ON SOME IMPORTANT ASPECTS IN PREPARING TEACHERS TO TEACH MATHEMATICS WITH TECHNOLOGY

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ABSTRACT

The introduction of technology in the classrooms at all levels of education has brought forth a need to change some teaching practices. Together with the modernization projects of undergraduate instruction for Engineering and Science courses, it is especially important to focus the attention on Courses for prospective (and in-service) teachers in Mathematics.

The use of technology in the teaching-learning activities can be regarded as a new communication language in developing the construction of knowledge. The recognition of this role of technology in education would contribute to a better preparation of the future teachers in selecting right teaching strategies, not only technology. This paper aims first to discuss this aspect of technology in the undergraduate instruction, through a systematized classification of the use of technology in the classrooms based on the forms of activities, illustrated with examples.

Furthermore, one of the advantages of the technology as teaching aid is the possibility of more realistic modeling in problem solving and interdisciplinary activities, so new and reformulated disciplines in the curriculum of teacher preparing courses come up. Regarding this aspect, we point out that the critical interpretation of the computer/calculator outputs demands an awareness of the kind of mathematics needed when using technology. That means that solving a problem with the use of technology requires from the user a deeper understanding of the importance of concepts like units, scaling of units, significant figures, approximation/numerical methods, parametric representation and implicit representation, interpolation methods, structure of algorithms, etc., along with the proper theoretic concepts underlining the problem. The careful use of technology as a teaching strategy would enrich in this way the lectures and the preparations of activities by teachers. The second aim of this paper is to illustrate these considerations exhibiting an example for teachers.

Keywords: Teacher Education Courses, Technology in Education, Mathematics of technology-based activities.

1. Introduction

The introduction of technology in teaching and learning environments brings up a general concern of educators of all levels of instruction, particularly of those involved with the formation of future teachers. The general issue is how to prepare the teachers for a generation for whom the technology is already familiar, and also to update the in-service teachers with new methodologies.

In this paper we will be focusing on the education of prospective and in-service mathematics teachers in the presence of technology.

On the importance of technology in professional development of teachers, Oldknow (Oldknow, 2000) says that "...the effective use of (Personal Computer Technology) in supporting the mathematics curriculum is in the hands of teachers. They need to know more about the use of technology than can just be found from manuals, teaching materials and other information sources."

Also, in the same Reference we find a quotation of Cornu:

"Mathematics is evolving and changing under the influence of computers and informatics. Therefore, teachers need to maintain their mathematics knowledge and to practice mathematics from an informatics viewpoint. Mathematics is becoming more experimental, more algorithmic, more numerical; teachers must be able to follow the evolution of mathematics, and to acquire new competencies and new attitudes and to be able to carry out new activities in mathematics."

The statements contained in the citations above are examples of recommendations alike that can be seen in many documents and papers requiring the change of attitudes of teachers regarding the use of the technology. Training the use of equipment and the particularities of some educational software are obviously not enough to achieve educational results in modern classrooms. Besides the necessary mathematical background, a question is what the prospective teachers should know about teaching with technology before trying the many existing materials or creating their own activities.

In (Lingefjärd & Holmquist, 2001) the authors say, "teachers of today need an understanding of mathematics that allows them to produce and interpret technology-generated results, to develop and evaluate alternative solution paths, and to recognize and understand the mathematical limitations of particular technological tools". Also they say "teachers must be well informed about its (of technology) place and role in a didactical process".

Therefore, some natural questions are posed:

What is teaching with technology? What may change if one uses technology to teach? What are the different ways of the use of technology in education, and which one is the most effective to reach educational objectives?

One great challenge that a mathematics teacher faces when he/she plans to introduce the technology in his/her classes is that, in general, he/she does not know when, what and how to use (sometimes why), even when he/she has previous knowledge about equipments and several educational software. This challenge is faced also by the faculty of Teacher Preparing Courses at university level, who has the responsibility to prepare adequately the future teachers with the mathematical as well as the pedagogical background required in a modern classroom.

This paper is based on the reflections of the author in introducing different teaching methodologies with technology to wide ranging classes, from engineering and sciences students to prospective and in-service teachers of basic level schools, and studying their responses.

The first aim of this paper is to discuss the different ways of communicating mathematical content with the aid of technology, proposing a systematic classification of the use of technology in the teaching context, in order to help the teachers to understand the role of technology in education and consequently facilitating the effective use in their classes of much information already available.

The second aim is to discuss the importance in the teacher preparation courses of mathematical concepts underlining some elementary activities suited for technology-based classes, indispensable to the formation of teachers as a user of technology. This is a key issue when teachers realize the didactical potential of mathematical modeling with the support of technology.

2. Different ways of communicating Mathematics with Technology

The activities done in the teaching/learning context are traditionally the following: a) expository classes in which the teacher introduces concepts and develops problem-solving, exercises, etc; b) working out problems, either individually or in group, repetition classes with drills, etc; c) homework, projects, etc; d) evaluation tests.

In each of these activities one can easily recognize who plays the active role, and also it is clear what are the objectives of each activity. The main difficulty of prospective and secondary level teachers is the perception about the possible changes of these activities into technology-based activities.

An effective teaching/learning process is a communication process between teacher and student that involves the principle of action and reaction, that is, each action taken by either a teacher or a student provokes a response from the counterpart that stimulates a new action. The completion and the repetition of this cycle as many times as necessary are actually required to the results been assessed properly. The technology may take part in this process as an asset to improve the communication between the teacher and the students.

We propose the following classification of the role of technology as teaching aid, based on the forms of communication and the recognition of the active-passive role of each part:

I- In a traditional expository class, in which *the teacher* is the active user of technology;

II- In a laboratory-type class and activity, where *the student* is the active user of technology;

III- In a different type of activity, where *the teacher and the students* **are** active users and together participate in the construction of knowledge.

When a teacher uses an **expository approach** to introduce and develop mathematical concepts, the student is a *passive* recipient of the lecture, and his/her understanding of the topic depends on teacher's communication skills and the interest of the class. The presence of technology in this type of class includes the slide-projection, the overhead projectors with transparencies, videotapes, computer software (CAS, DGS, GC (graphic calculators), etc.) combined with projectors, etc.

In particular, the possibility of using powerful educational software with computer or calculator to develop better examples and more realistic illustrations turns the ordinary exposition into a more exciting and meaningful class, where the main actor is the teacher. The computer-algebra systems and graphing capabilities, combined to fast calculation capabilities turns it possible to the teacher present examples where the use of technology is actually necessary, explore situations to confirm the theory being presented. The visualization effects and the animation feature found in much software are definitive allies to enhance the communication between the lecturer and the class, especially in basic level schools.

This is a nice role of technology that improves greatly the didactical transposition of mathematical concepts. Many experiences reported early on the use of technology in teaching environment started this way, in general. For example, the classical illustration of the slope of tangent lines to the graphic of functions related to the concept of the derivative, the classical illustration of the proof of Pythagoras' Theorem, the graphical study of the concept of limits and convergence, the integral curves and field-plot of differential equations, recurrent use of calculators to study progressive sequences and limits, and many others. Today there are many very good works using CAS, DGS, spreadsheets, calculators, etc.

Now, we point out that in this type of class the learning environment does not change much, the student is a passive observer of the technology and the evaluation of the achievement of the knowledge relies usually on traditional tests.

Other important observation in this type of class is that the teacher takes the most benefit of the technology in the sense that he/she uses the facility provided by the technology to deliver better lectures, and also he/she can feel the pleasure of creating his/her own activity. This last part show to the teacher the necessity of a good knowledge of mathematics, often more advanced than the topics he/she teaches, and of mathematical language of computing tools in order to create a good teaching material, therefore the importance of mathematics curriculum of Teacher Preparing Courses becomes clear.

Some ready-to-use programs, worksheets and files made available in the educational and personal websites are examples of the technology that are offered to those teachers who want to take the advantage of technology to enrich their classes but do not feel comfortable enough to make their own, or do not have time to develop them. Still, the knowledge about the mathematical limitation of the technology and properness of the activities to be used in the classes is required.

Soon it became clear from the experiences that the effectiveness of the technology in the educational context is to put the technology in the hands of the students. In (Waits & Demana, 2000) the authors say "change can occur if you put the potential for change in the hands of everyone".

Thus we discuss the **second category** of the classification. The technology makes possible new methodologies for teaching mathematics. The most important is its participation as a facilitator in those **student-centered** activities. In the laboratory-type classes the students actually manipulate computer software or graphing calculators, therefore playing the active role as user of technology and in learning process. The most representative activities of traditional teaching/learning strategies that can be compared in this category are exercises at the classrooms such as the activities of algebraic manipulations, homework with drill-exercises, etc.

The teaching material for this category, for either individual or group use, may be: a) a hypertext type programmed instruction, in which the student advances his/her understanding about some subject through step-by-step activities; b) an interactive worksheet allowing the student to manipulate the data, speculate and formulate conjectures; c) a worksheet to test the achievement of knowledge; d) laboratory-type activities for problem-solving and activities requiring the use of technology; e) homework and projects requiring the use of technology; f) games of mathematical content to stimulate the interest and to evaluate the mathematical abilities; and many others.

A look at this (incomplete) list shows the didactical potential that the technology can offer to do different teaching activities in or out of the classrooms.

Also, we classify in this category the student's use of technology to document and write reports on their activities, the Internet sites to be accessed by the student, the distance education material, and ready-to-use type didactical material.

The most important property of this use of technology is that the student changes the behavior from passive to active; he/she becomes a responsible participant of his/her instruction. The output produced by the student is his/her own effort and he/she can feel the satisfaction of accomplishment and being a real participant of his/her education. Also the teacher can accompany the rate of the learning progress continuously and in personal basis. The teacher plays an important role of advisor and supervisor of the activities.

The **third category** of the classification represents the most innovating feature of the technology as teaching aid and it is the most promising in making changes for the future strategies. Many researches in mathematics education related to technology point to the transition of traditional expository classes to those based on problem solving activities with mathematical modeling, and also technology aided development of mathematical reasoning and proofs of theorems.

The technology allows to teacher and student to communicate each other in a renewed process of understanding and constructing mathematical concepts. Through activities of modeling, visualizing, conjecturing, testing, confirming, etc, the teacher has the opportunity to show to the students the mathematical language and reasoning, building **together** the paths of the construction of results and connections to real life. This category actually reunites the properties of first and second categories of the classification, strengthened by the capabilities offered by the technology.

Each one of the categories described above has its importance and own place in teaching/learning process, and a teacher must be able to plan his/her class, choosing the right strategy. Yet, in every situation, either making his/her own activity or using the existing material, he/she must be prepared to use a specified software or equipment, knowing its mathematical capabilities and limitations, as well as be prepared with the mathematical language and concepts required to make (or use) the activities, sometimes beyond the content of the topic related to the activity. This is the case if one makes his/her own material, for example programming scripts with algorithms, numerical methods, or designing figures requiring notions of parametric and implicit representation, etc.

This subject is one of most important issues in mathematics education of teachers, and just problem solving or modeling strategy deserves a proper discussion, which is not the scope of the discussion brought in this paper. In (Baldin, 2002) we present examples of topics in teacher preparation courses regarding the limitation of technology.

3. Example

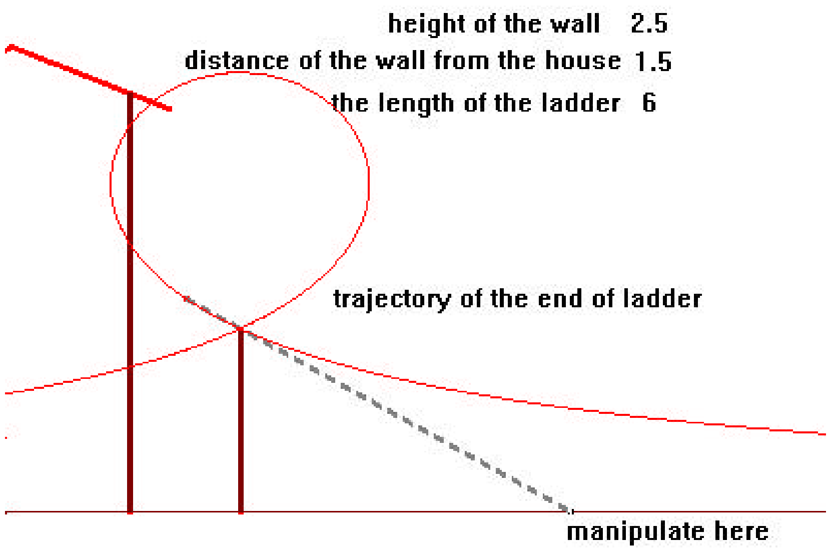
Due to the limitation of the pages, we exhibit one example of teaching situation in basic level in which the three roles of technology as teaching aid can be seen, and also the mathematical concepts and language in this activity that should be expected from a basic level teacher. Other examples from author's experiences can be found in the references.

Consider the classical "ladder problem":

"There is a two story house surrounded by a wall, height 2.5m. The wall is 1.5m distant from the house. Some firemen want to reach the house from outside, using a ladder of length 6m touching on the outside wall. How far from the wall the ladder must be put?"

Although very simple, the students (prospective and in-service teachers) are often surprised with the different use of technology that can turn the problem solving an amusing experience.

First, a DGS activity, previously prepared, illustrating the problem through the profiles of the house, wall and ladder is shown, in order to situate the problem and to facilitate the modeling.



A simple example like this, connected to real life situation, when illustrated with DGS, allowing a manipulation to experiment the possibilities can clarify some mistakes that one may make only by guessing, such as "the solution is unique" or "once the ladder touches the house, you may slide it along its wall to get other solutions in a continuous manner", etc.

The visualization of the problem suggests naturally a geometric modeling of the problem, recalling the concepts of "similitude of triangles" and "Pythagoras's Theorem". This produces a non-linear system of 2 equations, with appropriate variables.

$X*Y = (1.5)*(2.5)$ and $(X+1.5)^2 + (Y+2.5)^2 = 6^2$

Solving this system is clearly a task that needs the use of technology. The **algebraic approach** is the natural try of everybody. If CAS like Maple is used one can get the solution immediately, but this means that the user is not doing mathematics, is transferring the job to the software. A teacher can do better to explore the mathematics behind a simple problem with a pedagogical use of technology.

A student with the Graphic Calculator, (e.g. TI-92), is able to train his/her algebraic reasoning, by substituting Y= (1.5)*(2.5)/X into the second equation, which can be transformed and conducted to a polynomial function of 4^h order in X to get the final solution. The commands on the calculator follow the natural syntax, more friendly than those of Maple, and actually are very didactical to realize the mathematical language. The algebra of polynomial functions can be connected to this problem at this moment, and the teacher must be aware of The Fundamental Theorem of Algebra. The graphic plot of this polynomial function can show also the behavior of such a function and the meaning of the zeroes.

The problem can be treated through a **geometric approach**. From the first equation one get an explicit function Y (X) = (1.5)*(2.5)/X, defined on the open interval $(0,\infty)$. The second equation can be recognized as the equation of a circle with center (-1.5; -2.5) and radius 6 in a system of rectangular coordinates. Therefore the solution is given by the intersection of the graphics in such a system.

With the graphic calculator (TI-92), one can get one of explicit forms of Y (X) from the second equation, choosing the adequate formula. The simultaneous plot of two graphics give the solution displayed in an interactive screen, in which the cursor on the intersection points "reads" the solution. It is quite exciting to the students to solve an algebraic system without doing algebraic calculation!

We observe now that we have in hands an opportunity to explore the capabilities of DGS in exploring the concept of function arisen in the explicit formula of Y (X) from the first equation.

In Maple or the plot editor of TI-92, the graphics of functions are produced from the **expressions** Y(X), and this is the general understanding of students that leads them to make frequent confusions between the **concept of a function** and the **expression** that defines it. Using a DGS, like Cabri-Géomètre II, we can reconstruct didactically the concept of a function, following the order of mathematical elements of the definition as well as to study dynamically the dependence between the variables.

The strategy is to explore the "Locus" tool, summarized briefly as: 1) construct the *domain* of a given function as an object on the X-axis; 2) construct a point X **on** the domain; 3) calculate the abscissa of X; 4) calculate the expression of Y (X), **inserting** the value of X into the **interactive calculator** of the program; 5) construct the point Y on Y-axis with the result of the calculator; 6) construct the (X, Y) point in the plane; 7) construct the Locus of points (X, Y) in the plane, depending on X.

A teacher can follow together with the student the conception of a function and its graphic step by step in the procedure above, and study each part of a function (domain, correspondence law, image) in right order. With this construction, the function of the example can be explored as an **inversely proportional** function defined for $x > 0$. This property has a real meaning in the problem! The "Compass" tool provides the graphic of second equation as a circle, and its equation confirms the result.

The intersection points of two plots would give the solution. Yet, Cabri does not confirm on the screen the first equation, because Locus is not a constructed object. Can a teacher solve this trouble? Give up Cabri? A teacher should know that the first equation is a rectangular hyperbola, so the "Conic" tool can be used to get a conic constructed on the previous locus. The equation confirms the first equation. A teacher must know from Linear Algebra why this fact is true, and also that 5 points are sufficient to determine a non-degenerate conic to understand the "Conic" tool.

Connecting algebra and geometry is a very important aspect of basic education and the problem above illustrates how the technology can help in this task, using all the communication features.

4. Conclusion

The understanding of different ways the technology can be used in basic education would help the curriculum of teacher preparation courses to include analysis of the software in the light of mathematical foundation, as well as to in-service teachers to feel more confident in choosing the activities for their classrooms and to take profits from the literature on mathematical education.

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