, a detailed commentary to solved examples, this all can help students to deal with their studies easier and more effectively.

The development of the distance learning at the Technical University in Liberec induced the necessity of writing of mathematical texts several years ago. That is why some teachers of the Department of Mathematics and Didactics of Mathematics of the Faculty of Education started to observe/examine influence of mathematical texts written in structures on an acceptation of lectures and textbooks. It is a well-known fact that the reading of a mathematical text is for non-prepared readers generally and objectively difficult. We have investigated some graphic arrangements emphasising composition of the text and influencing the efficiency of learning. Similar principles could be used for an arbitrary vocational text.

Frequently external observers think that a typical mathematical explanation is of the form "definition – theorem – proof" with prevailing linear writing. But the practice shows that it is more suitable to state a well arranged summary of properties, a summary in tables, mini-graphs etc.

We can look at any given text from three dimensions - line, column, and the block one, and find relationships between them. This text can be structured along the string, across its lines, and on the long distance (between blocks). There exists a transformation between linear and structured text from this point of view. This transformation is based on in principle two opposite processes - aggregation and decomposition (see [Vil]). The structuring of a text means the usage of a natural destruction of the text linearity to emphasise differences, classifications etc. The means may be as standard (tables, Cartesian products of small sets, trees, graphs and mini-graphs etc.), as well as not so usual – different levels of formulations, parallel and/or alternative formulation (the so-called "storey notation") in definitions and theorems, accompanying solutions by intermediary comments, and so on.

The following examples demonstrate the difference between linear version and structured one.

Classical linear version [ThFi-90:$189_{8\text{-}6}$]

**Second Derivative Test for Local Maxima and Minima**
If $f'(c) = 0$ and $f''(c) < 0$, then $f$ has a local maximum at $x = c$.
If $f'(c) = 0$ and $f''(c) > 0$, then $f$ has a local minimum at $x = c$.

Commentary to C-version:
This is a typical partly formalised linear text where both of alternatives are in series.

Structured version with miniatures:

**Second Derivative Test for Local Maxima and Minima**

If $f'(c) = 0$ and $f''(c) \begin{cases} < 0, \\ > 0 \end{cases}$ then $f$ has a local $\begin{cases} \textit{max}\text{imum at } x = c. \\ \textit{min}\text{imum} \end{cases}$ 

Commentary to the S-version:
The linearity is survived only at the headline, the following two rows are rewritten almost table-like. The differences and alternatives are column close, they are seen at first sight. The miniature gives the eyes view.
The following (see [ViBi2]) deals with overview of possibilities of a function with respect to real/complex arguments, the number of its variables and range of it shortly and well-arranged.
The structure is Cartesian-like combination of adjectives {real; complex} and dimension {1; $n$}, giving 16 possibilities, these can be rewritten as a structure in only two lines:

$$\left\{\begin{matrix}(\text{real}) \\ \text{complex}\end{matrix}\right\}\left\{\begin{matrix}(\text{scalar}) \\ \text{vector}\end{matrix}\right\} \text{ function of } \left\{\begin{matrix}(\text{real}) \\ \text{complex}\end{matrix}\right\}\left\{\begin{matrix}(\text{scalar}) \\ \text{vector}\end{matrix}\right\} \text{ variable.}$$

The usually left out adjectives/parts are in braces. The symbolic version for a function $f$ with the domain $\mathrm{D}(f)$ and range $\mathrm{H}(f)$ can be concentrated into the following schema:

$$\left\{\begin{matrix}\mathrm{R} \\ \mathrm{C}\end{matrix}\right\}\left\{\begin{matrix}(1) \\ n\end{matrix}\right\} \supset \mathrm{H}(f) \overset{f}{\leftarrow} \mathrm{D}(f) \subset \left\{\begin{matrix}\mathrm{R} \\ \mathrm{C}\end{matrix}\right\}\left\{\begin{matrix}(1) \\ n\end{matrix}\right\}, \qquad 1 < n \in \mathbf{N}.$$

Python-like structure can be seen from time to time in textbooks – e.g. [MV-95:$261_{2\text{-}1}$]:

Die Summe aller $\left\{\begin{matrix}\text{auf einen Knoten} \\ \text{in einem Stab}\end{matrix}\right\}$ wirkenden Kräfte ist Null.

This can be applied in analysis where the following notion nest is discussed:

{(two); one}-sided (im)proper limit approached
from {(two); one} side(s) at an (im)proper point.

$\left\{\begin{matrix}(\text{two-sided}) \\ \text{one-sided}\end{matrix}\right\}\left\{\begin{matrix}(\text{proper}) \\ \underline{\text{im}}\text{proper}\end{matrix}\right\}$ limit approached from $\left\{\begin{matrix}(\text{two sides}) \\ \text{one side}\end{matrix}\right\}$ at an $\left\{\begin{matrix}(\text{proper}) \\ \underline{\text{im}}\text{proper}\end{matrix}\right\}$ point.

[IrRo-98:$51^{11\text{-}13}$] Hidden trichotomy (and three valued range).

**Definition.** The symbol $(a/p)$ will have the value 1 if $a$ is a quadratic residue mod $p$, –1 if $a$ is a quadratic nonresidue mod $p$, and zero if $p \mid a$. $(a/p)$ is called the *Legendre symbol*. ♠

In other books the three valued range is made clear and the meaning of the symbol commented. (Note. The fork with three teeth replaces the former parentheses.)
[Kob-98:$43_{13\text{-}7}$] Clear-cut trichotomy (and three values).

**The Legendre symbol.** Let $a$ be an integer and $p > 2$ a prime. We define the *Legendre symbol* $(a/p)$ to equal 0, 1 or –1, as follows:

$$(a/p) = \begin{cases} 0 & \text{if } p \mid a; \\ 1, & \text{if } a \text{ is a quadratic residue } \textit{mod } p; \\ -1, & \text{if } a \text{ is a nonresidue } \textit{mod } p. \end{cases}$$

Thus, the Legendre symbol is simply a way of identifying whether or not an integer is a quadratic residue modulo $p$. ♠

Commentary.

– the almost perfectness is disturbed only by a small *aaa* collision, distinguishing by italics only is often not sufficient.

# 2. The Realisation and Evaluation of the Research

The research started in 1999 with the goal to verify what type of a mathematical text is better for students – classical linear or a structured one. Writing the textbook for students we would also like to know the students' view. Therefore we prepared four students' polls to verify our hypotheses. These hypotheses were drawn from the long-term experience of significant psychologists and pedagogues, and also from our own practice, passed through in mathematical exercises at several faculties of our university. The first period of this research consisted of four parts and the end of it was in 2001. At the present times we continue in the second period. We show results and opportunities of the first period. The tenets and opinions of the students in four polls are presented. The discussions were organised in exercises of Mathematics. Hundreds of students of five faculties (three of them technical ones) at the TU in Liberec participated in them. The third (last 1999/2000) and the fourth polls of our research show a shift in the direction of the structured form. According to the student's answers the structured versions of the text are appreciated.

We prepared three different topics of Mathematics in classical versions and structured ones to verify type of mathematical text that our students prefer. The first theme was "mapping" (surjection and injection), the second one was "countable and non-countable sets", and the third theme was "Ratio Test and Root Test for number series". We also prepared a questionnaire for students. At the beginning of the lesson the students were divided to 2 groups – Structured and Classical. Then they got the questionnaires with empty upper parts where they wrote their answers to a task written on the blackboard. After 5 minutes they cut off these filled in parts of the questionnaires and gave them back to the teacher. If a subject matter was new and/or the students did not know it, they would give back empty papers. Immediately the teacher gave to each student one of two versions of the research text, structured or classical. After 10 minutes of their studying, the students completed the questionnaires. Then the exercise was running according to normal programme. In the last ten minutes, the students were asked to answer the same task as at the beginning of the lesson on the opposite side of their questionnaires. The teacher gathered them in 5 minutes, thanked the students for their favour and explained their prospective questions.

First of all we tried to present the pre-test to one group of students of the Faculty of Education to verify our questionnaire and the timetable. Then the first part of the own research took place in January 2000 at the Faculty of Mechanical Engineering (174 students) and the Faculty of Textile Engineering (212 students). First of all we wanted to obtain characteristics of students (types of secondary schools, level of their mathematical knowledge etc.), to obtain what forms of mathematical notation they prefer, whether students are able to read mathematical (and/or an arbitrary vocational) text. We wanted also to inform students with the intention of this investigation. It was very important for us to know if students would prefer a graphical

emphasising. That is why we chose the theme "mapping" well known from secondary schools and why we did not await considerable improvement. In the second poll (April 2000) students should study the definition of a (un)countable set. This topic was new for almost everybody and so the results were more credible. 241 students of technical faculties took part there. The third theme was investigated in May 2000. Students should acquaint with the Ratio Test and Root Test (105 students).

The experience with filling in questionnaires in the first poll was used to modify the questionnaire in the following polls. Several of them were impossible to evaluate. It was also a sorrow for us to find out that some students were not able to read any mathematical text at all regardless of its style. We did not find essential differences between the structured and classical groups in the first poll. Students must get used to mathematical notations and formulations independently on type of writing. It depends on the type of finished secondary school. Some students are not able to describe a term known from a secondary school and repeated in the first semester. There were cases when students saying, that this term is new for them, tried to formulate an answer before studying the given text. The second task was more interesting for them (we hope so) because it was something new. Many of students tried to explain these terms intuitively according to their names only. However the third poll passed through in accord with our expectations, although seven groups took part in. Several teachers had to finish classification and evaluation of students in the end of the semester.

Analysing the tests only, we see that the second poll does even not show essential differences between the classical and structured variants. However according to the student′s answers in their questions, the positive evaluations of the structured text have done. Looking at the "improvement" graph of the third poll of our research, a shift in the direction of the structured variant can be seen. Many students did not answer the first test but then they tried to formulate it. The most of them have got better in the second answers (the improvement about 5 points) but there were students who did not answer again. They said they had been tired. This theme was new for most of them and they did not want to study it after a semester test (students of two groups). It was very interesting that one of participants of our poll, who was in the classical group, used the structured form in her/his answers.

In 2000/01 (the fourth poll), we investigated somewhat-different view. Suppressing the concrete text in the Ratio Test and Root Test, we prepared three versions with them – all in the classical and structured form. One pair of them was without any background, one with an unmarked background, and the last pair had got the marked background. We wanted to obtain any information whether our students would choose the presented text on account of its subject only, and/or they look at the form of it. We expected influence of computers, websites etc.

The questionnaire contained also parts examining frequency of reading and browsing in websites. About 160 students took part in this poll.

The presented graphs express several views in this problem. Looking at the figures (see Appendix) we obtain the first information that our students prefer structures in texts. This evaluation corresponds with the Czech school scale (number 1 is the best). It means they like to study texts with appropriate applications of storey structure, and other underlining means etc.

The Fig. 2 shows the view according to investigated properties and also the ratio of linear or structured variants are seen there. We were interested in five (subjective) aspects – how is the given text (Classical and Structured) intelligible, objective, well arranged, in the ability to remember contents, and its aesthetics. Fig. 3 touches the fourth poll. Let us notice that lots of students prefer a simple background on the given text (an influence of websites?).

Fig. 4 shows the number of hours per week spared to Internet. Several interesting facts can be drawn. Our students think that more complete, lasting and also detailed knowledge can be obtained from books compared with Internet (Fig. 5). More than one third of students had read less than 100 books (fictions) during their whole life (Fig. 6). The minimal number of these fictions is one, maximal number 3 859. Minimum of vocational books during the whole life is also one, maximum 150 (15 in a year). Fourteen students dared to say that they had read more vocational books than fictions.

These graphs present the pilot view in this problem. It is necessary to elaborate assembled data and make the final evaluation more detailed. By that time we hope that the results of our poll will be useful not only for writing texts.

# 3. Conclusion

The modification of the text to be structured is asked at a practical view. We are going to realise further polls for students of the second years. They are more experienced not only in mathematics but also in other (special) subjects. We are convinced that students are able to understand how to read a structured text. Then they will appreciate its advantages. However, this process is long-termed, it needs teachers' systematic influence and students' practice.

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**Appendix – Illustrations of the Pilot Evaluation (Students' Polls)**
**Technical university in Liberec, Czech Republic**

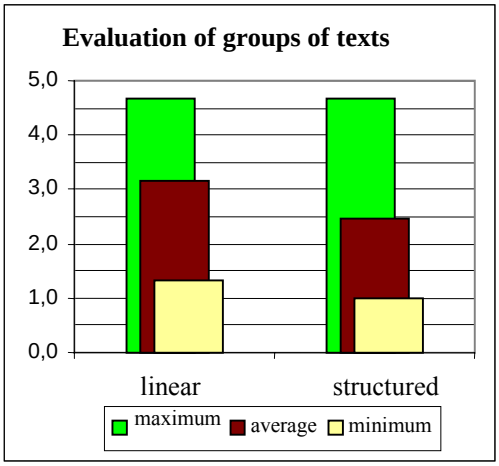



**Fig. 1** – Students' evaluation of linear and structured versions of a text

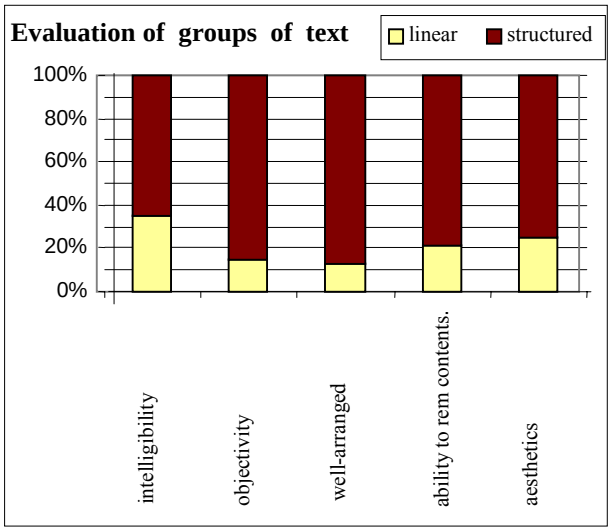



**Fig. 2 –** 5 aspects of the presented text

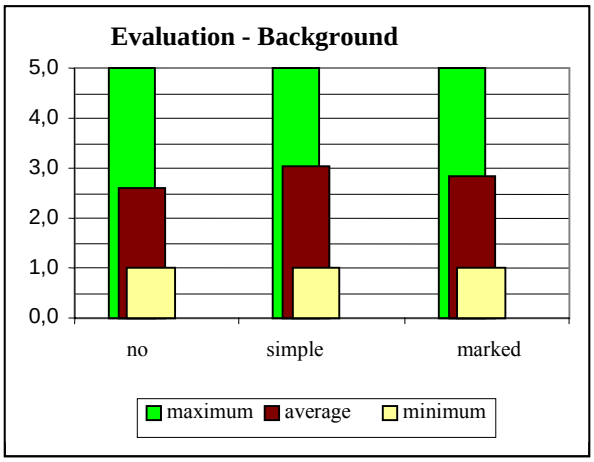



**Fig. 3** – Evaluation of linear and structured versions in 3 variants (no background, a simple background, and a marked one)

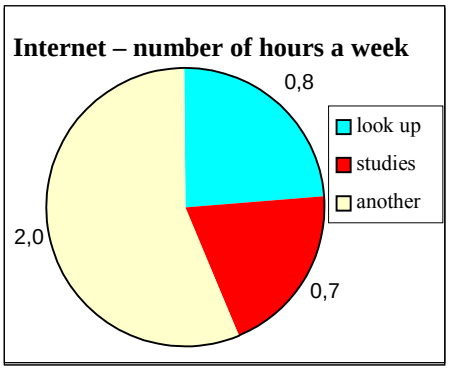



**Fig. 4** – The number of hours passing at Internet

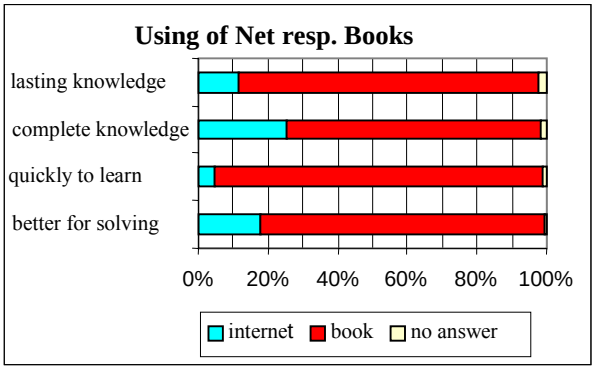



**Fig. 5 –** Properties appreciated by students using net or books

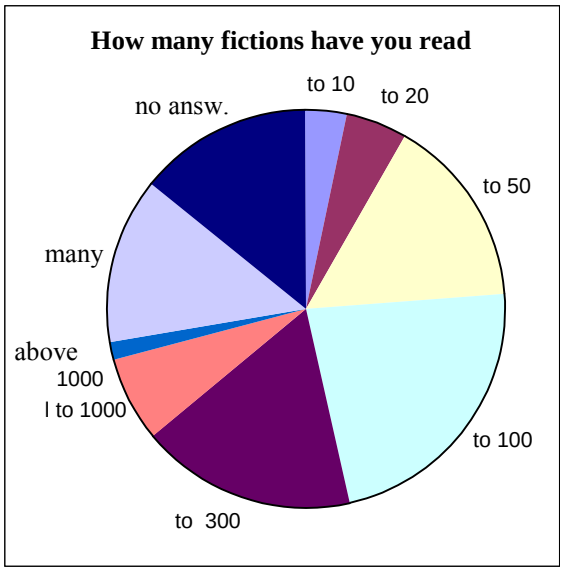
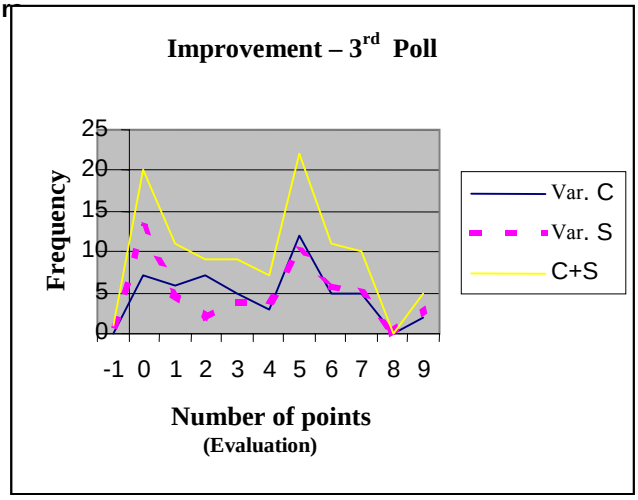



**Fig. 6 –** Number of books read by Czech students




| Improvement | *Var. C* | *Var. S* | *C+S* |
|---|---|---|---|
| -1 | 0 | 1 | 1 |
| 0 | 7 | 13 | 20 |
| 1 | 6 | 5 | 11 |
| 2 | 7 | 2 | 9 |
| 3 | 5 | 4 | 9 |
| 4 | 3 | 4 | 7 |
| 5 | 12 | 10 | 22 |
| 6 | 5 | 6 | 11 |
| 7 | 5 | 5 | 10 |
| 8 | 0 | 0 | 0 |
| 9 | 2 | 3 | 5 |
| **SUM** | 52 | 53 | 105 |

**Fig. 7** – Improvement of Knowledge in Dependency on the Form of Text