

# THE STUDY OF MATHEMATICS COMMUNICATION ON INTERNET WITH PALMTOP COMPUTER

**Masami ISODA**

Institute of Education, University of Tsukuba  
305-8572, Japan  
e-mail: msisoda@human.tsukuba.ac.jp

**Young-suk LEE**

Doctoral Program in Education, University of Tsukuba  
305-8572, Japan

**Kazuhiro AOYAMA**

Doctoral Program in Education, University of Tsukuba  
305-8572, Japan

**Manabu ISHIZUKA**

Tochigi Minami High School  
Tochigi-Shi, Japan

## ABSTRACT

Internet technology enables us to develop distance education system with the web site. A number of experimental studies for virtual university on web sites already existed. On the one hand, students need help of tutors or teaching assistants to learn mathematics collaboratively in each course. Instead of graphing calculator, the palmtop computer which enables access to the Internet is expected as strong next generations' mathematical exploration tools for collaboration in classroom (no computer lab) or for tutoring on distance education. For technological innovation of mathematics teaching on this context, the experimental research of mathematical communication with palmtop and Internet environment is necessary.

To design a palmtop environment for mathematics communication over the Internet as the newest mediational means for mathematics and to analyze how it works, this study developed and improved BBS sites. By experimenting with these sites, difficulties are clarified from the perspectives of grounding (Baker et al, 1999) and mediational means (Wertsch 1991). The different BBS designs strongly influenced the quality of communication. In the pilot study, two experiments illustrated that it is not easy for novice users of the environment to get the common ground such as image that is necessary to communicate mathematical ideas but we can communicate and collaborate on mathematics even in a small palmtop environment if we are accustomed to that environment or the environment is good designed for communication task. From this study, two no mathematical content factors were clarified for enabling communication with it. The first involves ways of communication in mathematics such as asking for better mathematical explanations, asking for conditions to be checked, confirming what the other party is saying, and general greetings. The second involves that users have to accustom to use palmtops such as BBS and DGS. Before the experiments, we expected that we easily collaborate as well as the communication on desktops but experiments well illustrated that the different BBS designs strongly influenced the quality of communication. These results implicated the specific environment help us to find how we depending on hidden common ground based on paper-pencil and face to face communication.

# 1. Introduction

Today, some universities request each student to bring laptop computer. On the other hands, most undergraduate students in Japan have their own mobile telephone which enables access to the Internet and their own electric palm size dictionary. By 2005, each classroom in Japanese schools must have Internet equipment and calculator companies expected the palmtop computer which enables access to the Internet, instead of the graphing calculator, as the next mathematical exploration environment in the mathematics classroom. There are a lot of research studies in education regarding using the Internet on the desktop or laptop environment. For example, we find studies described as ‘Computer Supported Collaborative Learning’ (Dillenbourg, P., 1999), ‘Distance Learning’ and ‘Distance Education’ (Fabos, B. and Young, M. 1999, <http://mcs.open.ac.uk/icme/>). However, mathematics education research on the palmtop environment has just begun with new palmtop computers for mathematical exploration such as the CASIO Computer Extender (CEX). Indeed, at the undergraduate level, every mathematics course has a lot of teaching assistants who help many students understanding collaboratively. The palmtop computer with mathematics exploration tools must be a strong for their collaboration in distance situation.

With this pilot study, we aimed to develop an experimental environment for mathematical communication on the palmtop computer, to analyse how it works and to recognize what kind of support is necessary. We developed the Bulletin Board Communication System (BBS) on the web site using CGI script for the CEX and researched how it works for mathematical communication. For this purpose, we analysed two experiments from two perspectives Socio-Historical-Cultural perspectives by Wertsch, (1991) of the functions and restrictions of mediational means for describing features of developed environment; and the perspective of collaboration as the grounding process for mutual understanding through communication (Baker, M. et al 1999).

## 2. Developed Environments and Setting

The Computer Extender (CEX) exists only on a palmtop computer in 2001 that is able to use Mathematics tools such as a Computer Algebra System (Maple), Dynamic Geometry Software (GSP) and Graphing Tool, and can connect with the Internet using Internet Explorer in Microsoft Office for Windows CE 2.0. Based on the experience of our previous study in which the Internet is used for collaborative mathematical problem solving between Japanese and Australian classrooms (Isoda et al 2000), the web pages of BBS for problem posing and communicating solutions were developed with the CEX’s window size of 640x240 in mind.



Figure 1. First Design of the Top Page.



Figure 2. Second Design of the Top Page.

In the first experiment, the page design consisted of two parts divided horizontally, the upper for reading the problem and the lower for communicating solutions (Figure 1). It aimed to show more messages at once because we expected long messages as well as the experiences of the previous

study on desktop computers. We found that it was difficult for users to read the problem while writing their solutions. In the second experiment, the page design consisted of two parts divided vertically, the left side for reading the problem and the right side for communicating solutions at the same time (Figure 2).

There are a number of restrictions with Internet Explorer (IE) on Windows CE 2.0. We can download the file through BBS but we have to use Outlook for sending the file. We have to inform recipients to renew BBS content by telephone because IE on the CE 2.0 does not accept automatically renewed settings. CEx with Windows CE 2.0 has a QT keyboard and we can input by Pen on display, but the drawing tool by Pen does not exist.

In the developed environment, BBS worked as the mediational means for communication between both sides. For the experiments, we preferred graduate students who had experienced learning mathematics in English (because CEx is only available in English fonts) and at the same time who were novice users of the desktop computer in mathematics (because experts can be expected to work well in any computer environment). Because we had to teach them how to use mathematics software, we set the communication between the teacher and a subject (student) with the help of a tutor. The subject used the CEx but the teacher used a desktop computer for sending the file through BBS. We recorded the student's working on VTR. We used the following problem which was expected to involve DGS. The easiest way for a novice to use mathematical software on CEx involves using a DGS file for simulation and it is necessary in mathematics communication to share visual images with mathematical language.

### *Problem*

In this picture; the rod CF is joined to the rod ED at the point E.

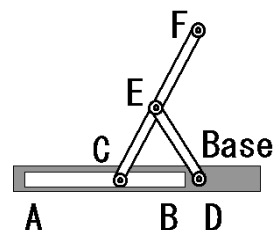
The point D is fixed on the base.

The length ED is equal to the lengths CE and EF.

When C moves between A and B, how does F move?

F moves along a

- a) sinusoidal path.
- b) curved path.
- c) circular path.
- d) straight line path.
- e) different path from a-d.



The problem were used for pre-service and in- service teacher program several times and most of teachers could not get right answer but the solutions are very simple. Thus, it is good for collaborative problem solving.

## **3. The Result of Experiment 1**

Experiment 1 using the first BBS design (figure 1) included four episodes (see Episode 1-1 to 1-4). In each episode, the left hand side activity is the subject person's activity based on observations by the researcher, who helps the operation of CEx, and the right hand side is the reported activity of the teacher (another researcher).

At Episode 1-1, the student (she) tackled the problem on paper as in figure 4 and selected c as the answer. From the reaction at S1, the teacher imagined that she had recognized a circle as in figure 5 and asked for reasons. Then, the student understood the conditions at S3 as in figure 6. Until the description of the conditions, the teacher believed they shared their images such as those of figure 5 and figure 6 (but her image is actually like figure 8). The teacher imagined figure 7 from S3's words of 'moving around AB'. Thus, the teacher confirmed the student's response and

asked for the center because the teacher wanted to change the student's image of figure 7. But the student's real image was figure 8 and she replied at S5 that C was the center point. The teacher recognized that the student had some mistaken image, and so asked her to read the problem once more at T6. She tried to read the problem but did not work through the whole problem and only read a part of it. At S7, she described her images, and at T8, the teacher lost ground in

	<p><b>Episode 1-1. Teacher began to confuse what student said.</b></p>	
<p>Figure 4 Drawing at S</p>	<p>S1(5/23,13:19) It will be a circle. (Like figure 4)  T2(5/23,13:23) Yes, Nh. Why did you choose the C(circle)? (teacher expected figure 5)</p>	<p>Figure 5  Expected at T2</p>
	<p>S3(5/23,13:28) Because <math>DE=CE=EF</math>, F is moving around AB. (Student drew the figure 6)  T4(5/23,13:39) Hi Nh? You thought point F is moving around AB, did you not? Please let me know which point is the center of the circle? (Teacher imagined figure 7)</p>	
<p>Figure 6 Drawing at S3</p>	<p>S5(5/23,13:44) Here C is a fixed point. (Student imagined the figure 8)  T6(5/23,13:50) My question is "Which point is the center of the circle?" You did not read the problem; the C of rod CF moves between AB. Please read the problem.</p>	<p>Figure 7  Expected at T4</p>
	<p>S7(5/23,14:01) EF is the fixed rod, C, D is the base point. So I think that the center is <u>03</u> C while CF is moving around AB. (Drew the figure 8)  T8(5/23,14:) Dear Nh. Thank you very much. We want to continue it next session. Please consider the problem for a while and let me know. With Best Regards, Maha.</p>	<p>?</p>
<p>Figure 8 Drawing at S7</p>		
	<p><b>Episode 1-2 Student used pencils for the model</b></p>	
<p>Figure 9 Modelling at S11</p>	<p>S9(5/25,9:29) Let me know what the problem is.  T10(5/25,9:35) Please let me know if your solution is changed or not.  S11(5/25,9:40) My previous answer was not correct. This will be a circle. (Used pencils to model the mechanics like figure 9 and drew figure 10)  T12(5/25,9:44) Please let me know your answer mathematically. If it is a circle, please let me know the center and radius. Please read the problem once more.</p>	<p>?</p>
<p>Figure 10 Drawing at S11</p>		


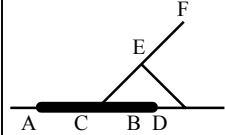
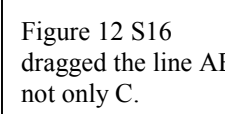
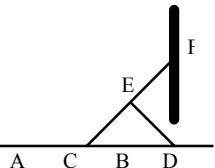
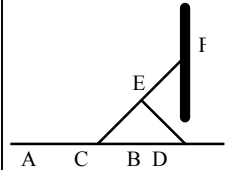
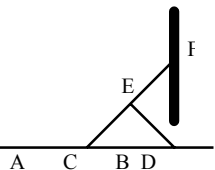
communication (Baker, M. et al 1999) and thus asked her to reconsider and redo the problem.

At S11 of Episode 1-2, the student reworked the problem with the help of a pencil model and got the locus as a circle. The teacher hoped she would change her invalid image at T10. At S11, the student replied that her answer was not correct, but the answer was still a circle. Thus, at T12, the teacher was unsure what the student imagined, and so asked her for a more mathematical description.

At Episode 1-3, she began to ground as well as the teacher, but they were not successful. Indeed at T13, the teacher began with a greeting as well as reference to previous episodes. Then, at

S14, the student responded with a greeting (it was the first time) and expressed her desire to solve the problem. At T15, the teacher sent her an attached GSP file because he felt it difficult to continue communication without correct grounding of their images. At S16, the student replied that she had found the locus was a line. At T17, the teacher believed that they shared the image of figure 13, but unfortunately the student's image at S16 involved the motion of C. At S16, the student dragged the line AB as well as the point C; she could not focus on the motion of F. Up to Episode 1-3, she had displayed skill in communication via the Internet, but had not displayed skill with GSP.

At Episode 1-4, both student and teacher succeed in synchronising their ideas. The student at S18 began communication with greetings and also displayed skill with GSP.

<b>Episode 1-3 Student simulated with GSP</b>		
	<p><u>T13(5/28,8:56)</u> Good Morning, Nh. Did you have any new ideas over the last few days?</p> <p><u>S14(5/28,9:02)</u> How are you? Yes, I am thinking about the previous matter. Please let me know the problem again.</p> <p><u>T15(5/28,9:05)</u> Yes, Nh. Please use the attachment file "GSP" to understand the problem. Move the point C! (Sent the file "55.gs4", figure 11.)</p> <p><u>S16(5/28,9:15)</u> Today in thinking about the previous problem, I reach the conclusion that if the point C moves along the rod AB, then it will be a line and coincide with the rod AB. (Drag as in figure 12 and figure 13.)</p> <p><u>T17(5/28,9:18)</u> Yes, Nh. If you move the point C, F moves along a line. Please let me know the reason tomorrow. (Teacher expected figure 13 but student imagined figure 12)</p>	
		
<p>Figure 11 Simulate with GSP</p> <p>Figure 12 S16 dragged the line AB not only C.</p>	<p>Figure 13 Teacher expected</p>	
<b>Episode 1-4 Synchronized communication</b>		
	<p><u>S18(5/30,10:17)</u> How are you? What are you doing now? I am now thinking about the previous matter. The previous matter I misunderstood a little, regarding point F. Let me write: If the point C moves along AB, then it is line that coincides with AB, but F moves along DF; it is a perpendicular. (figure 14)</p> <p><u>T19(5/30,10:23)</u> Yes, I am fine and you. Oh, you were thinking about only point C. The problem is the motion of point F, is not it? Thus, as you wrote, the point F moves on the perpendicular line to AB. (figure 15)</p>	
<p>Figure 14</p>	<p>Figure 15</p>	

### 3. Discussion for Experiment 1

Episode 1-1 to 1-4 illustrate the difficulty of mathematical communication in a developed environment, mediational means (figure 1), the selectable strategy for sharing ideas, and where difficulties arise. We will analyze these points from the grounding process for collaboration and Socio-historical-cultural perspectives.

Michael Baker et al (1999) defined grounding as the process for reaching common ground of mutual understanding, knowledge, beliefs, assumptions, presuppositions, and so on that were claimed to be necessary for many aspects of communication and collaboration. A number of research studies report on the difficulty of communication or collaboration over the Internet due to the lack of common ground. Episode 1 also illustrates this difficulty. In episode 1, the most influential grounding factor is the difference between the images of the student and teacher. At episode 1-1 and 1-2, the teacher could not picture the student's images and thus asked her to explain mathematically and read the problem once more. However, the student could not easily begin the problem over the web and explain the motion with appropriate mathematical conditions on the problem. At this stage, the teacher's strategy for grounding is to ask the student to explain the image mathematically and to read the problem to confirm the conditions. The teacher did not succeed, and then, at episode 1-3, preferred using a file for sharing the image as the next strategy. Use of the DGS file was expected to lead to a sharing of the image and the answer. The teacher hoped it would help to construct a pseudoconcept of mechanics before mathematically explaining the mechanical motion of F. Indeed, we had other good experiences to suggest that it helped in explaining the motion without mechanics. But at episode 1-3, the student dragged C and responded regarding the motion of C rather than that of F, because this was her first experience of using DGS. After she became accustomed to using DGS, she found common ground in the images at episode 1-4.

Roschelle and Teasley (1995) defined collaboration as a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of a problem. Lee (2000) illustrated that collaboration in mathematical problem solving is analyzed from two aspects: object-oriented activity and interaction-oriented activity. Until the collaboration of episode 1-4 with common ground, there are some remarkable changes in the student's responses. At episode 1-1 and 1-2, the teacher gave a lot of interaction-oriented messages such as greetings as well as object-oriented messages such as asking the student to explain mathematically and to read the problem. Interaction-oriented messages are important teacher's strategies to continue communication using restricted mediational means; unfortunately, the student just responded with object-oriented answers. At episode 1-3, she began to reply with interaction-oriented messages as well as object-oriented answers and at episode 1-4, she began the interaction-oriented message by herself. The changes illustrate that the student needed these experiences of Internet communication on the BBS to synchronize with the teacher. At episode 1-4, the student became a user of the mediational means for mathematical communication.

From analysis of the difficulties, the following functions and restrictions of developed mediational means (Wertsch 1991) are clarified. First, the BBS in the design in figure 1 is functional for posing problems and text communication, and enables file download but not file transmission. At the first stage, the miscommunication of images which is not easy to explain by text is unavoidable. Thus, there is a need for a grounding activity to promote the sharing of each other's images. The teacher preferred the strategy of asking for conditions, but until episode 1-4, the student failed to understand the conditions precisely, because she could not read the problem and messages simultaneously. Second, DGS on CEx is designed for sharing images. But if sharing is to occur, the user has to recognise ways of using and observing. If the novice user cannot use the DGS file, she cannot see the same point as clearly as the expert. Third, it is necessary for the student to use traditional tools such as paper and pencil for reasoning. In particular, at episode 1-2, we see that the pencil also functioned as an aid in representing a model of mechanics. The pencil

model helped the student's images, but the mediational means (BBS) restricted its use to communication.

## 4. The Results of Experiment 2

We determined that it was difficult to read both problem and messages on BBS simultaneously with the first BBS design (figure 1). This design was not suited to confirming the conditions of a

### Episode 2-1. It looks flexible shape

T1(9:10) Hello Pusan. My name is Mathe. I am looking forward to your reply.

S2(9:21) It may draw a circle.

T3(9:33) Good morning, Pusan. OK, you chose C, a circle. Please let me know the centre of it.

S4(9:37) Oh sorry, I didn't read problem carefully. I would like to change my answer to a or b.

T5(9:40) Hi Pusan. You changed your answer from c to a or b. Can you tell me where F goes when C moves B to A?

S6(9:45) Cause of Point D is fixed on the base and it is a rod. It's not flexible so may be F will may draw a line.

T7(9:49) Oh you changed your answer from a or b to d, right? Why did you image the motion a or b and now you changed the image d. Could you explain me why d must be answer?

S8(9:53) At first I thought it like a flexible shape so it will be the motion like a wave or curve. But it's a rod it is not flexible so when it move, it will move in a straight way so I prefer chose d.

T9(9:57) Aha, Pusan. You thought the motion of F based on the motion of rod. I attached a very interesting file by "GSP". Please explain your result mathematically.



Figure 16

### Episode 2-2. Students looked other part of figure.

S10(10:07) Cause at first I didn't understand what I have to do. Sorry. Well after I view a figure that you sent, I think my answer should be change to b. F may be curve (figure 16)

T11(10:12):Pusan, did you drag the point C? Please use the locus command. Firstly, delight the point c and f. Secondly, chose the locus command from the construction. Then, please drag the point c, again.

S12(10:22):It's very interesting. F may draw a circle and E is a center of it. (figure 17)

T13(10:27):Yes,  $EF=ED=EC$ . Thus, there is a circle that the center is point E and radius is EF, ED and EC. Please read the problem once more with comparing the GSP.

S14(10:38):F move with "curve line." (figure 18)

T15(10:48):Pusan, please let me know the locus on GSP, mathematically. I do not think it is a curve.

S16(10:50):F moves on a line

T17(10:53):Yes, F moves on the line, which is perpendicular to the base AD. Can you prove why F moves on the perpendicular line using the conditions you already knew?

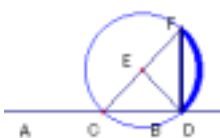


Figure 17



Figure 18

problem while communicating. Thus we changed the design from that in figure 1 and

experimented with how the second BBS design, shown in figure 2, works. Episodes 2-1 and 2-2 took place within the second design.

At episode 2-1, the effect of the new design is illustrated from the beginning. At S4, the student replied that she read the problem once more without the teacher asking. From S2 to S8, she changed her answers, because she reflected on her solution with the conditions of the problem based on the teacher's questions. At S8, she misunderstood the conditions but then understood that the rod is not flexible. Because the teacher believed they already shared the same image, the teacher sent her the GSP file.

At Episode 2-2, the student changed her answer again at S10. The teacher asked her to use the drawing and locus functions of GSP at T11. At S12, she replied with a different observation of the drawing. Then, at T13, the teacher asked her to read the problem again for reconsideration. At S14, the student changed her answer again and at S16 obtained the correct answer.

## **5. Discussion for Experiment 2**

Comparing experiment 2 with 1, communication was synchronized from the beginning of Episode 2-1, but the GSP file is not helpful for sharing ideas. These results gave us some view of grounding and the function and restriction of mediational means.

First, the different BBS design altered communication significantly. From episode 2-1, the student could review her ideas based on each message from the teacher and the conditions of the problem. We cannot see such synchronized communication from the beginning in experiment 1. The BBS of figure 2 functioned on text as well as the BBS of figure 1, but the design in figure 1 did not enable messages and the problem to be compared simultaneously. The design of figure 2 enabled simultaneous comparison and functioned better for communication because this new BBS supported the student's reasoning. Indeed, even if student and teacher could not share their images, the teacher succeeded in the grounding of images at episode 2-1 without DGS because the teacher's strategy for sharing images functioned well in this case. It was easy to compare the student's images with the teacher's questions and the conditions of problem. In addition, the teacher's strategy in the second experiment changes for the better compared with the first experiment. At episode 2-1, the teacher began his message by confirming what the student said. It enhanced both object-oriented and interaction-oriented collaboration. Second, DGS on CEx also did not work from the beginning in Experiment 2 but did work at the end. Because it was also the first time the student had used DGS, she did not know which part of the figure to observe in the situation. It is difficult for the novice to know what to observe even if we tell them by text. Third, traditional tools are necessary even when the DGS file is made available. Indeed, at episode 2-2, the student used the pencil model as well as the DGS file. For the novice, traditional tools have an important role.

## **6. Conclusion**

In order to design a palmtop environment for mathematics communication over the Internet and analyze how it works, this study developed and improved BBS sites. We successfully experimented with how such sites work and clarified difficulties from the perspectives of grounding and mediational means. Due to the mediational means developed, BBS sites functioned well with respect to text communication but were not easily able to exchange mathematics software files. The endeavor of grounding for sharing images is necessary for communicating



mathematical ideas. The different BBS designs strongly influenced the quality of communication. In order to share images, it is necessary to have a simple means to compare the conditions of a problem with questions posed in communications from the teacher. The teacher's strategies of asking the student to provide mathematical explanations and to read the conditions of the problem worked only when the subject could easily compare them. On the other hand, it was difficult for novices to share images with DGS. Thus DGS use could be also seen as a grounding factor in these experiments. The pencil model as a traditional mechanism was a common ground for face to face communication but it is impossible to use over the Internet. These results are in agreement with the idea of affordance from the general theory of cognitive design science (Norman, 1992).

From the pilot study, both experiments illustrate that we can readily communicate and collaborate on mathematics in a palmtop environment if we are accustomed to that environment. Because the Internet provides a new form of communication, users need to accommodate to this environment. This study clarified two factors regarding this. The first involves methods of communication such as asking for better mathematical explanations, asking for conditions to be checked, confirming what the other party is saying, and general greetings. The second involves methods of using the CEx, such as how to use BBS on the Internet and how to use DGS.

It can be expected that the palmtop will evolve into the equivalent of today's desktop. At the same time, we expect that findings relating to the palmtop, such as the design of the BBS, will remain valid into the next generation.

#### REFERENCES

- Anderson, C., 2001, "The Computer as Mediator in the Development of Mathematical Concept", in M. Heuvel-Panhuizen (ed.), *Proceedings of the 25<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education.*, vol.1, 283, Utrecht; Freudenthal Institute
- Baker, M., Hansen, T., Joiner, R. and Traum, D., 1999, "The Role of Grounding in Collaborative Learning Tasks", in P. Dillenbourg (ed.), *Collaborative Learning: cognitive and computational approaches*, Amsterdam: Pergamon, 31-63
- Dillenbourg, P. (ed.), 1999, *Collaborative Learning: cognitive and computational approaches*, Amsterdam: Pergamon,
- Fabos, B., Young, M., 1999, "Telecommunication in the Classroom: Rhetoric Versus Reality", *Review of Educational Research*, vol.69, 3, 217-259
- Lee, Y., Nohoda, N., 2000, "The Process of Collaborative Mathematical Problem Solving: Focusing on Emergent Goals Perspective", Japan Society of Science Education, *Journal of Science Education*, vol.24, 3, 159-169
- Isoda, M., McCrae, B, Stacy, K. 2000, "The Internet Project", the paper presented at the TSG 6 at the 9th International Congress of Mathematical Education, (see <http://www.mathedu-jp.org>, <http://mcs.open.ac.uk/icme/>)
- Wertsch, J. 1991, *Voices of the Mind*, Cambridge, Mass; Harvard University Press.
- Roschelle, J., Teasley, S. 1995, "The Construction of Shared Knowledge in Collaborative Problem Solving", in C. Malley (ed), *Computer-Supported Collaborative Learning*, NY: Springer-Verlag, 69-97
- Norman, D., 1992 *Turn Signals Are the Facial Expressions of Automobiles*, Mass: Addison-Wesley