PROBLEM SOLVING FOR FUTURE TEACHERS - AN INDIVIDUAL LEARNING COURSE

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ABSTRACT

At the Faculty of Education, Charles University, Prague, problem solving represents one of the key subjects in the preparation of future teachers. For four years, the first of a series of problem solving courses has been organised as an individual learning course as the only course during the study. By that we mean that students have no scheduled classes, they work individually and meet their teacher for consultations. The main aims of this form of study (besides the obvious goal to teach different strategies of problem solving) are to acquaint students with the range of mathematical books and textbooks and to develop their ability to (a) work independently, (b) take responsibility for their learning, (c) critically evaluate mathematical texts, (d) write mathematically.

The course comprises three topics: Equations and their Systems, Number Theory and Plane Geometry. Students have to submit one seminar work for each topic which includes solutions to (a) problems given by the teacher (different for each student), (b) problems chosen by students from the assigned literature, (c) an 'extra' problem chosen by students from any book but with a short justification of their choice. The fourth and last seminar work has a different character – it is an essay in which at least two books or textbooks used during the course are evaluated according to a student's criteria. Finally, students sit for a short test.

After the term, students are asked to write a short anonymous evaluation of the course (they mention advantages and disadvantages of an individual form of work and give suggestions for improvement). These written evaluations and their analysis contribute to the running modifications and improvements of the course over time.

Keywords: individual learning course, problem solving, evaluation, design of a course, student teachers

1. Introduction

Students at the Faculty of Education, Charles University in Prague, start their studies with the aim of being mathematics teachers from the outset. This means that they do not study with future mathematicians as in some other countries. They will qualify to teach students in the age range 11-19 years.

When students leave secondary school and enter the university, they have to learn a different way of working. They are supposed to take responsibility for their learning and rely less on their teacher, to organise their study themselves, to study literature independently and choose relevant information, to be able to communicate their mathematical ideas in writing and as future mathematics teachers, to be able to explain their solving procedures. At the university, more stress is put on home study rather than class learning. This change is not always easy for students. Taking their difficulties in the above areas into account, it was decided in the Department of Mathematics and Mathematical Education that a new type of course should be designed – an individual learning course. When looking for suitable subject matter for this type of the course, we concentrated on less formal courses which could include the study of a variety of books (so, for instance, abstract algebra or calculus were ruled out). Finally, the course 'Problem Solving I' was chosen. The course is offered in the fourth term of study, i.e. students are in the second year of their five-year study.

2. Framework

At the university level, many types of teaching-learning situations can be determined, some similar to those in elementary or secondary schools, others specific to university teaching: tutorials, lectures, seminars, individual instruction, demonstration, class discussion, home study, etc., among non-standard teaching-learning situations we have, for instance, scientific debate (Alibert, Thomas, 1991), and using constructive, interactive methods involving computers and co-operative learning (Leron, Dubinsky, 1995).

In recent years, problem solving has become one of the most important activities of school mathematics, the main reason probably being that it "places the student in the role of actor in the construction of his/her own knowledge" (Grugnetti, Jaquet, 1996). There has been a considerable body of research concerning its use in teaching mathematics (see e.g. Frank, Lester, 1994, Schoenfield, 1992). Problem solving at the university level is explored e. g. in Yusof, Tall (1998).

In this article we will present a non-standard way of teaching problem solving to future mathematics teachers which we call an <u>individual learning course</u>. By that we mean a course which does not include any scheduled classes and consists mostly of individual home study (even though students can co-operate) and consultations with the teacher.

3. Design of the individual learning course

3.1 Aims of the individual learning course

The course has two types of aims and goals. First, there are the goals specific to the *content* of the course: that students are aware of various techniques of problem solving and learn to solve problems out of context[1]. Second, there are aims specific for the *form of work*, i.e. the individual learning course:

- to widen the range of different forms of work with students
- to develop a student's ability to take responsibility for his/her own learning
- to acquaint students with the relevant literature which can be used both for their problem solving at the university, but also for teaching problem solving at school
- to develop a student's ability to write mathematically and formulate a mathematical text
- to develop a student's ability to work independently
- to develop a student's ability to read and understand mathematical texts written for different purposes and audiences, and critically evaluate them in terms of their suitability for a certain purpose
- to enable students work both individually and in teams

3.2 Content of the course

The course 'Problem Solving I' is the first of a series of problem solving courses which focus on basic methods of problem solving. It is the only one which is organised as an individual learning course, the others are organised in the classical way via seminars. It comprises three topics – Equations and their Systems, Number Theory, Plane Geometry. Its content will be briefly illustrated by several problems from individual topics which are taken from a <u>Booklet</u> for students (see below).

Topic and subtopics		**Illustration**
Equations	systems of equations solvable by a 'trick'	Solve a system of equations using a method other than the Gaussian elimination method: $x_1 + x_2 + x_3 = 6, x_2 + x_3 + x_4 = 9, x_3 + x_4 + x_5 = 3,$ $x_4 + x_5 + x_6 = -3, x_5 + x_6 + x_7 = -9, x_6 + x_7 + x_8 = -6,$ $x_7 + x_8 + x_1 = -2, x_8 + x_1 + x_2 = 2$
	equations which include the integer part of a number	Solve the equation in **R** ([x] is the integer part of x): $[(5 + 6x) / 8]=(15x - 7)/5$
	graphical solution to a system of equations	Solve the system of equations graphically: $x^2 + y^2 < 11 - 2(x - 2y),\ x^2 + 4x \geq 2y - y^2 + 4$
	system of equations with a parameter	Discuss the number of solutions of the following system of equations in terms of a real parameter m, x is the unknown: $x^2 + y^2 = 4, (x + m)^2 + (y - m)^2 = 1$
	equations solvable by a suitable substitution	Solve in **R**: $2x^2 + 6 - \sqrt{2x^2 - 3x + 2} = 3(x+4)$
	more difficult systems of equations with parameters	Solve in **R** the system of equations with the unknown x, y, z in terms of real parameters a, b, $c > -1$: $y + z + yz = a, \quad z + x + zx = b, \quad x + y + xy = c$
	more difficult systems of equations of a higher degree	Solve the system of equations in **R**: $x^4 + y^4 + 3x^2y^2 = 109$, $x^2 + y^2 + xy = 13, z(x + y) = z + x + y$
Number Theory	systems of Diophantine equations	For which x will the numbers $(x - 3)/7$, $(x - 2)/5$ and $(x - 4)/3$ all be whole numbers?
	Proofs of theorems on the divisibility of numbers	Prove that for all natural numbers n $3^n \mid 111...11$ (there are 3^n of ones).

[1] In view with Arcavi (1998) we believe that this goal is very important as in "traditional courses problems and exercises are often sequenced in such a way that students can easily find solution techniques".

	‘algebrograms’ – looking for numbers with certain properties	Find all four-digit numbers which are the squares of a natural number so that their thousand’s digit is the same as their ten’s digit and their hundred’s digit is bigger by one than their unit digit.
	least common multiple, greatest common divisor	For which natural numbers n is (a) NSD(n + 6, n + 2) = 4? (b) NSD(6, n + 3) = 3?
Plane Geometry	construction of a triangle – non-trivial problems	Construct a triangle *ABC*, if we know: $b - a$, v_c, r (r is the radius of the inscribed circle, and v_c the height from *C*)
	construction of a quadrilateral and other polygons	Given points *S*, *O* and a line *p*, construct a triangle *ABC* so that the centre of the circumscribed circle is *S*, the centre of the inscribed circle is *O* and its side lies on the straight line *p*.
	problems on proofs of relations between elements of polygons	Consider a triangle *ABC* in which the angle *ABC* is not right. On side *AB* construct a square *ABKL*, which does not lie in the half-plane *ABC*. Similarly, on side *BC* construct the square *CBMN*, which does not lie in the half-plane *CBA*. Prove that the triangles *ABM* and *KBC* are congruent.

3.3 Set literature

The set literature consists of about sixteen books which range from secondary school collections of mathematical problems to books for university students organised in the way ‘definition – theorem – proof – problems’. Many of them are organised as ‘exposition – examples – exercises’. Students can also use various collections of Mathematical Olympiad problems and some journals on mathematics education. The books are also from different times so that students get to know the style of writing from different periods of the development of mathematics. Books do not only include problems from the three topics above, but other topics too, so that students have to choose parts relevant to their course.

It is important to stress that these books are not specifically designed for individual learning. Some of them are meant for the classroom use, while others are for tutorial/seminar use.

3.4 Organisation of the course

The core of the course work lies in the student’s independent work and his/her solving of mathematical problems and then summarising their solutions for a seminar assignment. The interaction with a teacher is limited to his/her office hours when a student may but does not have to come to see him/her. The organisation of course work will become clear in the next section on the course implementation.

4. Implementation of the individual learning course

The course was first implemented in the school year 1996/97 in the spring term and has been offered in the spring terms in subsequent years. Its content and organisation differed a little from one year to another according to the students’ and teachers’ evaluation (see below). The description below fits the current state of affairs.

4.1 Students’ work

A student is given a <u>Booklet</u> (prepared by a teacher) which includes: (a) worksheets with problems from each of the three topics, (b) details of literature he/she should study for each topic: which parts, which problems, to what extent, (c) details of the seminar work for each of the three topics,

(d) the assignment of the essay (fourth seminar work), (e) details of the written test, (f) deadlines for submitting all seminar assignment.

Each of the three seminar assignment has the following content:

- compulsory problems from the worksheets (a); these are assigned in such a way that if possible, no two students have the same problems
- problems chosen from a certain part of the recommended literature (b); for instance, for the topic Equations students have to study a certain booklet and choose two problems which are not solved there and solve them
- an 'extra' problem which can be chosen according to the student's liking from any literature, but which must be relevant to the topic and at an appropriate level. The student must justify his/her choice

The fourth seminar assignment, the essay, differs from the previous ones. It includes a student's evaluation of the literature (at least two publications) from the point of view of their use in the subject 'Problem Solving I'. There is no limit set for the extent of this project.

During the term, students work on the four seminar assignments individually or they can co-operate. They study at home, go to the library or can ask the teacher to be allowed to study in the department library where all the literature is available for them. While doing so, they can go to see the teacher and consult their work.

During the examination period, a test is written which consists of six problems (two from each topic) which are chosen from a given set of problems (e) (some from the worksheets, some from the literature). An example of such a test is given below.

4.2 Teacher's work

The teacher prepares the Booklet and gives it to students at the beginning of the term. This booklet is constantly revised after each term. He/she sets four deadlines during the term by which students have to submit their seminar assignments[2] and the date of the written test. He/she sets office hours and is available to students during these times to discuss their problems. He/she gives a test to students during the examination period.

The focus in the subject lies in seminar assignments. It is a teacher's task to assess them as students hand them in[3]. Each seminar assignment is evaluated by means of points and is accepted if a minimum number of points is gained. If this is not so, the teacher discusses the work with the student and he/she can correct it and submit it again.

This course represents a student's first opportunity to 'write mathematically'. They gradually learn how to do it. Among the most frequent problems is their inability to explain in a logical way their solution strategy. They sometimes write in a too succinct a way, omitting important parts of the explanation because they do not realise that writing mathematically has different rules than when they directly explain their solution to the teacher.

The assessment of the essay is a subjective one. The teacher takes into account:

- if the choice of books is appropriate
- if the text is structured clearly
- if the mathematical language used is accurate
- if the student chose appropriate criteria for the evaluation of the books

[2] The setting of deadlines is essential, otherwise students tend to hand in their work towards the end of the term and the teacher is not able to correct them all at once.

[3] It is our desire that students get feedback on their work as soon as possible and that the teacher speaks with each student about at least one seminar assignment during the term (unless the work must be redone, of course). However, it is not always possible and it depends on the number of students who enroll in the course.

- to what extent the student evaluates the books critically (if he/she expresses his/her opinion and not only lists the content of the books, if it is clear from the text that he/she knows the books sufficiently well, etc.)

The test comprises six problems from three topics and each topic must gain at least 60 percentage points in order for the test to be accepted. Students can write the test three times[4].

Example of the test:

1. Solve in **R**: $x_1(x_1 + x_2) = 9,\ x_2(x_1 + x_2) = 16$
2. Solve in **R**: $\sqrt{x^2 + x + 7} + \sqrt{x^2 + x + 2} = \sqrt{3x^2 + 3x + 19}$
3. Prove that for each n natural is $57 \mid 7^{n+2} + 8^{2n+1}$.
4. Find all primes which are at once a sum and a difference of two suitable primes.
5. Construct a triangle *ABC*, if we know v_c, γ, $\omega = |\angle BCD| - |\angle ACD|$, where *D* is the foot of the altitude v_c.
6. What is the sum of the inner angles of a polygon, which has 52 diagonals more than sides?

Students get a credit provided their three seminar assignments and the essay have been accepted and that they successfully wrote the test.

5. Evaluation of the individual learning course

After its first implementation, the course was evaluated both by the students and by the teachers. The results of this evaluation led to the redesign of the course both in its content but mainly in its organisation. This evaluation has been repeated several times since and each time led to additional changes in the organisation. Below we present both the students' and the teachers' evaluations.

5.1 Students' evaluation

Several years ago, when the individual learning course took place for the first time, we were not sure about its positive and negative aspects. Therefore we started with the students' evaluation of the course. They were asked to submit a written anonymous evaluation of the course. Their comments were taken into account and the course was redesigned. Moreover, in the essay students sometimes spontaneously express their opinions and suggestions. Some of them, which we consider to be typical and important for the course, will be given below. Some are positive, some are negative, some include suggestions.

- *For the first time I was forced to look up literature and solve independently problems which I had not met before.*
- *We are at least forced to search through literature which we would not normally see.*
- *The advantage is that one has to study independently, learn how to work with literature, to be active and patient.*
- *One can solve problems when it is most convenient and this is not dependent on the timetable.*
- *One can study at home 'in peace' and can consult a teacher if necessary.*
- *I believe that the course's main goal was not to solve the problems accurately, but to learn where the problems can be found and to look at them from two perspective: as a problem solver and as a teacher who will be using similar problems him/herself during teaching.*
- *It is very subjective to evaluate books.*
- *It was difficult to get some books from the set literature.*
- *It was too much work to get a credit, in other courses it is easier.*

[4] In the Czech universities, students usually have three attempts to pass an examination.

- *It is a disadvantage that we could not feel the presence of a 'mediator' of knowledge, someone who could react immediately to our questions.*
- *The test should be abolished, seminar assignments themselves are enough.*
- *I would like to know if the problems I solved could have been solved differently.*
- *It is not assured that the students solve the problems independently. The teacher should speak to each student and ask him/her how they solved the problems. Then he/she could be sure that the student understands the problems.*
- *It might be good to include several teaching lessons during the term.*

5.2 Teacher's evaluation

Here, we will summarise the main features of the course.

From the first time it was used, the individual learning course has been constantly evaluated by both students and teachers and has been redesigned several times to meet students' need and the course aims. Therefore, we can claim that it is flexible.

Even though students do not meet in scheduled classes, they know each other (unlike in traditional distance learning), they meet during other courses and often co-operate when working on their seminar works. Such co-operation is desirable provided that it is meaningful for all participating students and that one does not merely copy the other's work.

There is a limited amount of interaction between the teacher and the student which means that students cannot benefit from the immediate exchange of ideas with the teacher. They do not get an immediate response to their queries. However, on the other hand, they are made to try to find the answers themselves or look them up before approaching the teacher. This contributes to the students' ability to study independently and organise their own learning.

It is important to strengthen the feedback – to speak with the students mainly about the mistakes and imperfections in their work. Sometimes to let them explain their thinking, to show them that there was a more 'elegant' solving strategy, etc. However, the individual learning course is time consuming not only for the students, but for the teacher as well. Therefore, it is necessary to find a balance between the number of students and the number of teachers.

6. Conclusions

When we take into account both the students' and teachers' evaluation, the students' results in the written test and seminar assignments, we believe that the individual learning course has its place among other more traditional courses in the preparation of future mathematics teachers and that it serves our aims well. However, we are well aware of its drawbacks and try either to remove them or to compensate for them in other courses. For instance, in the course 'Problem Solving II' more stress is put on the teacher-student interaction and group work. At present we are considering some changes in the written test. Instead of using problems from a set of problems, which students know in advance, we would like to use problems similar to those in the books. Students could then use any literature they want during the test.

The contribution was supported by grant GAUK 316/2001/A PP/PedF.

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