INTEGRATING WEB-BASED MAPLE WITH A FIRST YEAR CALCULUS AND LINEAR ALGEBRA

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ABSTRACT

This paper reports on extensive work carried out over the last eight years at the University of Queensland to adapt Maple to each of the topics of a first year Calculus and Linear Algebra and the results of this implementation.

The course has about a thousand students mainly engineering and science students with a few from biological science or arts. Most students start with little if any CAS skills, though some have used Derive or graphics calculators at school.

Each topic in the course is introduced by discussion until the analytical background is established. Once this has been covered and digested Maple applications are illustrated on the computer in the lectures and then students work through similar ideas and extensions in their next lab tutorial.

Each student has a one hour computer lab every week. From week two students are introduced to Maple and they can work through the twelve tutorials at their own rate, though one a week is recommended. The tutorials are on the web and students can download them. Week one provides an introductio n to Maple followed by introductions to arithmetic, algebra and calculus so that, by week five, students have some understanding of Maple commands and syntax.

The next tutorials take students through Taylor and Maclaurin series and their uses in approximating π and e and sine, cosine and log functions. Tutorial 7 is a tutor marked test which allows students to judge their progress. The last tutorials cover numerical integration and then Linear Algebra, including vectors, matrices, linear independence, Leslie matrices and the start of programming and finally eigenvalues and eigenvectors.

Projects include practical applications to numerical approximations and using Leslie matrices for predicting changes in populations and dominating eigenvalues to estimate asymptotic distributions.

This paper reports on the evaluations undertaken over the last eight years of the advantages and disadvantages of such an approach.

Keywords CAS, Web-based material, Algebra and Calculus

1. Introduction

At the University of Queensland we have large first year Calculus and Linear Algebra classes of 600 – 700 students comprising students from Science, Engineering and Information Technology. Each student in these courses has three hour lectures, one hour tutorial and one hour computer laboratory every week for thirteen or fourteen weeks of semester.

Peter Galbraith of Graduate Education and I have monitored the introduction of CAS to students for over ten years now and this has changed my attitudes. Initially I was so pleased that CAS had become available (first in the form of our own package and later Maple) that I rushed in and tried to introduce students immediately to its uses and applications with poor results – students could not cope with the syntax and complexity and soon became frustrated with the whole experience. We instigated student surveys and discovered that students felt that computing, as presented to them, was not helpful to their understanding of the course material. I then reacted and probably introduced Maple too late in the course to be effective. For the last eight years I think I have probably got it right (according to the student surveys) by having Maple on the web and starting computer labs in week two of semester with ample introductory explanations.

For their first lab session students have a printed sheet, which tells them how to get into the web site. Once in, there is an explanation of what Maple is and then four introductory sections on arithmetic, algebra, calculus and graph plotting. Following this are nine tutorials illustrating where Maple can be used in conjunction with the lecture material. Students have printed notes for each session but can download any section and print them in the lab. They can work at their own pace – many work at home from their own computers or come in to other lab sessions when there is space available - but it is suggested they do one section per week. There is also a Maple quiz after week six, which we mark and return to students together with solutions, so that they can judge how they are progressing.

Maple is also presented in every lecture to illustrate its applications where appropriate and we discuss in the lecture any problems associated with its implementation and the advantages of using Maple for each application.

2. Content of the Maple tutorial sheets

Tutorial 1 introduces students to the basic syntax of Maple, the system constants for π and e and the five operators for +, -, x , ÷ and exponentiation, with several exercises. Then the difference between exact integer arithmetic and floating point and how to convert to floating point and how to choose the number of digits required. More practice problems.

Tutorial 2 introduces students to algebra and how to assign and unassign variables and the setting up of Maple expressions and functions and the difference between them and composite functions. Examples. Then the six useful algebraic operators *expand, factor, simplify, normal* (combining expressions over a common denominator), *sort* and *collect*, with examples of how Maple can use these commands to perform many useful algebraic manipulations and simplifications. Finally, the use of *solve* for exact solutions (where they exist) and *fsolve* for floating point solutions (where they exist).

Tutorial 3 is the first tutorial, which mirrors what is being presented concurrently in lectures – limits (finite and infinite), differentiation (the *diff* and *D* operators) and integration (definite and indefinite, finite and infinite ranges).

Tutorial 4 supplements work in lectures on sequences (several ways that Maple can create a sequence), infinites sums (especially $\sum \frac{1}{n}$) and products. Then there is a section on various ways of plotting functions (the course does not specifically contain this, but it is so useful) – simple and multiple plots, parametric and implicit plots, with many examples and practice examples.

Tutorial 5 is really the start of serious uses of Maple and again mirrors lecture work on Taylor series. It first introduces students to finding Taylor series of several common functions and how to convert them to polynomials and the use of the *op* command to extract terms from these polynomials. We then see how to approximate e and π as accurately as required, using the above.

Tutorial 6, as with lectures, is about Maclaurin series and error estimates for series of positive terms and Alternating series and using Maple to approximate various trig and log functions to prescribed accuracy and then examples for students to try for themselves.

Sheet 7 is a quiz which gives students twenty questions with no hints (they can, of course, use the web material) to be done in 50 minutes. These are marked and returned with full solutions.

Tutorial 8 mirroring lectures uses Taylor series to approximate definite integrals, which have no closed form to, prescribed error. This is the end of the calculus section.

The second half of the tutorial starts with the Linear Algebra part of the course by introducing students to Maple's *linalg* package and how to set up vectors and matrices and perform addition and scalar multiplication and matrix multiplication. Several examples and exercises on this and whether certain given matrix products are defined.

Tutorial 9, following lectures, takes students through linear systems, augmented matrices and the start of Gaussian Elimination and the use of Maple's *linsolve* to solve systems of linear equations. Then several ways to look at linear independence and bases. Again many examples and exercises.

Tutorial 10 has a nice illustration of the use of Leslie matrices applied to a marsupial population (of course!). The students are introduced to programming in Maple and this is applied to setting up a generational profile for the population.

Tutorial 11 mirroring lectures is about setting up Gaussian elimination step by step and all the commands needed to do this (augmentation and various row manipulations) and then examples for students to do for themselves.

Tutorial 12 as in lectures takes students through five different ways of establishing whether a set of vectors is linearly independent or not, or whether the matrix formed from these vectors is singular or not.

Tutorial 13 is the last topic of the course and shows students how Maple can find eigenvalues and eigenvectors and the characteristic polynomial and factorize this polynomial.

Assessment I have used two variations – three assignments worth 3%, 3%, and 4% and two worth 5% each. This is the same weighting as given to the course tutorials – 10% each. Among topics included are: solving systems of linear equations; finding conditions for sets of equations involving a parameter to have a unique solution; no solution; an infinite number of solutions; simplifying complicated algebraic expressions; zeros of functions; areas under graphs; problems with $\sum_{n=1}^{N} \frac{1}{n}$; using Taylor series to estimate functions and integrals; problems with Leslie matrices using the dominating eigenvalue result to find the final relative age distributions.

3. Discussion

With Peter Galbraith we set up various instruments to monitor problems encountered by students and applications which helped students. The results were as follows.

General Remarks We discovered that the timing of the introduction of a CAS is important – if students encountered a CAS too early they will only treat the whole experience as irrelevant and confusing and if too long after the material is discussed in lectures they will miss the relevance of the applications. Again, if a CAS is merely kept to the computer laboratory, students will believe that the lecturer feels it is not an integral part of the course. It is essential for the lecturer to bring the CAS into lectures and demonstrate it so that students can see its applications and how to implement it successfully and **discuss** any problems they might have as it is presented. I always have **three** forms of information in the lecture theatre while demonstrating a CAS – the actually computer display, one overhead for the CAS commands and one overhead where we can make remarks and answer student queries.

Our surveys also showed there are essentially three types of students – the purists (usually female and good students) who refuse initially to do any computing as they believe all mathematics can be done (and **should** be done) **analytically**. To convert these I always show them the depot problem, which demonstrates clearly that you cannot solve even some of the simplest problems analytically. To the other extreme there is always a group of students (usually male) who love to spend vast amounts of time fiddling around with computers (left over from their computer games) who will lose the main analytical thrust of the course in their quest for arcane computer methods – it is important not to let a CAS take over a course. In the middle is the largest group who are willing to try it out but whose confidence in the outcome is always in the balance of how well you implement the presentation and timing and relevance of your CAS programme.

Specific remarks.

Syntax is the main source of problems and frustrations for students. Those students who always had troubles with brackets soon find they cannot even enter simple expressions like $\frac{2^9 + 3^8}{4^7 + 5^6}$ without getting a syntax error. Worse cases were e.g. $(a + b)^3$,where they say to themselves: “a + b cubed” and enter a+b^3 , which, of course, returns a **wrong** expression with **no** syntax error! We actually interviewed students and discovered that those who have always had poor algebraic skills merely carried this incapability through to their computing – the problem goes back to Grades 8 and 9 (13 and 14 year olds). Counting the number of opening and closing brackets sometimes helped but their main problems always were actually getting their first expression “ to work”.

Choice of algebraic commands was another source of problems e.g. to find the zeros of a polynomial they would use the *simplify* or *expand* command and, of course, Maple merely returns the original polynomial. Knowing that zeros are associated with **factors** is a vital piece of information – if you do not possess this you will get nowhere. However, those with adequate algebraic skills who could enter expressions correctly found the capability of simplifying complicated expressions very useful and soon realized that Maple could save them literally hours of grinding away through lengthy algebra.

Problems and advantages with the calculus section Students found it helpful to be able to find derivatives of complicated functions (especially quotients) and where possible simplify them, provided, of course, they could enter the function in the first place. Typical here was the exponential being entered as e^x instead of exp(x), **despite** the introductory warning about the representation of e.

Confusion also occurred when trying to evaluate the derivative at a given point, as they were not sure if the result of using the *diff* or *D* operators were **expressions** or **functions**. What they found extremely useful was the capability in maximum and minimum problems of finding the derivative (however complicated), plotting it (so they could see how it behaved) and using *fsolve* to approximate where the derivative was zero.

Probably **most useful** was evaluating integrals that students always find difficult like $\int x^n e^{ax}\,dx$, $\int e^{ax}\cos bx\,dx$, $\int x^n \sin bx\,dx$ where n is sufficiently large to make it tedious. Also for integrals involving square roots (what to substitute?) and those involving partial fractions, which students find fraught with numerical mistakes – Maple's capability of finding partial fractions helps simplify things greatly.

This section brings up an important point for discussion – after doing it students will ask **why do we have to learn how to do these integrals when Maple can do them for us? – aren't we wasting our time? – this, of course, is one of the big debates surrounding the teaching of any CAS – how much material should we remove from our traditional syllabus and replace by CAS?**

Plotting Students are very poor at sketching graphs of even relatively simple functions and all report this is **one of the greatest aids**. It also means that you can use more complicated and realistic functions for your examples.

Taylor series and numerical approximation Every year students find understanding Taylor series the most difficult part of the course – until they actually use them to approximate various expressions with desired accuracy they cannot envisage how they work. **All reported this was the most useful area for them especially as Maple can easily produce Taylor series to any order and evaluate them to any given accuracy.**

The Maple quiz (after six weeks) acts as a good guide for their progress – we get them to print out all their attempts so we can see where their approach goes wrong. The most common errors were still **syntax errors,** using the **wrong algebraic commands**, **confusing solve and fsolve** and **expressions and functions**. In general, though, most have mastered the content so far quite adequately.

Vectors and matrices Many students report being alarmed when they load the *linalg* package and forget to end their command with a colon resulting in a vast description of the package in front of their eyes – they felt as though they have done something terribly wrong. Once this was rectified they found vectors easy to enter and perform addition and scalar multiplication with them.

Many found it difficult to enter matrices, usually leaving out one of the final] brackets and if they achieved this successfully they forgot to include *evalm* to view the results of their matrix calculations. However, once this was overcome, they found the **capability of finding inverses, determinants, powers etc. very useful**, especially when large size matrices were involved. To be able to find the determinant of a large matrix, factorize it and hence discover for which values of the parameter(s) it is singular (or not) was very helpful.

Gaussian elimination They found setting up the process step by step pretty difficult, but liked the *gausselim* command, which did it all for them, very helpful as they could easily see the three possibilities for solutions – unique, none and infinite.

Bases Again, students found the whole concept of a basis very difficult and they reported that the sheet which took them through several ways of attacking these problems were helpful.

Eigenvalues and eigenvectors I always find I have to return to eigenvalues and eigenvectors in second year before students fully understand the concepts. By hand students can only really manage to find them for 2X2 and 3X3 matrices and problems are always encountered if there is multiplicity. Maple provides the characteristic polynomial which can be factored easily to give the eigenvalues and thus the eigenvectors via *linsolve*. Maple can also return eigenvalues and eigenvectors immediately together with the multiplicity – this is an easy way out for students.

Assignments We mark all assignments and monitor students' problems. After one semester students still exhibit problems with syntax – I expect they will retain this problem for all their undergraduate days. Most other problems stem from their misunderstanding of the course material which flows through to their incapability of using Maple successfully. Whichever CAS one must use there will always be these problems.

4. Conclusion

Whatever CAS you might choose I suspect there will always be advantages and disadvantages associated with its implementation. The timing is vital – it must be introduced after the relevant material is covered in lectures but sufficiently soon enough afterwards that students still have it fresh in their memories. Further it is essential to reinforce its impact by having CAS in lectures where we can all discuss problems associated with its application.

Our studies showed that if students either have CAS introduced too quickly or in not sufficient explanatory detail they will react negatively and find the whole experience worse than having no CAS at all. Again, if CAS is not presented as an integral part of the course it appears to students that it cannot be relevant and they will not treat it seriously.

There will always be some students who will encounter problems with syntax and this leads to frustration and withdrawal from the programme. For those who overcome the various pitfalls there can be many advantages and in many cases CAS can illustrate and extend ideas which students otherwise find hard to grasp. We found this especially true with these topics: simplifying complicated algebraic expressions; finding derivatives and integrals of complicated functions; graphing difficult functions; setting up Taylor series and using them for numerical approximations; finding inverses, determinants and powers of large matrices; finding eigenvalues, eigenvectors and characteristic polynomials.

In conclusion, I think the whole exercise is probably worth the considerable effort of setting up the required materials.

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The relevant web sites are:-

Introduction www.maths.uq.edu.au/computing/local/maple/m.html

Tutorials www.maths.uq.edu.au/~mrp/mt151/index.html#maple