TEACHING CALCULUS WITH DIGITAL LIBRARIES

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ABSTRACT

A few years ago I was shown some Java applets that were of sufficient educational interest to cause me to begin a modest search for material I could use in my elementary calculus class. The search was frustrating—a lot of widely scattered material and much more chaff than wheat, but some of the good material showed real promise. This lead, with a lot of help from my friends, to a successful proposal to the National Science Foundation's Digital Libraries Initiative for the Math Forum along with the Mathematical Association of America to create the Journal of Online Mathematics and its Applications, http://joma.org. Part of the goal of JOMA is to search out and peer review mathlets—applets and other interactive web-based teaching tools for mathematics.

The MAA and Math Forum soon received a further grant to create the MathDL site, http://mathdl.org, for which JOMA is the cornerstone. In this talk I'll examine the advantages to using digital libraries for finding calculus resources and how this material can be effectively used for teaching calculus. In addition to JOMA, a number of other digital libraries now contain mathlets and more extensive teaching material. These include Merlot and iLumina. Such libraries will be surveyed, and how new users might find them will be discussed. I have taught calculus using these resources and will be doing so this fall. My talk will update this rapidly changing area through June, 2002.

Keywords: calculus, digital library, interactive web

Teaching Calculus with Digital Libraries
Part 1: My Experience

Introduction

Technology brings exciting possibilities for visualizing mathematical concepts and for student interaction with meaningful calculations and images. But even though computer algebra systems have become simpler, most of us are unwilling to spend the time in a calculus class to develop student's necessary expertise unless our department or institution has a commitment to a particular system. Thus, the advent of very simple to use, interactive web-based material brings wonderful possibilities to the rest of us.

Additionally, the US National Science Foundation and other funding sources have for some years been financing the development of web-based educational modules aimed at providing supplementary teaching resources and student projects. Some are directed toward calculus. Some involve computer algebra systems and some are based on free software.

The problems lie in finding good quality and also appropriate material. Searching the web for a given topic produces a flood to wade through, almost every drop irrelevant, much of the relevant of unacceptable quality, much of the remainder redundant repetitions of the same few themes (check out Java function graphers, for example).

Digital libraries are an effort to collect and sort out the wheat from the chaff, to switch to a drier metaphor. Spurred on by visions such as Frank Wattenberg's in

http://www.dlib.org/dlib/october98/wattenberg/10wattenberg.html,

the NSF has now had several initiatives that have resulted in the construction of digital libraries and at least three have some mathematical content. Moreover, it has become common for popular texts to have web sites devoted to supplementary resources and commercial digital libraries have begun to spring into being.

What is out there and how can we harness it to the teaching of our courses? (Calculus has been chosen for focus and because it's what I'm currently up against.)

What this paper is about

This paper is concerned with teaching calculus making use of resources taken from digital libraries. It does *not* address online courses, it is about using online material to enhance one's non-virtual course.

How it will keep from being out of date

This paper is being submitted in January and by July when it is presented, the World Wide Web is likely to have changed dramatically. I wish to give would-be readers an up-to-date version and also seek their ideas and information on the subject—I plan to teach calculus in the fall and would like my resources and techniques to profit from this. Consequently, I am taking this paper and placing it at http://mathforum.org/wiki/CalcWithDL as a "wiki." This is a collection of easily editable web pages, so step right up and add your comments or change what I've said to match your ideas.

Background and possible biases

A few years ago I was shown some Java applets that were of sufficient educational interest to cause me to begin a modest search for material I could use in an upcoming elementary calculus class. The search was frustrating—a lot of widely scattered material and little of quality, but some of the good material showed real promise.

This lead, with a lot of help from my friends, to a successful proposal to the National Science Foundation's Digital Libraries Initiative for the Math Forum along with the Mathematical Association of America to create the Journal of Online Mathematics and its Applications, http://www.joma.org/. Part of the goal of JOMA is to search out and peer review *mathlets*—applets and other interactive web-based teaching tools for mathematics. Another goal is to publish high quality *modules*, online learning materials whose scope goes beyond that of the simpler mathlets. The MAA and Math Forum soon received a further grant to create the MathDL site, http://www.mathdl.org, for which JOMA is the cornerstone.

In this talk I'll examine the advantages of using digital libraries to find calculus resources and how this material can be effectively used for teaching calculus. In addition to MathDL, a number of other digital libraries now contain mathlets and more extensive mathematics teaching material. These include Merlot and iLumina. I will look at all these sources, and others below with dispassionate eyes, pointing out warts as well as virtues.

My Experience Using Mathlets in Class

In the fall of 2000 with the MathDL project just off the ground I was again scheduled to teach elementary calculus so I looked over what was available and decided to use some of the few applets relevant to my teaching. No other types of mathlets were then available but now other platforms such as Flash are showing some advantages in terms of loading speed and stability.

At first I explained to my students that they needed to be patient because the applets were slow to kick in and that some wouldn't even work with certain browsers (a little test information was available in JOMA). Then I demonstrated an applet showing an interpretation of the derivative as the slope of a surfboard as a wave the shape of the given function was surfed. (I didn't expect it would be necessary for the students to further play with this simple applet, but I gave them the url in case they wished to.)

For homework, I provided the url of an applet which gives a quiz on the shape of the derivative of a function, giving them some functions randomly chosen from a list and asking them to choose the corresponding shapes. Throughout, I put all the material on a course management system to which they had access, although a simple web page would have been sufficient. I warned the students that they would need to know this material in a few days for their first quiz. For that quiz I printed out some images from the applet. Perhaps half the class showed they had not put in enough time on the assignment to adequately master the concept. Student comments indicated that those having difficulty thought they had learned the concept on their own and had no need of the applet, or else they couldn't get the applet to work. Sigh.

I promised them another opportunity to demonstrate mastery of the material on the next quiz, and most had indeed come to grips with the underlying tricky concept. Students indicated that once they got going on the applet they found the experience both valuable and enjoyable.

I also used applets later in the course, for example for demonstrating Riemann sums.

Lessons (Re-) Learned

First of all, technology has its difficulties, not only for students but for classroom demonstrations as well as I (re-re-re- …) discovered during my first presentation when computer-projector difficulties ate up more time than I had allotted. One should *always* go *completely* through a demo on one's own before subjecting others to it. Nor should we take for granted all students' ability to work with even very simple technology on their own—over the course of this experiment I had several come in and work with me before they developed real understanding of how to deal with the interactive applets.

Secondly, simple interactive tools can apparently meaningfully contribute to mathematics education. This statement is based on informal evidence, but it is backed by some later preliminary research the Math Forum has done with school students; we have more ambitious plans for research on the educational impact of digital library material, as well. My calculus class was quite small, one of the joys of teaching at the earliest morning hour at my institution, and I got to know most students reasonably well. The class was varied in ability and effort put forth. Grades ranged from A to D, with an average a bit above B-. Based on discussions with them, most students found the experience of using the applets a useful addendum to their regular work, and one that provided them with new understanding. One student, who had struggled all along regardless of the learning medium, found this experience frustrating and not worth her while.

I would also offer my subjective conclusion that based on quiz results, students ended up with a better mastery of the material where I used applets than they normally would have.

Conclusions

My conclusion here is that the problems and successes in using web-based technology is pretty much the same as the more familiar problems and successes in using computer algebra systems with students in labs. There is the added difficulty that the students encounter the material when not under supervision. There are also the added liberating factors that they can work flexibly on their own and after learning to use one simple interactive program they can then use most. Moreover, most of the resources are free, and they offer great variety because of the large number of developers and lack of program constraints.

Part 2: A Survey of Digital Libraries with Math Content

(A) "Early" Digital Libraries

Educational Object Economy (EOE) http://www.eoe.org/

EOE was founded in the mid-90's, not just as a digital library but to develop and distribute tools to enable the formation of communities engaged in building shared knowledge bases of learning materials. It lists some 82 applets devoted to calculus in its Java Applet Library. These are nicely browseable, often with good descriptions and images, but since these are freely exchanged resources, postable by any visitor, there are some very curious objects classified under calculus. Nonetheless, this collection was one of my original inspirations and their goals of

developing an authoring community and of making tools that are reusable and interoperable remain laudable. (In fact, they are reflected in the Developer's Area of JOMA).

Math Archives http://archives.math.utk.edu/

The Math Archives predate the current NSF digital library initiatives, but they do have selected calculus resources as a very long but searchable list at http://archives.math.utk.edu/topics/calculus.html. Additionally, Larry Husch's *Visual Calculus* http://archives.math.utk.edu/visual.calculus/ is a collection of modules that can be used for studying or teaching calculus. They use a variety of media—LiveMath, Java Script, Flash, Java, Maple, etc., frequently with options. They are clearly done and presented and objectives are articulated. The Calculus Resources Online, http://archives.math.utk.edu/calculus/crol.html, features material available from various universities, along with some other material.

Math Forum http://www.mathforum.org/

According to The Calculus Page, http://calculus.org/, This "may be the most comprehensive, up-to-date calculus website anywhere on the internet." See

http://mathforum.org/calculus/calculus.html or http://mathforum.org/library/topics/svcalc/

for different entry points. There are many carefully annotated links to calculus material on the web. One of the distinguishing features of the Math Forum is that there are also human mediated interactive resources such as Ask Dr. Math and the Problems of the Week, both of which have calculus components. Moreover, these resources and the Internet Mathematics Library—the entire site, in fact—is developed as a whole with interconnecting parts.

(B) NSDL Libraries, that is to say Science, Technology, Engineering, and Mathematics Libraries funded by the National Science Foundation

In general, with a few lacunae noted below, all have these common features: peer review, searchable and browsable, detailed information about materials, not just lists of lists but genuine teaching material available.

MathDL http://www.mathdl.org/

MathDL is a digital library managed by the Mathematical Association of America and hosted by the Math Forum. It has a number of areas of potential interest to the calculus teacher. The Journal of Online Mathematics and its Applications (JOMA) features

- Mathelets, small interactive web-based tools, such as applets, for use in teaching mathematics
- Modules, larger teaching units and student projects that may require computer algebra systems

There is also a Digital Classroom Resources section that at the moment contains no calculus oriented material.

At the moment the more extensive mathelet material contains a browse structure, and a search engine for the whole site is expected in the near future. The project has been highly selective and thus far out of some 900 mathlets examined, only around 16 have been published. In addition

there are a dozen modules, some of which could be included in a calculus course. Many of the modules are available for a number of different computer algebra systems, see for example this table from an article in the current issue

http://www.math.duke.edu/education/ccp/materials/intcalc/index.html.

Information provided for each mathlet includes author, intended uses, appropriate courses, software specifications (results of some platform testing), author's statement, avalibility of code, and acknowledgements.

Merlot http://www.merlot.org/

The Multimedia Educational Resource for Learning and Online Teaching (MERLOT) is an international cooperative for high quality online resources to improve learning and teaching within higher education. It has some 47 objects classified as calculus materials. Each peer review is conducted by at least two higher education faculty members who, from their individual reviews, compose a "composite review" that is posed to the MERLOT website, with an Amazon-style star summary and possibility of user reviews.

The reviews focus on quality of content, potential effectiveness as a teaching tool, and ease of use. Resources vary from applet simulations through tutorials, quizzes, reference materials, and websites.

The brows structure is a bit coarse grained—one can obtain a list of all 541 mathematics materials, and sort by title, author, date, rating, or item type, but that's all. The entries also give web institution, and location. Searching for calculus produced 154 items, undifferentiated between single and multivariable. Advanced search allows you to make this distinction and also allows one to specify:

material type, title or name, content url, description, primary audience, technical format, language, whether cost or copyright, source code available, authors' name, email, and organization, whether peer or user reviews are available, date restriction, and assignment or advanced assignment search (which includes learning objectives and education level.)

iLumina http://turing.csc.uncwil.edu/ilumina/homePage.xml

iLumina is a digital library of sharable undergraduate teaching materials for science, mathematics, technology, and engineering. There are some 72 calculus entries. One can carry out quick and advanced searches, and also browse (first by "taxon path" which leads to a list giving date, title, author, resource type and data type; the latter not too clear to me). Browse entries and searches lead to especially thorough descriptions of resources:

title, authors, download location, description, keywords, taxonomy path, type of learning resourdce, level of interactivity, difficulty, end user role, structure, cost, copywright, data type, size, tech requirements, other platform requirements, "is part of", and contact information.

The Advanced Search allows one to search on most of this information, but doesn't allow one to search via type of software, e.g. applet, or for tech and platform requirements—key user needs in my opinion. The review process is not clear.

(C) Other Digital Libraries and Sites

Commercial Digital Libraries

There have been a number of commercial digital liberary ventures, most of which have yet to prove very successful. These include Questia, eBrary, XanEdu, NetLibrary, and JonesKnowledge. None appear to have any real math content, nor STEM content for that matter.

Publisher's Resources/Digital Libraries

Most publishers now offer some sort of online support for their texts and/or CD-rom material. I've not attempted to survey the latter but have attempted to look over what is available for some of the leading texts. At the moment my quest has not had much success. I've found the Internet arm of the publishing industry to be in a bit of disarray. One site, for the text I'm planning to use, had instructor materials for the several variable text for the single variable page. Some editor's did not have up-to-date information on their sites and some publishers are proving to be not forthcoming with necessary information. I will persist. I welcome your knowledge.

Some non-digital library calculus sites of note

It is worth remembering that there are calculus sites that do not aspire to the digital library format and nonetheless have much to contribute to some calculus teachers, even though they go beyond the scope of this paper. For example, for students seeking practice exams there is the venerable COW (Calculus on the Web) site of Gerardo Mendoza and Dan Reich of the Mathematics Department at Temple University, http://www.math.temple.edu/~cow/, and also Mike Gage's WeBWorK, http://webwork.math.rochester.edu/docs/docs/, of the University of Rochester.

An emerging site of some interest is Calculus@Internet, http://www.calculus.net, a potpourri of everything calculus-related. At this time it needs some digital library organization and clarity to be really useful.

How are the NSDL digital libraries doing?

Imagine that you are preparing to teach a calculus course and are searching for help over terrain known to be rocky, or perhaps you are in the midst of your course and are looking around for first-aid when both you and your text have not got through to as many students as you think you should. Here are some issues that are likely to be confronting you:

1) hardware constraints; perhaps your institution uses Macs, perhaps you have ancient PCs.

2) software constraints; perhaps a particular browser is the only one supported, perhaps you have either a single computer algebra system available or no such system.

3) specificity; you need material that deals with a specific topic, say the chain rule, and need a search engine to take you directly to this.

4) general direction; you are willing to browse around in a general area to see what is available.

5) quality; you are only looking for really high quality material that gives students real insights.

6) quantity; you would like a number of items from which to choose.

How are the digital libraries doing to help you in your quest? (1) & (2): Alas, a mixed bag. No library tells you upfront about hardware or software constraints, nor at this time allow you to search on these, but MathDL and iLumna tell you when you get down to looking at individual items. (3) All will have search engines and iLumina already has a detailed search (although it doesn't give you exactly the choices I'd like). On the other hand at this time the value of these search engines is weakened because of (6): the libraries lack quantity so that you can't expect to find anything when you make a very specific search by topic. (4): None have particularly good browse facilities at this time, although MathDL plans to allow users to use a generic calculus text table of contents. Perhaps the libraries are doing best with quality (5), since all have given considerable thought to acceptance criteria and most are using peer review.

At this time the digital libraries can be likened to open stack libraries where it is necessary to browse to find good fits to your needs, rather than closed stack libraries where the "card catalog" contains enough information that you can confidently order from the circulation desk. Moreover, the shelves are not very full at this time, although the digital librarians are eager to fill them up.

Nonetheless, there *are* interesting materials in these libraries and careful thought has gone into the catalogued materials, the information gathered for users, and the user interfaces.

Thus, at this time each of the three main NSDL libraries discussed above have valuable features and lack certain things that my generic calculus teacher above would find important in looking for appropriate material. Roughly speaking, the libraries have three distinct and useful models that can be vastly oversimplified to

- journal publishing (MathDL),
- a super-Amazon with both peer and user reviews (Merlot),
- the best of current digital library notions about metadata (iLumina).

I repeat: this is a vast simplification and all the approaches are valuable.

Big questions for the digital libraries:

I. Will they package their material in a manner users find convenient?

II. Will they be able to fill their shelves? Is there enough good material out there? Will they be able to find it before search engines catch up? I tried "chain rule applet" on Google and it seemed to give back more interesting material earlier in the vast list of matches than heretofore, and the popularity measure it employs may mean that the cream will rise to the crop.

III. Will meta-digital libraries be necessary? That is, will it be necessary to have an über-library that one searches for the material one wishes in the various digital libraries? One hopes not. Fortunately, the NSF is funding attempts to allow easy searching across all digital libraries. Unfortunately, we're not there yet and if a digital library does not keep track of information the user finds important, such as necessary platform, it won't be searchable for.

IV. Is it possible for the digital libraries to give us genuinely valuably distinct look, feel, and material, or are they doomed to expend the immense amount of effort and time already put in to producing products that are so similar that users would have been better off if they combined forces?

Coda

What will I do in my calculus course this fall?

Ah, through this talk I have become much more familiar with the various libraries and how to use them. I've tried to impart this to you as well. By the deadline for this paper I have not been able to make detailed choices of particular learning materials, but I will continue to work on this and the paper I present will contain references to the material. Moreover, as mentioned, this paper is being set up as a wiki at http://mathforum.org/wiki/CalcAndDL, its existence will be made known to as many calculus teachers as I can find, and I hope that some will contribute their ideas and suggestions so that I can do a good job in my course.