SYMBOLIC MATH GUIDE : AN INNOVATIVE WAY OF TEACHING AND LEARNING ALGEBRA USING TI-89 AND TI-92+ GRAPHING CALCULATORS

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ABSTRACT

During the past decade, handheld graphers have fundamentally changed the teaching and learning of many school mathematics concepts – particularly those dealing with graphical representation and visualization (Demana and Waits, 1992). Graphing calculators have enabled many students to experience mathematics as a more dynamic, interactive, and visually-appealing area of study. Yet, because graphers have historically lacked symbol manipulation capabilities - relying on numerical approximations to calculate - their impact on the teaching and learning of *equation solving* and *symbolic manipulation* has been minimal. While many secondary school teachers and students use calculators to study graphs, they continue to examine algebraic manipulation using pencil-and-paper or chalkboard-based activities.

However, a powerful new generation of graphing calculators equipped with symbolic manipulation capabilities is likely to change this situation. Handheld *Computer Algebra Systems* (CAS) – including the Casio FX 2.0 and Hewlett Packard 49g – will likely prompt instructional changes that mirror those precipitated by handheld graphers a generation ago.

In the following article, the authors discuss features of *Symbolic Math Guide (SMG)*, a CAS designed for use with Texas Instruments TI-89 and TI-92+ graphing calculators. Unlike earlier CAS, *SMG* was developed primarily as a pedagogical teaching and learning tool for high school mathematics students – not a research tool for university faculty. In the first sections of this document, the authors present research findings suggesting a need for such pedagogically-oriented CAS. In subsequent sections, the authors provide sample calculator exercises that highlight *SMG*'s ability to simplify algebraic expressions, exploring differences between pedagogical and traditional CAS (e.g. *SMG* and TI-92 CAS). The calculator exercises are provided as an introduction to *SMG* for both teachers and researchers.

Keywords: Educational Technology, Graphing Calculators, Computer Algebra Systems, Algebra

1. Introduction

> *As educators, we must prepare our students and ourselves for new and exciting forms of technology that take the best of what we have to offer as teachers and apply it to our subject matter.*
>
> *(Diem, 1992, p.109)*

Today, we live in a world significantly different from that of only a generation ago. Over the past two decades, technology's influence on everyday life has been pervasive and powerful - challenging our notions of human interaction, communication, and learning. Incorporating previously unthinkable tasks into daily routine, technologies such as word processors, electronic mail and the internet have made life richer, more convenient, and more productive. In a similar way, handheld graphers have profoundly transformed many aspects of school mathematics. Graphing calculators have enabled students to experience mathematics as a more dynamic, interactive, and visually-appealing area of study. Graphing tools have heightened the importance of graphical representation and visualization in mathematics classrooms (Demana and Waits, 1992).

Despite the revolutionary role that graphing calculators have played in the past, their impact on the teaching and learning of *equation solving* and *symbolic manipulation* has been minimal. Because graphers have historically lacked symbol manipulation capabilities, many teachers have used the devices to study *graphical concepts* - while continuing to examine algebraic manipulation using more traditional *pencil-and-paper* activities. The introduction of a powerful new generation of graphing calculators (e.g. Texas Instruments TI-89 and TI-92+, Casio FX 2.0, Hewlett Packard 49g) promises to change this situation. Equipped with symbolic manipulation capabilities, these handheld Computer Algebra Systems (CAS) challenge popular notions of algebraic manipulation in school mathematics. While providing students with powerful means of investigating the richness of mathematical symbolism in more dynamic and interactive ways, they call into question the continued role of pencil-and-paper in school algebra instruction.

Although studies of CAS with secondary school students have existed since the early 1990's (Aldon, 1996; Hirlimann, 1996; Klinger, 1994), early investigations have typically taken place in school computer labs using CAS on desktop computers. Important distinctions exist among CAS studies using calculators and computers.

- CAS-equipped graphing calculators may be used in traditional classroom settings on an "as-needed" basis. Unlike school computer labs, the use of CAS-equipped calculators requires no interruption in classroom instruction and no special trips to a remote lab.
- Calculators are more portable and more convenient. Students can use handheld CAS tools in other classes or to do homework without installing additional computer software or hardware.
- CAS-equipped calculators integrate symbolic manipulation functionality within an environment with which many students are already familiar – that of graphing calculators.

Portable CAS-equipped devices have only recently begun to appear in school classrooms. Therefore, research studies involving the use of *CAS-equipped calculators* in school settings are not commonplace. Preliminary research involving the use of CAS-equipped calculators with secondary school students has indicated that the tools are useful as "conjecture building" devices (Edwards, 2001). However, research also indicates that CAS-equipped devices have a tendency to perform "too many steps" for novice algebra students, while employing symbolism that is unfamiliar or even contradictory to that found in school textbooks (Edwards, 2001). The findings of Edwards (2001) have findings suggested a need for CAS tools designed primarily as pedagogical teaching and learning tools – not as a tool for researchers and mathematicians. Texas Instruments *Symbolic Math Guide* (*SMG*) was developed to address issues such as these.

2. The Need for *Symbolic Math Guide*

During a year-long study of CAS use with secondary school students, Edwards (2001) found that CAS students were dissatisfied with emphasis on calculator-based methods when solving manipulation-intensive problems. CAS student dissatisfaction appeared to be related to the calculator's tendency to complete large portions of problems for students.

> I think most knowledge about math is learned through hand-written work. Hand-written work gives the student a visible and mental track of what work was done and how the problem is solved. Calculators don't always show the individual steps to solving equations (Mike Fine, second-year algebra student).

The screenshots highlighted in Figure 1 illustrates the results of entering the equation $\frac{x^3 - x}{x+1} = \frac{x^3}{x}$ on the home screen of a TI-92.

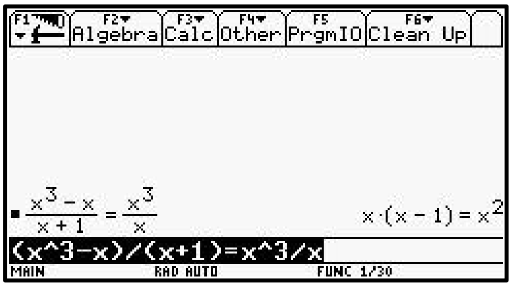


Figure 1: Steps automatically performed upon entering equation into TI-92 CAS

As Figure 1 suggests, the TI-92 homescreen CAS automatically performs the following tasks:

1. re-expresses $x^3 - x$ as $x \cdot (x^2 - 1)$
2. re-expresses $(x^2 - 1)$ as $(x+1) \cdot (x-1)$
3. re-expresses $\frac{(x+1)}{(x+1)}$ as 1
4. re-expresses $\frac{x^3}{x}$ as x^2

After a student decides to subtract x^2 from each side of the equation, the calculator automatically performs several more steps. These are highlighted in Figure 2.

1. Expands $x \cdot (x-1)$ as $x^2 - x$
2. Simplifies $(x^2 - x) - x^2$ as $-x$

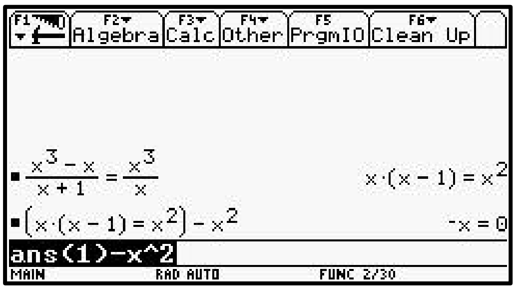


Figure 2: More calculations automatically performed by TI-92 CAS

In addition, Edwards' students complained that calculator notation differed significantly from notation typically found in school textbooks. Several major differences are highlighted in Figures 3 and 4.

For instance, unlike conventional mathematical text, in which algebraic steps are written one below the next, TI-92 output is read from left to right, then from top to bottom (like sentences in a book). This is shown in Figure 3.

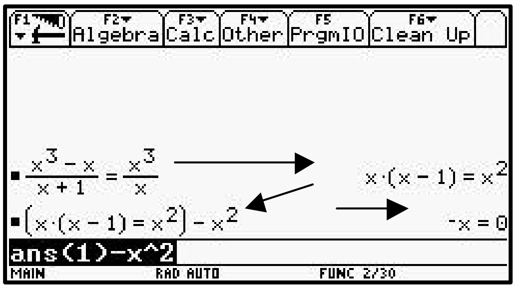


Figure 3: Algebraic output is read like "sentences in a book" on the TI-92 home screen

Additionally, the manner in which the TI-92 homescreen CAS simplifies expressions suggests to students that transformations are applied to *entire equations* (rather than to each side of an equation). This tendency caused confusion with novice algebra students. An example is provided in Figure 4.

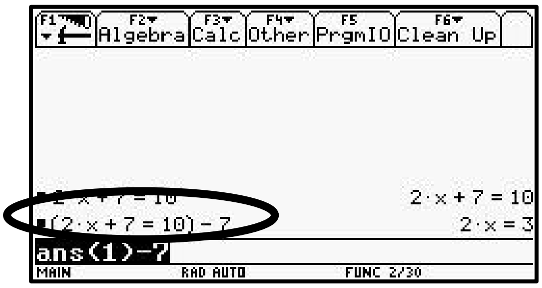


Figure 4: The TI-92 CAS applies a single transformation to an entire equation rather than performing separate transformations to each side of an equation

Edwards concluded the following at the end of his study:

> CAS based equation solving does not appear to support conceptual understanding to the same extent as traditional by-hand equation solving. The awkwardness of the TI-92 output as well as the calculator's tendency to perform "too many steps" automatically may have contributed to students' preference for by-hand methods. (Edwards, 2001, p. 299)

Traditional CAS were designed as tools for researchers – not as learning tools for young students. Thus, they tend to perform algebraic steps automatically – with little explanation provided to the user. In addition, CAS often display algebraic information in non-standard formats. Although these tendencies may suffice for university researchers who need fast answers and *already know significant mathematics*, they render CAS unsatisfactory as a learning tool for beginning algebra students. As we note in the following section, tools such as *SMG* provide students with access to the computational power of CAS, while at the same time providing an environment explicitly designed to *teach*, not confuse.

3. Features of Symbolic Math Guide

A primary purpose of *Symbolic Math Guide* (*SMG*) is to help students develop a deeper understanding of various algorithms used to solve algebraic manipulation-style problems. Unlike the raw symbolic manipulation utilities studied by Edwards (2001), *SMG* is more faithful to the mathematics and mathematical notation found in school textbooks. *Symbolic Math Guide* was built first and foremost as a pedagogical teaching tool - not an answer generator. The program encourages teachers and students to solve problems in a step-by-step fashion in a manner similar to traditional pencil-and-paper methods. Several features of *SMG* are listed below.

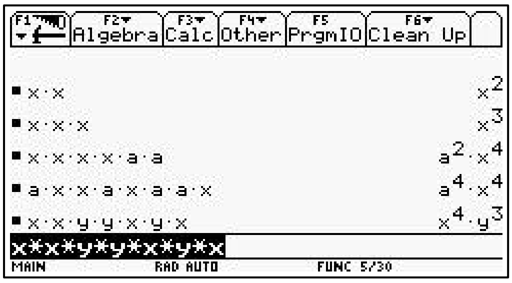
- Student exercises are organized by problem sets. Teachers, publishers, and students are able to create problem sets for particular lessons or activities. The sets may be easily shared online or in class.
- As they select algebraic steps from menus and dialog boxes, students solve algebraic problems in an interactive manner.
- While considering the results of students' most recent calculations, *SMG* generates intelligent problem-solving options that focus student attention on new material being learned.

- Because *SMG* simplifies arithmetical expressions automatically, students may focus more attention on theoretical aspects equation-solving. Student work is not unduly hampered by arithmetic and lower-level algebra mistakes.
- While using *SMG*, students are encouraged to consider algebraic expressions and equations as mathematical objects. *SMG* encourages students consider appropriate transformations to apply to these mathematical objects to solve problems.

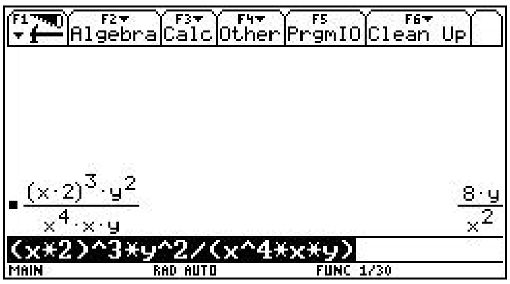
SMG is a self-paced learning tool to help students in learning symbolic manipulation. It offers a source of extra problems for students who haven't mastered a certain symbolic manipulation skill and can be used as a quick review for exams or a quick review of previously learned symbolic manipulation skill. The authors of this document have informally used *SMG* with students when introducing new classes of problems.

4. Simplifying Expressions With Powers with TI-92 CAS and *SMG*

4.1 Exploring Powers with Traditional CAS

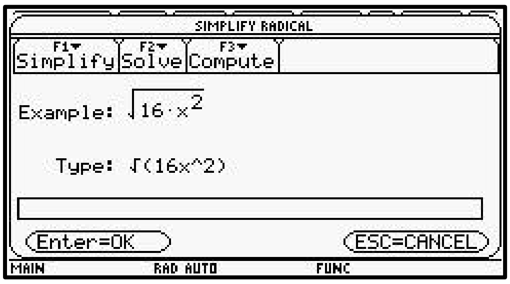


CAS allows students to discover rules about simplifying powers. By typing in several related examples into the TI-92 home screen, students form conjectures regarding algebraic rules. The examples to the left suggest a well-known "exponent multiplication" rule.

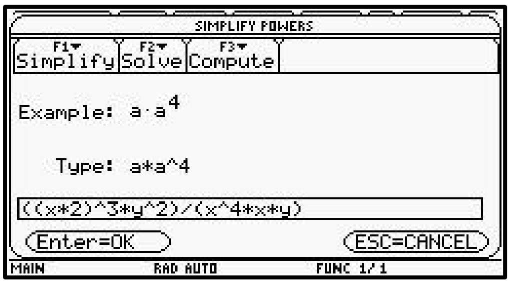


Unfortunately, the TI-92 CAS has a tendency to simplify more complicated expressions in one or two steps. This tendency creates confusion for inexperienced students, impeding their understanding of algebraic equivalence.

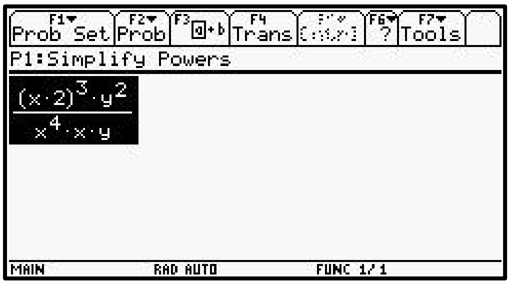
4.2 Exploring Powers with *Symbolic Math Guide*



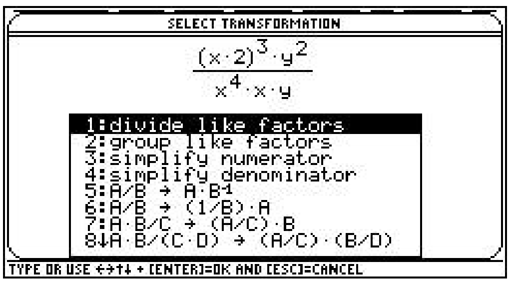
After starting *SMG* and upon selecting the `New Problem` option, *SMG* prompts the user to select a problem type. For instance, if a student wants to simplify an algebraic expression, he or she should press F1. Equation solving options appear under F2. Computational options appear under F3.



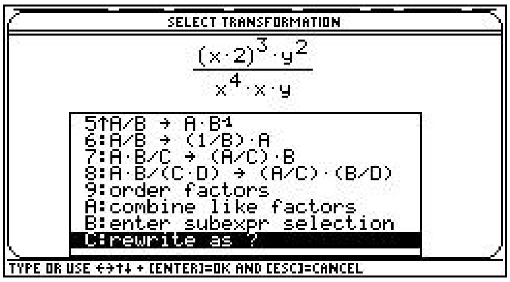
Inside the data entry line (at the bottom of the screen), type in the expression `((x*2)^3*y^2)/(x^4*x*y)` then press enter. The problem is now entered into the *SMG* main work screen.



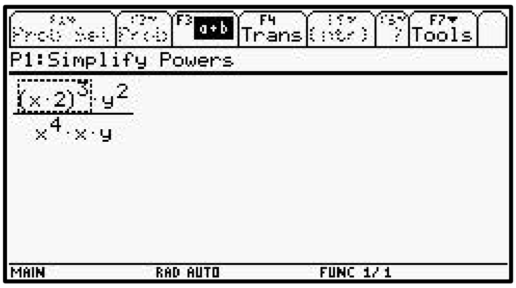
Several tools are available to the user at this point. In particular, the F3 menu option allows the user to select subexpression. The F4 menu option provides the user with different algebraic transformations that may be applied to selected expressions.



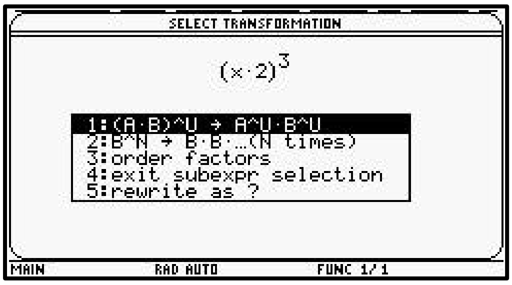
When the problem is entered into *SMG*, twelve legal choices are provided for the user. The student can choose any of them - although some selections lead to more efficient solutions than others. By offering legal steps, the *SMG* strengthens student understanding of rules used in simplifying powers.



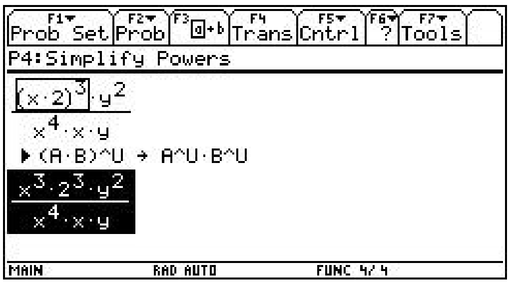
Since the twelve legal choices do not include a "power of a power" rule, students are encouraged to look at subexpressions within the problem. Students may use the subexpression feature of *SMG* to choose a smaller portion of the problem to simplify first.



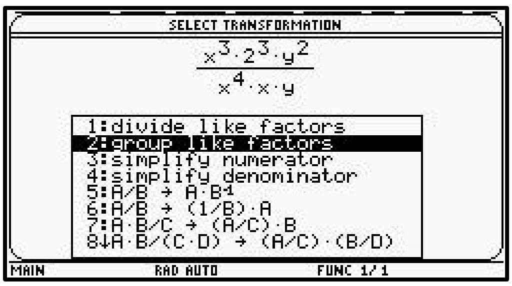
The screenshot to the left shows the selection of the subexpression $(x \cdot 2)^3$. Sub-selection is accomplished by pressing F3 and highlighting an expression with the calculator's keypad.



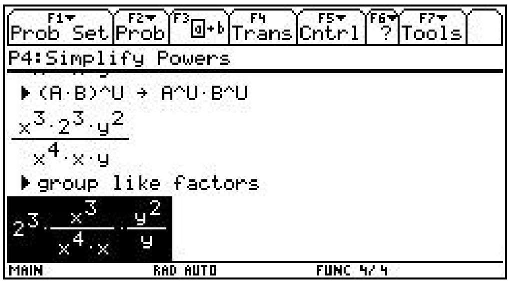
When the subexpression $(x \cdot 2)^3$ is selected and F4 is pressed, a different list of algebraic options is made available to the user.



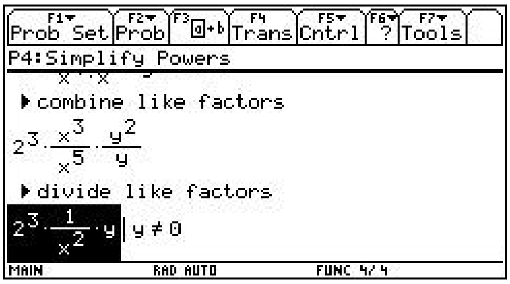
The first option - `(A•B)^U  A^U•B^U` - distributes an exponent across factors within parentheses.



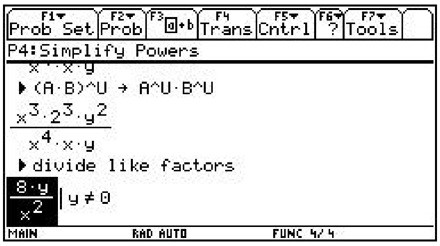
After $(x \cdot 2)^3$ is re-expressed as $x^3 \cdot 2^3$, a new listing of algebraic options is once again provided to the user. The `group like factors` and `divide like factors` options are both reasonable selections.



By selecting the `group like factors` option, one is able to look at different variables combined.



The application of the `combine like factors` and `divide like factors` options makes it easier for many students to understand what is meant by "cancelling out."



oHowever, if students are already familiar with "canceling," one step cancelation is accomplished by omitting the application of `combine like factors`.

4.3 Anecdotal Evidence regarding *SMG*

Using *Symbolic Math Guide* informally with second-year algebra students, Edwards notes that the tools do offer some benefits over traditional CAS. Specifically, *SMG* does not seem to skip algebraic steps. Students have the ability to think more critically about algebraic transformation when simplifying algebraic expressions with *SMG*. In addition, the application offers classroom teachers an instructional partner in the classroom. Students needn't wait for teacher approval when checking the correctness of various algebraic manipulations. As one of Edwards' students notes:

> I like the math guide (*SMG*). It doesn't do as much work for you as other calculators, so you still have to think about the algebra. That's a good thing for future classes. Plus, I like the fact that the program (*SMG*) lets us find our own solutions. I think it makes algebra a little more interesting because we can experiment. The teacher doesn't have to lecture to us so much (Zak Stevens, second-year algebra student).

Nevertheless, the application isn't a perfect learning tool, and it certainly isn't as flexible as a seasoned classroom teacher. For instance, when using *SMG* in classroom situations, Edwards noted the following problems related to *SMG*:

- Selecting "subexpressions" within a term (e.g. highlighting $(x \cdot 2)^3$ within the expression $\frac{(x \cdot 2)^3 y^2}{x^4 xy}$) requires manual dexterity not required with pencil and paper. Some students become frustrated with the "subexpression" selection features of *SMG*.
- After selecting a specific expression to simplify, menu options do not always contain desired transformations. Students are left wondering "what to do next?"
- Inconsistencies exist with regard to domain restrictions. For instance, when simplifying the algebraic expression $\frac{(x \cdot 2)^3 y^2}{x^4 xy}$, SMG notes that $y \neq 0$ but no such restrictions are generated for *x* (see last screenshot).
- Functionality does not exist for roots other than square roots.

5. Discussion

SMG has an important role in helping students to give meanings to the algebraic transformations they frequently employ in secondary mathematics classes. In addition, *SMG* provides students with a more interactive method for learning concepts of symbolic manipulation than possible with pencil and paper. While using *SMG*, students are less preoccupied with calculations – spending more time considering algebraic transformations and concepts of equation solving. The authors of this document have found that learning is maximized when students are encouraged to anticipate the result of each transformation they select before pressing the ENTER button within *SMG*. By using *SMG*'s 'Press ENTER' mode, the application provides students with extra time to write down predictions – showing results to students only after ENTER is pressed again. The physical act of writing down each step with pencil and paper appears to help some students as they learn appropriate manipulation steps. While less effective when reviewing previously learned material, we've found the 'Press ENTER' mode to be quite useful when teaching material to students for the first time.

Because *SMG* allows students to select a variety of legal algebraic steps that are not necessarily the "best" or "most efficient" steps, students often construct different methods to solve individual problems. By comparing different solution strategies, students begin to appreciate the richness of algebraic problem solving (a subject which many students see as having "only one right way of doing things"). On the other hand, if legal algebraic steps do not lead to a solution, *SMG* makes it easy for students to go back to any previous step and try different transformations. To accomplish this task, students press the "up cursor" to get back to the step they wish to change. Then they choose a new transformation from a variety of menu options. In addition, SMG allows students to select subexpressions and replace them with an equivalent expressions from the keyboard. For instance, if a student knows that x+x is equivalent to 2x, the student can highlight "x+x" and choose a "replace with equivalent expression" menu option. *SMG* tests for equivalence of original and the user-defined expressions.

We always discuss that it is necessary to connect mathematics with real life situations. However, algebraic manipulation is one of the areas in the secondary mathematics curriculum that can be very abstract and very monotonous for students. Because students' minds and attention are always busy with calculation details, it is all too easy for them to lose sight of general equation solving techniques - particularly those involving algebraic transformation. *SMG* attempts to address this problem by offering teachers and students a novel approach to learning algebra. The CAS makes it possible for students to focus on the transformations in a visual, interactive and technology-rich environment.

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