ANIMATIONS TO ILLUSTRATE ILL-CONDITIONING AND AN INTRODUCTION TO MATRICES USING MAPLE

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ABSTRACT

Computer packages, usually Maple, are used in most of our undergraduate courses but less so in the lower level courses. Students of geospatial sciences (surveying) undertake the equivalent of first year engineering mathematics in a year and a half. In the fourth semester they undertake the course MA067 designed specifically for them consisting of two thirds numerical linear algebra and one third introduction to complex variables and conformal mapping.

In the third year the surveying students take a course “Geometry of Surfaces” which is a classical differential geometry course with one lecture and one lab session (using Maple) per week. All calculation and assessment is done using Maple in the computer lab. This course has run successfully for a couple of years. However the students find that it is difficult at the start of the course with the necessity of learning differential geometry and Maple concurrently.

In MA067, some Maple was introduced. Only one hour per week was available for the classwork practice sessions and the Maple lab sessions. This resulted in the availability of five sessions for Maple. These were devoted to a general introduction to Maple and then work with entering matrices (using the Matrix data type in Maple 6) and solving linear systems of equations (matrix equations). Students used Maple to solve exercises that they had solved the week before “by hand” in the practice class.

To add interest, following an expository lecture, a Maple presentation was given to reinforce the understanding of ill-conditioning. Maple animations illustrated ill-conditioning and this was contrasted with an animation of a well conditioned system. These animations were strikingly effective and appreciated by the students. Besides anecdotal feedback, we report on a feedback survey designed to investigate student attitude to the Maple component of the course.

1. Introduction

The Computer Algebra System (CAS) packages Maple and Mathematica have been used in teaching and research in our department for about a decade. Due to the success (from the perspective of both staff and students) of our CAS laboratory sessions, we have progressively increased the usage of these in our teaching and are fundamentally changing the way we teach. Since 1998, our mathematics program has included computer laboratory sessions (usually using with Maple) for almost every course. Teaching students of other departments, particularly from engineering, geospatial sciences and computer science, is a core activity of our mathematics department. In consultation with these departments, we have also increased the use of various software packages as appropriate to the course. Where the demand is for specialist packages (such as for Matlab toolboxes or commercial software Finite Element Method packages …) they are used, otherwise Maple is incorporated in the course. This usage of Maple varies a lot from a little "tutorial" support, to major assignment work, to Web delivered material, to "immersion" with 100% of the assessment in an examination in the computer laboratory.

This paper discusses the introduction of Maple (in "support" mode) in the course MA067 for the surveying students in the semester before they undertake the Maple "immersion"course: Geometry of Surfaces. Geometery of Surfaces is a third year classical differential geometry course with one lecture and one lab session (using Maple) per week. All calculation and assessment is done using Maple in the computer lab. This course was initially developed by one of the authors using Mathematica and ran successfully in 1998 (Blyth 1998). With a site license for Maple, this course was rewritten using Maple and run from 1999 onwards. By the end of the course, students are quite positive about this approach. However the students find that it is difficult at the start of the course with the necessity of learning differential geometry and Maple concurrently.

Since MA067 is a prerequisite for the Geometry of Surfaces course, and a review of this course was required in 2001, we took the opportunity to provide an introduction to Maple in MA067. With an allocation of 3 contact hours per week for one semester, two hours were used for the classes (the lectures – with less than 50 students) and the third hour alternated with a tutorial / classwork exercise followed by a Maple laboratory session on (essentially) the same topic. We also presented Maple animations in a lecture demonstration to illustrate ill-conditioning (of matrix equations). The examination is a traditional one (with no Maple) and contributes 80% to the assessment.

Introducing students to Maple was an objective, but it was important to do so in a way that did not negatively affect their attitudes with respect to using Maple. This is clear since the Geometry of Surfaces course follows MA067, but a further incentive for us is that Geometry of Surfaces has a slightly insecure status (sometimes compulsory and sometimes an elective - which the students have been strongly encouraged by their home department to select). We designed and administered a comprehensive questionnaire to investigate the attitude of the students to the use of Maple. This survey provided evidence that most students had a positive to neutral attitude to using Maple.

This MA067 Maple Questionnaire was a modification of the Maple Questionnaire designed and used for the course MA910 that is offered in the first semester of first year. MA910 is primarily the first semester of a traditional calculus course taken by science students. It is closely related to the course taken by the engineering students, but one of the topics studied by the engineering students at the end of the semester has been replaced by a Maple topic. Because of the module structure used in this and related courses, the Maple module is dealt with in one block near the end of the semester (in contrast to the Maple work being distributed throughout the semester as in

MA067). The students enrolled in MA910 were in two groups, the first of which was studying a multi-major degree in science and will probably not take a major in mathematics. This student group had not already used Maple, so they were provided with an introductory program. The second MA910 group consisted of students studying our specialist mathematics degree or our Dual Award of our mathematics degree and a diploma in information technology. This second group of students took an additional mathematics subject and had been using Maple in two one-hour laboratory sessions per week. The more advanced work done with this second group of students, already experienced with using Maple, is described in Blyth and Naim 2001. A discussion of the Maple survey results for both of these different groups is provided in Saunders and Blyth 2001.

2. Maple and MA067

The course MA067 replaced a previous course: the content was the same but the number of contact hours increased from 2 to 3 hours per week. We used two lectures weekly to cover the same content as previously and introduced a new weekly "tutorial". Usually a topic was discussed in the lecture and then followed by a practice class (where students completed some problems by hand on a worksheet which was handed in for marking) and repeated a similar Maple worksheet (also handed in) the following week.

An outline of these weekly tasks follows:

Week 3.	Maple 1.	Introduction to Maple
Week 4.	Tutorial 1.	Error / relative errors / error bound & loss of significance
Week 5.	Maple 2.	Matrices and solving Systems of Equations
Week 6.	Tutorial 2.	Direct & Iterative Methods
Week 7.	Maple 3.	LU Decomposition
Week 8.*	Surveying Camp	
Week 9.*	Tutorial 3.	Residual Correction Method
Week 10.	Maple 4.	Residual Correction
Week 11.	Tutorial 4.	Interpolation and estimating derivatives
Week 12.	Maple 5.	Intro to Complex Analysis (Not marked)
Week 13.	Tutorial 5.	Complex problems and past exam

The Maple worksheets were handed out as hard copies to students in the lab. Students were required to enter all code themselves and were assisted whenever they required help. Assessment consisted of 10% for the "by hand" work, 10% for the Maple assignments and 80% for the examination (in which Maple was not available). With just as few sessions available for using Maple, the emphasis was placed on the numerical linear algebra (and an introduction to complex analysis) rather than having a (trivial) introduction to lots of topics.

Maple's new linear algebra package **(**LinearAlgebra**)** contains commands for matrix and vector manipulation. This was used, not the original (and still provided) Maple package, linalg. This led to some improvements, particularly with respect to display and manipulation of the matrices and vectors. However for the worksheet on the residual correction method, the default use of hardware floating point arithmetic made it difficult to change the precision for the calculation of the residual. Students were led through these issues with their Maple worksheet in the lab and asked to do the following Maple assignment:

The ill-conditioning assignment

Consider the 2 equations in 2 unknowns:

$$\frac{x}{1001} + \frac{y}{1003} = \mathbf{0.2002}$$

$$\frac{x}{1003} + \frac{y}{1005} = \mathbf{0.1998}\ .$$

This can be written as the matrix equation **A** x = b where

$$\mathbf{A} = \begin{bmatrix} \frac{1}{1001} & \frac{1}{1003} \\ \frac{1}{1003} & \frac{1}{1005} \end{bmatrix}\ ,\ \mathrm{b} = \begin{bmatrix} .2002 \\ .1998 \end{bmatrix}\ \ \textbf{and}\ \ \mathrm{x} = \begin{bmatrix} x \\ y \end{bmatrix}\ .$$

Given an approximate solution $\mathrm{y} = \begin{bmatrix} -100 \\ 310 \end{bmatrix}$, calculate the residual correction **y1** and two further iterations of residual corrections (giving **y2** and **y3**).

An additional Maple worksheet on vector and matrix norms was developed but not used due to time constraints. The students are not available during their surveying camp, but in 2002 we will start the Maple labs earlier and we will be able to include the norms worksheet.

3. The Maple animation illustrating ill-conditioning

Before the residual correction method was covered, an introduction to ill-conditioning was given in the lectures and followed by a Maple presentation in the next lecture. This used Maple animations to illustrated ill-conditioning and was contrasted with an animation of a well conditioned system. These animations were strikingly effective and appreciated by the students according to the anecdotal feedback specifically on this session.

The model problem considered was the solution of the 2 equations in 2 unknowns:

$$\frac{x}{2} + \frac{y}{3} = 1$$

$$\frac{x}{3} + \frac{y}{4} = 1\ .$$

This can be written as the matrix equation A **x** = **b** where both off diagonal elements are 1/3. Finding the algebraic solution has the graphical representation of finding the point of intersection of the two lines. Ill-conditioning is exhibited by small errors in the off diagonal elements leading to larger errors in the solution: this was explained in terms of finding the intersection point of nearly parallel lines.

An animation to illustrate the ill-conditioning was run with the error in the off diagonal elements changing from 0% to 10% in steps of 1%. The changing lines were shown along with a blue dot at the solution for the exact case. Two of these frames are given in Figure 1.

Graphically, ill conditioning corresponds to the lines nearly parallel. So the best behaviour should be when the lines are nearly perpendicular. If the lines (the equations) are

a x + b y = k1

c x + d y = k2

the slopes of the lines are - a/b and - c/d. The lines will be perpendicular if c/d = - b/a (WHY?).

We construct an example: if we keep a=1/2, b=1/3, then c/d = - 2/3. We also keep c=b=1/3, then d= -1/2. We also choose the right hand side constants so that the solution point remains the same, to obtain A x = b where

$$\mathbf{A} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{-1}{2} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$$

These animations were strikingly effective and appreciated by the students (as reported by them at the time).

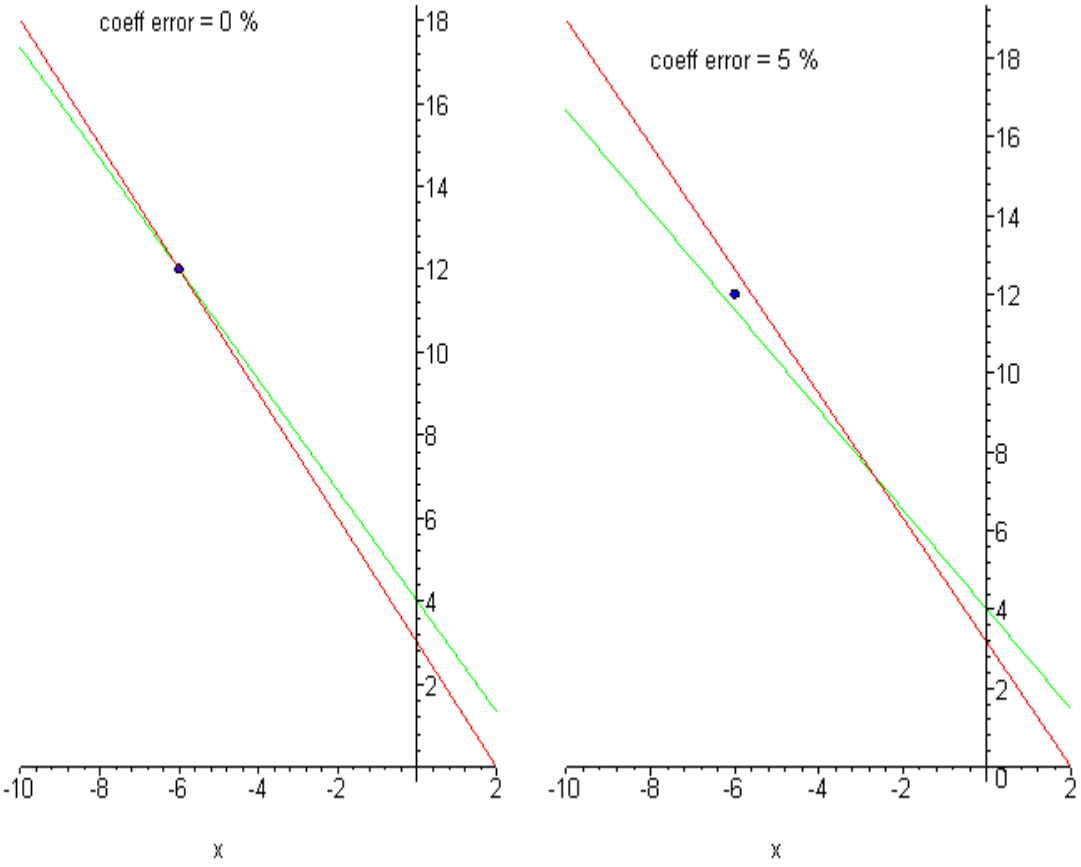

Figure 1: Two of the frames from the ill-conditioning animation.

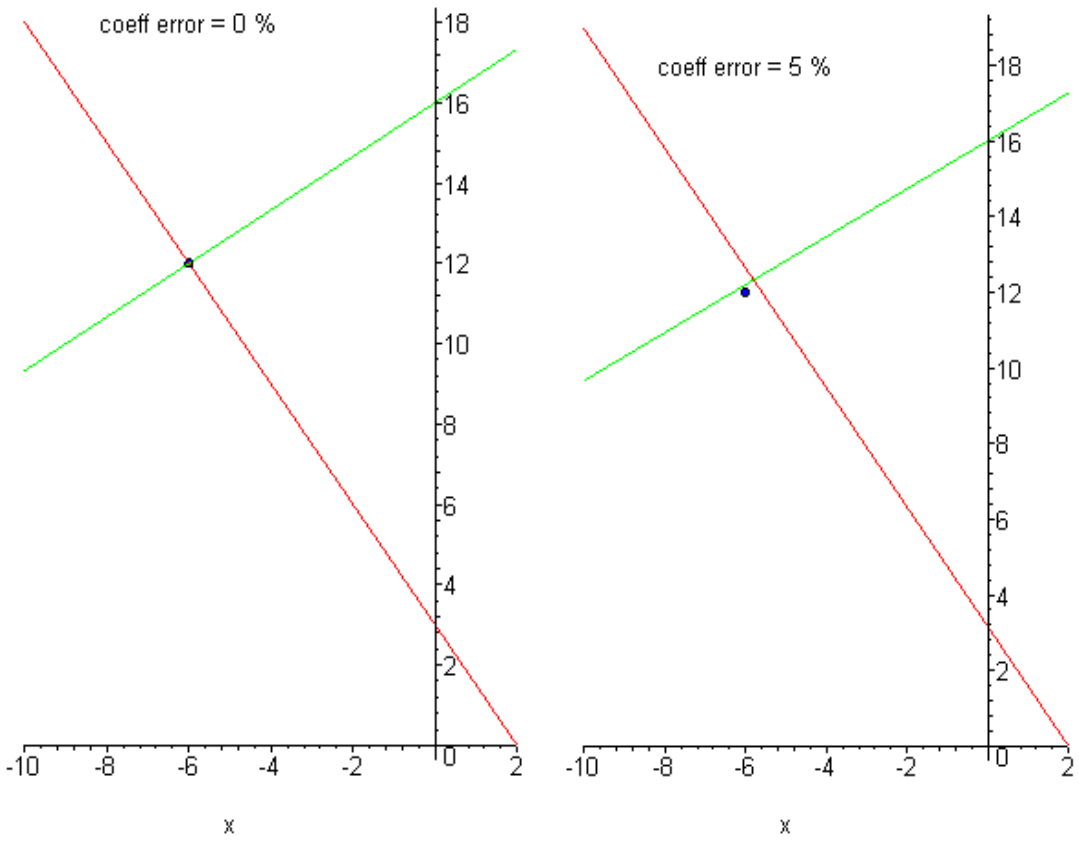

Figure 2: Two of the frames from the well-conditioned animation.

4. The Maple questionnaire

We designed a comprehensive questionnaire to investigate the attitude of the students to the use of Maple. For a report on related work, see Galbraith et al. 2001. Our survey was administered in the last teaching week and a copy of the questions is given in the Appendix. The questionnaire items were presented on a Likert scale with 5 points from strongly disagree to strongly agree with neutral in the middle.

Factor analysis of the responses led to grouping into the following five scales (where the responses are indicated as negative, neutral, positive):

The impact of the Maple component on motivation and interest (0, 18, 4)

The impact of the Maple sessions on mathematical understanding (3, 17, 2)

The level of appreciation of Maple's usefulness as a tool (2, 11, 9)

The ability to use Maple as a problem-solving tool (6, 10, 6)

The appreciation of the features of Maple (3, 13, 6)

It is clear that most responses were neutral or positive – so we didn't do much damage! Our objective was to introduce Maple without disaffecting students' attitude to the use of Maple (since the following course on Geometry of Surfaces uses Maple comprehensively in teaching and assessment). In this modest aim, we were successful, but we would have liked to see more positive responses. It appeared that the very positive anecdotal feedback after the animation demonstration in the middle of the semester was a dim memory by the time of the survey (at the end of the semester).

Reponses concerning aspects of teaching methods and delivery indicated that the students liked to receive the worksheets in hardcopy form and to be able to sit down (in the lab) and get on with the worksheets straight away. The students did not want Maple to be included in the exam but they did want the Maple component to be worth more of the total mark for the subject. There was some indication that the students would have liked more help in the Maple sessions (although the worksheets were not seen as being too difficult). The students did not find working in the lab difficult but there was a mixed response to the level of distraction there.

5. Conclusions

The introduction of Maple (in "support" mode) in the course MA067 for the surveying students in the semester before they undertake the Maple "immersion" course: Geometry of Surfaces achieved its objectives. The questionnaire provided useful feedback, which will lead to some minor changes such as some more tutorial help in the lab and increased marks for the Maple work. We will also start the "tutorials" earlier, which will allow for the inclusion of an extra Maple worksheet on vector and matrix norms. We will pursue further development of both the Maple work and mechanisms for student feedback.

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APPENDIX

The Maple Questionaire (the first page cover sheet and the boxes for the responses are omitted)

Section B – Response to Maple

Please place a tick against each statement according to the scale strongly disagree (SD) up to strongly agree (SA). Several items are used to gain an indication of your response to similar aspects of the Maple sessions so some items may seem to be repetitive. There are an equal number of positively and negatively phrased items.

1. I am glad we used computers as part of the course
3. Seeing how Maple solved problems helped me to understand methods better
4. I think that software like Maple would rarely be used in industry
5. Maple is too slow
6. Maple should be included in the exam
7. I would find it difficult to use Maple to solve a random set of problems
8. Maple was just one more thing to learn
9. Maple is not particularly useful as a Mathematics tool
10. The individual Maple sessions need to be longer
11. There were too many distractions in the lab
12. The Maple help files are good

13. It takes more than one semester to feel competent using Maple as a tool
14. I think Maple would be useful for Maths research
15. It was easy to save my work
16. The worksheets were too difficult
17. The graphs produced in the Maple sessions were helpful to my understanding
18. I would rather do the Maple classes in one block
19. I found it quite easy to edit commands to solve a new problem
20. I think I understand the topics we covered with Maple better than other topics
21. The computers were unreliable
22. I liked seeing how software could be used to carry out computations
23. I found the Maple sessions uninteresting
24. Maple is useful for doing long or complex computations
25. I was unclear what commands to use to get Maple to do what I want to do
26. I found that I was generally confused when using Maple
27. The Maple help files and examples were all we needed to learn the commands
28. Calculations done on paper are easier to check
29. I like being able to sit down and get on with the worksheets straight away
30. I understand better when I do the whole solution by hand
31. I would have liked more help in the Maple sessions
32. The Maple component should be worth more of the total mark
33. The Maple edit features are useful
34. I think that spending the time practising questions would be more useful
35. I could access the Maple work quite easily
36. Maple is useful as a checking tool
37. I like to see the Maple worksheets in printed form
38. I find working in a lab uncomfortable
39. I find Maple's language difficult to use
40. I can enter a method using Maple commands quite easily
41. I would have liked to have done more Maple work on other topic areas
42. The Maple graphics I have seen are good
43. I would rather have continued with normal lectures
44. I know what Maple needs to do to solve the problems

Section C Overall, how did you find the experience of using Maple this semester?

Thankyou for your time.