

USING TECHNOLOGY AND COOPERATIVE GROUPS TO DEVELOP A 'DEEP UNDERSTANDING' OF SECONDARY SCHOOL GEOMETRY

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ABSTRACT

In Fall 2001, the Conference Board of the Mathematical Sciences (U.S.A.) released a list of recommendations on the mathematical preparation of prospective teachers. These included recommendations that prospective teachers take courses that “develop a deep understanding of the mathematics that they will teach” and that “prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.”

In 1997, Ohio University significantly revised the ‘Foundations of Geometry’ sequence taken by prospective secondary teachers. The revised course uses a significant amount of group work and technology to plant the seeds for a deep understanding of the geometry taught at the secondary level. By using software programs and manipulatives, students begin building an understanding of non-Euclidean and Euclidean geometry from the outset of the course. The use of cooperative group work and written reports on group projects develop student’s writing and oral communication skills.

A major goal of the course is to give the students the experience of ‘doing mathematics’. During the course, the students use the experience gained using software programs and manipulatives to develop their own axiom systems and use these systems to prove theorems. This paper describes the overall structure of the course, how and where various learning aids are used, and discusses the effectiveness of the course in promoting a ‘deep understanding’ of the secondary school geometry curriculum. The assessment is based on student work and journals collected during the first four years the course was offered in its current form. The evidence suggests that the students improve their ability to prove theorems and develop a good understanding of models and axiomatic systems.

Keywords: Geometry, Non-Euclidean Geometry, Geometer’s Sketchpad, Secondary and Middle School Teacher Preparation

Introduction

In its Summer 2001 report, *The Mathematical Education of Teachers*, the Conference Board of the Mathematical Sciences (U.S.A.) “calls for a rethinking of the mathematical education of prospective teachers within mathematical science departments.” [CBMS, pg. 3] The report is sweeping in scope and makes eleven recommendations related to the mathematical training of prospective teachers, cooperation among the parties involved in teacher education, and the support of high quality school mathematics teaching.

The first recommendation is that “Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.” (Recommendation 1) They note that K-12 teachers need a mathematical foundation that will help them assess errors, nurture talented students and recognize their students’ level of understanding. Proof and justification are also emphasized. The fourth recommendation asserts that mathematics courses should “. . . develop the habits of minds of a mathematical thinker and demonstrate flexible, interactive styles of teaching.” In a discussion of technology, the report states that prospective teachers should be given experience with technology with two goals in mind: the short term goal of using it in teaching and the long-term goal of helping them “become thoughtful and effective in choosing and using educational technology.” [CBMS, pg 48]

In Fall 1997, Ohio University began to offer a revised geometry course that meets many of the criteria suggested in the Conference Board (CBMS) report. This course is taken primarily by prospective middle and secondary school teachers. Motivated by the desire to have a course aligned with the NCTM standards [NCTM], to have the students gain the experience of ‘doing mathematics’ and develop the material in a manner consistent with pedagogical styles that they will eventually have to adopt for licensure, the course rigorously develops much of the content of the secondary school curriculum using structured cooperative groups working in a computer lab.

In that the revised course used a significantly different teaching style and added some content, it was natural to wonder how students would respond to the course and whether or not the course was effective in meeting its goals. From the outset, the instructors kept copies of student work and journals with a view towards assessing the effectiveness of the course.

Course Description

The revised geometry course was designed to give students a strong understanding of the content of a standard secondary school geometry course and to build connections between geometry and other areas of mathematics. Although developed prior to the release of the CBMS report, it is consistent with many of the recommendations the CBMS report makes regarding geometry courses for prospective teachers. In particular, it provides a solid understanding of core concepts of Euclidean geometry; an understanding of the nature of axiomatic reasoning and facility with proof; multiple representations; an introduction to transformations; and uses dynamic drawing tools to conduct geometric investigations. [CBMS, pg.41] In that teachers who are taught using ‘reform’ techniques tend to use them more than teachers taught using traditional techniques ([Jo91], [MTH95], [SS94]), the revised course was taught in a computer lab using structured cooperative groups.

The course provides an in-depth discussion of the axioms used in some standard secondary geometry texts. In the United States, many secondary school geometry texts include a development of Euclidean plane geometry (loosely) based on a set of axioms developed by the School Mathematics Study Group ([WW98]) in the early 1960's. (It should be noted that these texts also often discuss transformational geometry and introduce non-Euclidean geometry.) In the revised course, we develop this axiom system and establish the standard results of Euclidean geometry.

There are several ways in which the revised course is meant to sharpen the preservice teachers understanding of Euclidean geometry. First, Euclidean geometry is developed in greater detail than the typical secondary text. In order to simplify the mathematics at the secondary level, secondary texts often incorporate theorems into the axiomatic system. For instance, it is not unusual for each of the ASA, SAS, SSS and SAA criteria for the congruence of triangles to be assumed as axioms (cf., [Sch01], [Ser97]). In the revised course, one of the group projects requires the students to establish that SAS implies ASA and SSS. Secondly, students explore the validity of familiar Euclidean propositions and concepts in non-Euclidean settings. For instance, they also justify that AAA is a criteria for the congruence of triangles on the sphere and in the Poincaré disc using Lenárt spheres and a software program.

Third, the course develops some topics from multiple viewpoints. Transformational geometry is introduced via MIRAs, using matrices and vectors, and from an axiomatic approach. The result that the composition of three reflections with concurrent axes is a reflection is discovered in a group project using a manipulative (the MIRA) and GSP, revisited as a result on matrices on \mathbf{R}^2 with the Euclidean metric, and then given an axiomatic proof which is valid in elliptic, hyperbolic and Euclidean geometry.

Lastly, the topics are developed from a constructivist viewpoint. At the beginning of the course, while working in groups, students are asked to develop their own axioms and definitions in order to establish some well-known results from geometry. This is consistent with the 'Necessity Principle' suggested by G. Harel: "Students are most likely to learn when they see a need for what we intend to teach them, . . .", where the 'need' is an intellectual need [Ha00]. After class discussion, the students use their definitions and axioms as the basis for explorations into some models of non-Euclidean geometry. Most lectures are based on group projects that introduce the topics covered in the lecture.

The content of the course is introduced via group projects; approximately 70% of the class time is spent having the class work in structured cooperative groups. Each project consists of two or three progress reports, which require each group to write-up the results of their investigations. The progress reports are collected and returned with written comments. Although the comments address writing style, minor and major errors, points are only deducted for major errors. Each project ends with a final report in which the students rework some of the results of the progress reports and synthesize the results of the progress reports and lectures related to the project; points are deducted for both minor and major errors in the final report.

In addition to the group work, students are assessed via traditional exams (two midterms and a comprehensive final), individual quiz scores, individual homework and journal entries. Group work accounts for 40% of the student's final grade; the remaining 60% is based on individual work.

The first project introduces students to axiomatic systems by having them develop an axiom system that will allow them to establish the standard formula for the area of a trapezoid. The project combines the use of technology, lectures and cooperative group work in the following manner:

- Progress Report 1: Students describe a procedure for finding the area of a polygonal region assuming they know the formula for the area of a square and a triangle. They then use one of these procedures to ‘justify’ the standard formulae for the area of a triangle, square, rectangle and trapezoid.

- Lecture on axiom systems: Students are introduced to the idea of an axiomatic system and a model of an axiomatic system. In particular they are introduced to axioms, primitive terms, definitions, and theorems. Students do a homework set based on the material introduced in the lecture.

- Progress Report 2: Based on their work in progress report 1, the groups develop a set of axioms for a theory of area and definitions for rectangles, trapezoids, et cetera. They verify that GSP is a model of their axiomatic system and then prove the standard area formulae using their axiomatic system.

- Progress Report 3: Students are introduced to spherical geometry. Using Lenárt spheres, they explore the validity of their area axioms on the sphere. They modify their area axioms and use the modified axioms to obtain the formula for the area of a triangle on a sphere.

- Final Report: A class discussion leads to a consensus on a set of area axioms. The students write up their axiomatic systems for area in the plane, provide proofs of the standard area formulas, and prove a formula for the area of a triangle on a sphere.

The second project proceeds in much the same way as the first. Students are given a set of axioms to produce rays and measure angles and the groups prove that the angle sum of a triangle is 180. To do this they need to add an axiom equivalent to the Euclidean parallel postulate. They are then introduced to the Poincaré disc via a software program (NONEUCLID) and establish that it also satisfies the axioms used to construct rays and measure angles. In the second progress report they explore the Poincaré disc via several statements equivalent to the Euclidean parallel postulate. The defining characteristics of absolute, elliptic, hyperbolic and Euclidean geometry are then introduced in a lecture. In a final report, the groups establish that one can construct parallels in absolute geometry and, that if the Euclidean parallel postulate holds, the angle sum of a triangle is 180.

At this point, the students are about a third of the way through the two-quarter sequence. They are working with software models of Euclidean and hyperbolic geometry and a physical model of elliptic geometry. During the remainder of the sequence, they study congruence of triangles, similarity, circles, the ruler postulate and given an axiomatic introduction to transformational geometry using a similar pattern of progress reports and lectures.

In order to encourage student participation, each final report and progress report has a quiz associated with it. The ‘correct’ answers for the quiz are based on the group’s work on the report and the total of the quiz scores consists of 30% - 40% of the grade for the report. The intent of the quizzes is to keep individual group members engaged in the project and to prevent one person from dominating the group and submitting work only he/she understands.

Student Response and Performance

Overall, the students respond to the course in a positive fashion. Initial concerns about group work and writing proofs diminish as the course progresses and, at the conclusion of the course, (anonymous) student evaluations for the course are nearly entirely positive.

As students enter the course, although they had taken an introduction to proof course as a prerequisite for the course, their journal entries contain spontaneous remarks indicating concern over their ability to create proofs and/or their ability to communicate their proofs to others. By the fourth week of the course (journal 2), some positive comments are made regarding proofs and by the end of the ninth week (journal 4), far more positive entries than negative entries occur. As the course continues into the second quarter, fewer entries regarding proofs, both positive and negative, occur.

One of the main goals of the course is to have the students learn to create and write proofs. The students generally view working in groups and using technology as having a positive effect on learning how to do proofs. In student journals from 6 two-quarter sequences, students made 134 comments on these issues; 106 were positive and 28 were negative. Student journals indicate that working in groups is beneficial in that it allows brainstorming, peer instruction, group checks of proofs and confidence building. Negative comments included that 'time pressure' sometimes did not allow individuals time to understand the entire project, that there was difficulty transferring skills from group work to individual work, sometimes the groups developed and learned incorrect arguments, and that group work slowed progress through the material. The primary positive theme regarding use of technology is that it provides a context to do explorations and build intuition with different geometries. The negative comments included that it was hard to move from the dynamic drawings to axiomatic arguments, and learning the programs took time away from the mathematics. Most student comments regarding the use of technology and group work are made before the tenth week.

The quality of proofs submitted during the group projects improved over time. In order to test whether or not proof creation and writing ability improved, three proofs were identified, each of them appearing in a progress report and final report. For each proof, 5 or 6 key steps or issues were identified and the submitted work was evaluated as follows:

1. In each proof and for each issue, it was determined whether the issue was partially identified or clearly *identified*; $\frac{1}{2}$ point was given in the first case and 1 point in the second. These points were then summed for each issue over all of the proofs reviewed. (For instance, if looking at the work of 9 groups, 2 had missed the issue X, 4 had partially identified the issue X and 3 had clearly identified issue X, a total of $2 \times 0 + 4(\frac{1}{2}) + 3(1) = 5$ 'identification' points would be associated with issue X.)

2. In each proof and for each issue, it was determined whether an issue was partially (1/2 point) or correctly (1 point) *resolved*. For each issue, the 'resolved' points were summed as above.

Figures 1 - 3 show the results for the proofs that appear in the progress reports. Each letter indicates an issue or step related to that particular proof. Note that at week 2 the groups first have trouble identifying issues and then resolving them. In week 5, they are better at identifying issues that need to be resolved. At week 12 of the 20-week sequence, scores for identifying and resolving issues are about the same. In addition to doing these proofs in a progress report, the same groups were asked

to redo them in the final report for the project. Performance did not significantly increase and, in some cases, performance actually declined on the second attempt.

At the conclusion of the first course in the sequence, anonymous student evaluations often contain comments indicating that they found the combination of technology and active learning led to a better understanding of the material than a traditional lecture based course.

In order to test the validity of this perception, exam performance was compared on topics developed in lecture vs. topics developed in groups. Using 6 sets of final exams, it was found that the students earned 68.7% of points possible on group based questions and 72% of points possible on lecture based questions. (There were 248 responses to 25 questions, 134 responses to lecture topics and 114 responses to group topics.) The distribution of scores appears in Figure 4, which shows the percentage of responses that earned a particular score. For instance 47% of the responses to the lecture-based questions scored either 9 or 10 on a scale of 0-10. Note that the 'group' performance is slightly better in the middle scores.

Several topics developed in the revised course had also been covered when the course was taught in a traditional lecture format. These topics had been developed in class and stressed as important. Student performance was analyzed using final exams from the 'revised' course and from the 'traditional' course. Figure 5 shows the relative frequency of scores for 159 responses to 4 test questions, 68 from the revised course and 91 from the lecture course. As before, performance on the exam appears to be approximately the same for the traditional and revised course.

The final exams from the traditional and revised courses also had some 'novel' questions that had not been discussed in class; the intent of these questions was to test the students' ability to apply the content of the course to an unfamiliar problem. The students in the traditional course earned 47.1% of all possible points and in the revised course they earned 50.4% of all possible points. The performance of the two groups on these questions is shown in Figure 6; note that performance in the revised course is slightly better in the middle scores.

Discussion

The most remarkable aspect of the above analysis is that there does not appear to be a substantial increase in student performance when a topic is developed in groups instead of a lecture format. Students spend far more time with a topic when working in groups, have discussions with the instructor on parts they are having difficulty with, receive comments on their written work, and, in this course, eventually receive a solution sheet with a correct version of the argument. When a topic is developed in lecture, it is discussed once and lecture notes are distributed. That students appear to do as well in a traditional course as the revised course on both topics developed in class and 'novel' problems is equally remarkable; especially in light of student comments (and the instructors' impression) that student's in the revised course develop a superior understanding of the material in the revised course.

There are some possible reasons for this appearance. One is that students may have a clearer understanding of lecture topics than group topics in the revised course. Material developed in groups often contains a number of minor errors; once learned, students may not correct these errors and hence reproduce them on the exams. Topics developed in lecture, on the other hand, have fewer errors.

Also, since the lectures build on the group experience, the students are still getting the benefit of the group work during the lecture.

A weakness in the analysis of performance in the traditional and revised courses is the simplistic method of comparing performance. First, the analysis is based on the grades assigned at the time; the problems were not graded using a common rubric (over all sets of exams). Also, the analysis cannot distinguish between memorized proofs and proofs that the students genuinely 'understand'. Note, however, that this does not seem to explain the similarity of performance in the traditional and revised class on 'novel' problems.

Given the discrepancy between the above analysis and the impressions of students and the instructor, it seems that an in-depth qualitative study should be done. In particular, students should be interviewed regarding their work in groups and on exams. These interviews could indicate at what point in the course to collect quantitative data.

Conclusion

The revised course has many of the features of a course intended to lay a foundation for a deep understanding of curriculum content. In particular, while being centered on the secondary school curriculum, it expands on the content discussed in the secondary curriculum. It appears that students become more comfortable with the notion of proof and the proofs done in groups improve over the duration of the course.

The analysis discussed in this paper, however, does not suggest that the revised course is superior to the traditional course in helping students create and write proofs at the time of the final exam. This analysis will serve as the basis for a more rigorous study of the effectiveness of the course.

The revised course may offer a variety of benefits not discussed in the analysis. It is at least the instructors' impression that students leave the course with a good intuition for hyperbolic and spherical geometry and have a solid understanding of the wide ramifications of the different parallel postulates; it is the hope that this broader perspective of geometry will give them a context to think about axiom systems, models, and Euclidean geometry in particular. In the end-of-course student evaluations, the students report a strong increase in their appreciation of geometry; hopefully, they will convey this appreciation to their own students.

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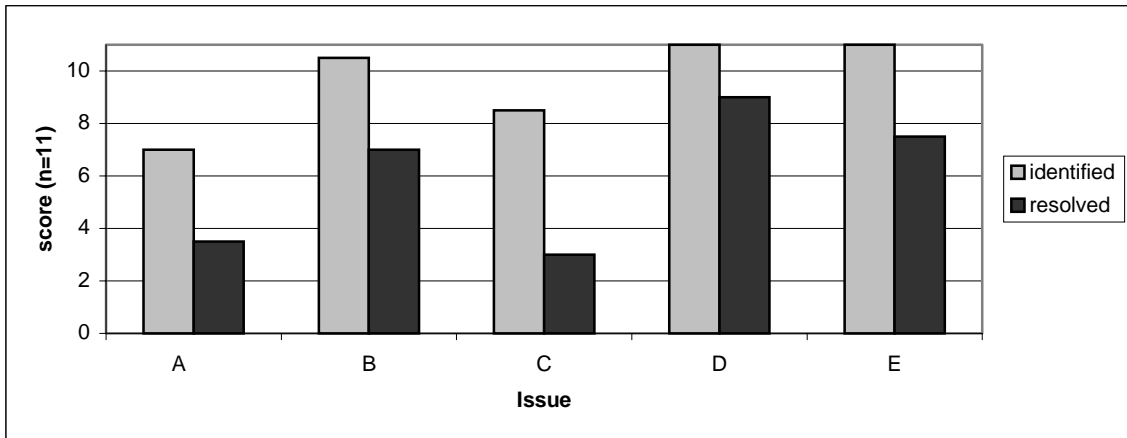


Figure 1: Proof Performance, Week 2

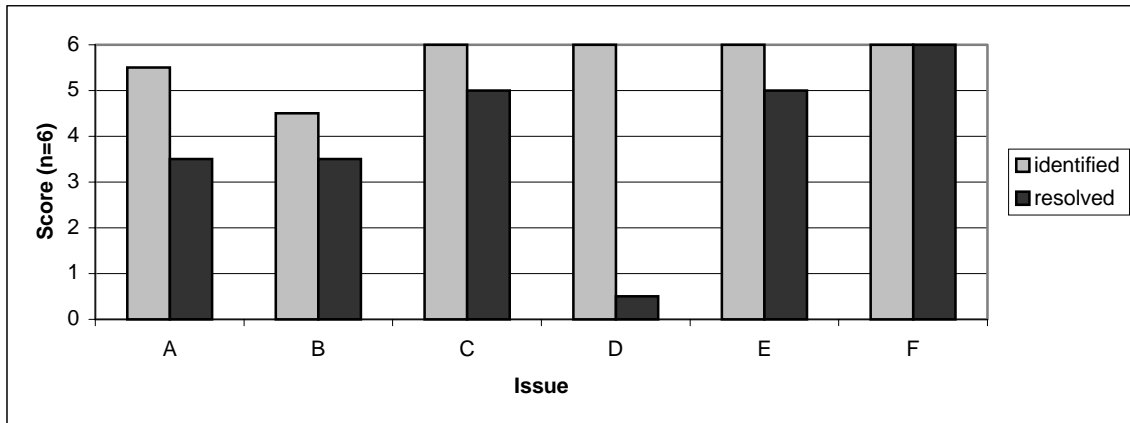


Figure 2: Proof Performance, Week 5

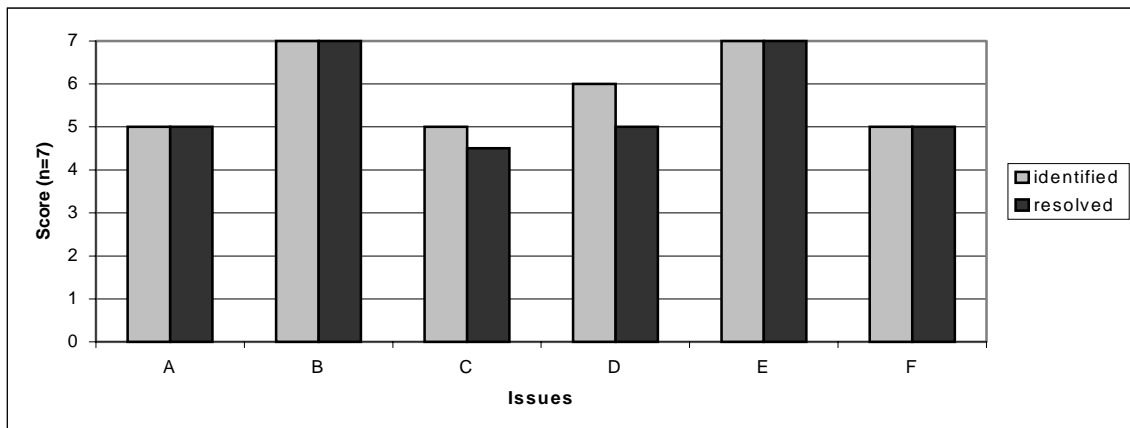


Figure 3: Proof Performance, Week 12

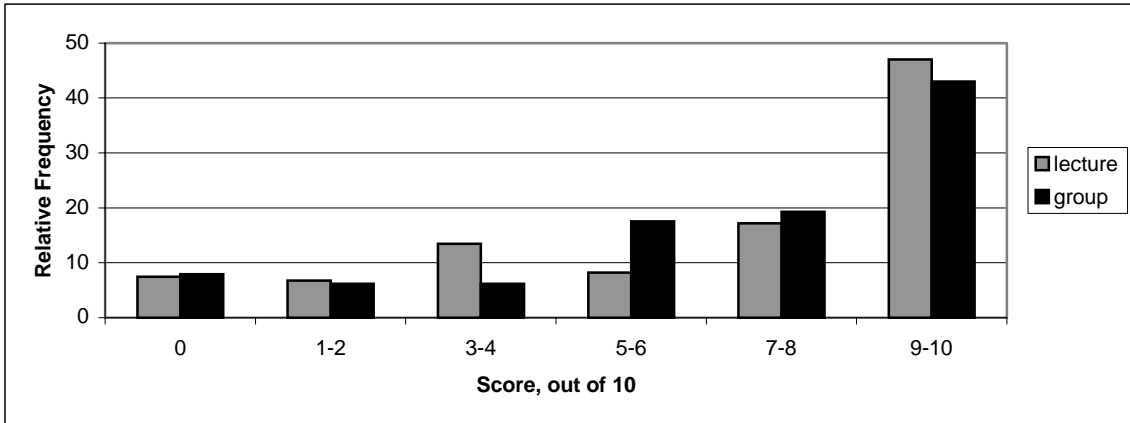


Figure 4: Exam Performance on Lecture and Group Topics

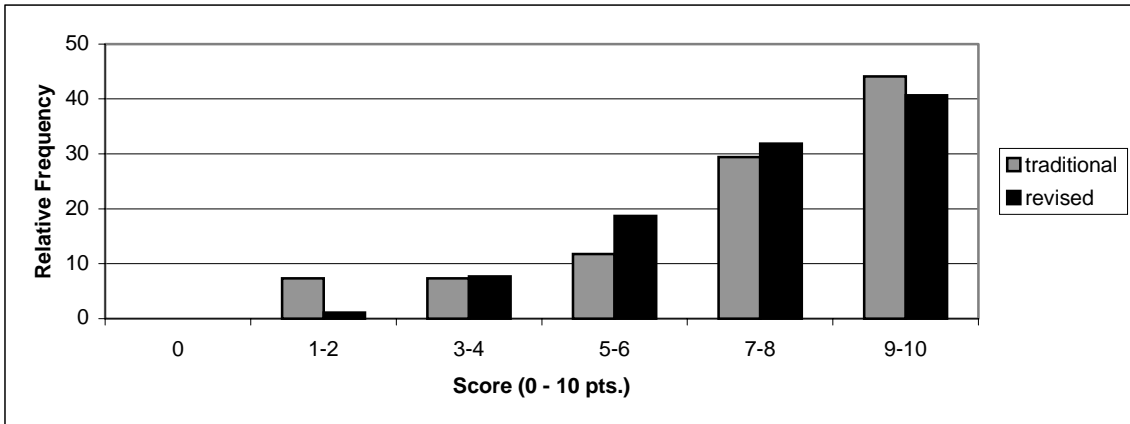


Figure 5: Exam Performance in Traditional and Revised Course

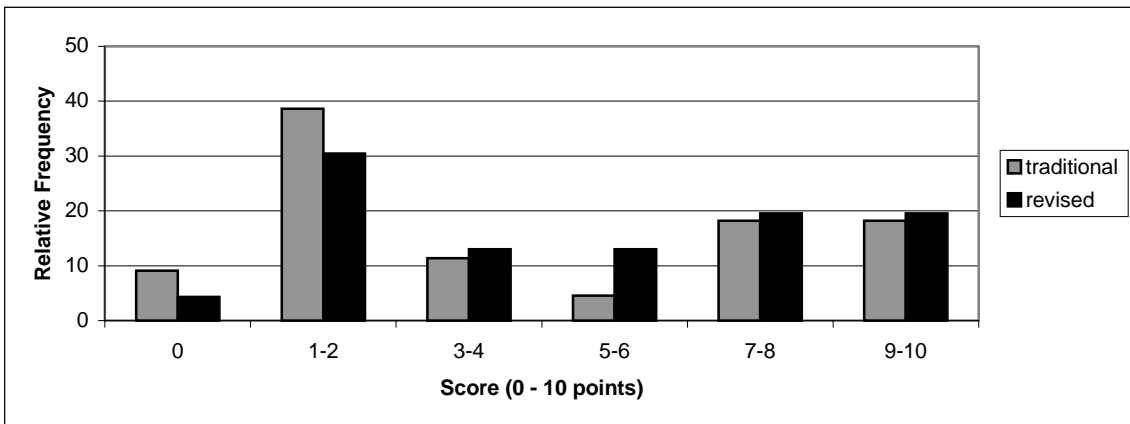


Figure 6: Exam performance on 'novel' items in traditional and revised course