PRECALCULUS WITH INTERNET-BASED PARALLEL REVIEW

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ABSTRACT

An innovative precalculus course with an integrated Internet-based component will be described[1]. The course: (1) has an Internet-based "just in time" review component, (2) has Internet–based weekly tutorials, practice, and testing, (3) is designed for science and engineering students, (4) integrates the study of functions of two variables $f(x,y)$ and other basic three dimensional ideas, and (5) incorporates the use of symbolic algebra systems and other innovative pedagogy.

The study of multivariable functions is traditionally postponed until multivariable calculus. However, with the aid of a set of low cost manipulatives that we have developed to aid in the visualization of three dimensions, multivariable topics are being effectively incorporated into the precalculus curriculum. The goal is to build an early geometric intuition in three dimensions in students. This goal directly addresses a common concern voiced by our colleagues in engineering departments.

The Internet component of the project allows the establishment of a weekly practice, tutorial, and quiz system that helps students review the pre-requisites for upcoming material and review the material just covered in class. This component consists of a large and highly organized data bank of questions, a set of accompanying tutorials, and the software necessary for generating and administering quizzes on-line. The Internet component is being designed to facilitate its implementation in a wide variety of institutions. Interested faculty will be able to easily edit, contribute to, and adjust the data bank of questions to suit their needs.

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1. Introduction

This article addresses the precalculus course typically taught in many universities in the United States. The course is essentially an introduction to the function concept and a study of the main properties of polynomial, rational, exponential, logarithmic, and trigonometric functions. It usually also includes an introduction to vectors, matrices and linear systems, as well as several other elementary topics. Students who plan to follow careers in science or engineering and who do not have an adequate preparation in pre-college mathematics will take this course in their first year of university studies prior to enrolling in the calculus sequence.

The traditional precalculus course starts with a review of pre-college mathematics including properties of real numbers, algebraic expressions, solving equations, the Cartesian coordinate system, and lines. For an underprepared student, this introductory review attempts to teach in a very short time everything that the student did not learn in two or three years of pre-college mathematics. It is not surprising that it frequently fails to do so. Another problem with the traditional course is that it tries to cover too much material leaving little time for experimentation, collaborative learning activities, and other innovative pedagogies. Time constraints will frequently force professors, even those who would prefer alternative more student-centered approaches, to lecture a substantial amount of the time. In the past few years there have been several prominent attempts to redesign the course: by giving it a more modelling oriented approach, by incorporating graphing and sometimes symbolic computation, and by placing more emphasis on numeric and geometric representations while building connections with the traditional algebraic approach. Some of these course restructurings managed to introduce innovative elements into the course essentially by de-emphasizing algebraic computation skills. For students majoring in some fields this is a perfectly reasonable thing to do as one may argue that calculator and computer technology can be used for many of the routine calculations that students need to do in the course. But some students, particularly those planning to follow careers in engineering, do need a substantial amount of algebraic manipulation skills. Engineering and some professional licensing programs require these skills. The problem is how to restructure the precalculus course so that alternative pedagogical approaches can be incorporated into the course, and so that incoming students with a weak mathematics background can succeed in the course while still developing a substantial amount of algebraic manipulation skills. This is an important problem in universities where many of the science and engineering students start at the level of precalculus and universities wishing to open the doors to engineering careers to students with weaker mathematics backgrounds.

The course described below addresses the above problem by taking most of the basic algebraic computation skills out of the classroom but not out of the course. This frees enough class time for innovations without compromising algebraic computation skills. Basic skills are reviewed in parallel to the course using an Internet-based system of weekly quizzes.

2. Internet-based Component

Every week the precalculus students take a quiz, which is available via the Internet. Most of the questions have randomly generated parameters so the students may practice taking the quiz as much as they want prior to the "real" quiz taken for a grade. In fact, they must attain a professor-defined expertise level before the system allows them to take the weekly quiz for a grade. In the practice quizzes, the system tells the students which problems he/she answered incorrectly and allows the student to link to the appropriate web-based tutorial (see Figure 1).

Weekly quizzes have two parts (see Figure 2): a *Review Topic Section* consisting of the prerequisite material needed for the next week of classes and a *Precalculus Topic Section* consisting of the course material covered in the previous week of classes. The *Review Topic Section* of the quizzes allows distributing the review of basic material throughout the entire course, covering what is needed for the course just at the time it will be used in the course. This is in contrast to the common practice of attempting to do all the reviewing of prerequisites in a block of time at the beginning of the course. With the weekly quiz system, reviewing is done as needed throughout the course via the Internet and, more importantly, without taking any class time. The *Precalculus Topic Section* of the quizzes is meant to help students keep up to date with the course material.

Each week, every student in the precalculus program enters the principal page for the given week via the Internet (quiz.uprm.edu). They may enter this page from any computer laboratory in the university or from their home. This can make it possible for a department of mathematics to support the intensive use of computer resources from a large quantity of students without placing too much of a burden on already existing departmental resources. It also allows a more efficient use of computing resources campus wide. From there, they may enter either the *Review Topic* section or the *Precalculus Topic* section. Upon entering the *Review Topic* section, the review topics necessary for the following week's precalculus topics will be outlined with links to pertinent tutorials. Should they wish to review these materials, they may proceed to the online tutorials. If they feel that they already understand these review topics, they may proceed to a practice quiz which will evaluate how well they comprehend these topics. Upon submitting their answers for the practice quiz, the computer generates a report indicating which topics they did well on and which topics require further review. This report includes links to pertinent tutorials. When the practice quiz indicates a sufficient degree of understanding (exactly what is "sufficient" is at the discretion of each instructor), the student is cleared to take a real quiz which counts toward their final grade. In our case, the real quiz may only be taken in the Testing Annex to the Mathematics Tutoring Laboratory where their identity is verified. Also, in our case the weekly quizzes count for 15% of the grade. Of course, anyone adopting the system will choose to give whatever credit they deem reasonable.

The three major components of the Internet component are: (i) the highly organized databases of questions that may be used both for quizzes and practice; (ii) the tutorials of review material to which students are referred, and (iii) the software used to administer the quizzes and refer the students to appropriate tutorials. These components have been class-tested for one year and are presently at different stages of development.

In order to make the quiz system useful to a wide range of precalculus professors each having their own idea of how to teach and what to teach in the course, large and comprehensive databases of questions are being developed to cover the topics normally taught in the precalculus course per se, and the review topics needed to understand the course. The guiding principles in their development are:

- Professors will be able to edit the databases, adding or removing questions as they wish.
- Professors may contribute to a central data bank. This will be available to any educational institution and will be maintained by the project directors.
- Databases will exist for each topic of precalculus. The organization will be Topic_Focus_Degree-of-Difficulty. For example, Lines_geom_easy will be a database which will contain questions concerning lines which have a geometric focus and are easy.

- Databases of questions which require an extensive written response will be created; however, these will not be graded by the system. Student responses will be forwarded to a professor's chosen address. If professors do not want to use these questions, they need not include them. Other than the open response questions, multiple-choice questions are also available on the quizzes. While this presents somewhat of a limitation, it is perfectly adequate for most of the topics dealing with basic algebra skills.
- Databases of questions which require that the students have graphing and/or computer algebra system technology available will be available for each topic. Once again, if professors do not want to use these questions, they need not include them.

As an example, for an introduction to linear functions we would end up with perhaps 12 databases each of which would contain 100 questions. The titles of these databases might be

Lines_algebraic_easy, Lines_algebraic_int, Lines_algebraic_hard, Lines_geom_easy, etc. Professors may then use the quiz generating software described below to indicate how many questions from each of these databases should be placed in their quizzes.

The software works in the following manner: (1) professors fill out an electronic form indicating the content of the quiz; (2) students receive a practice quiz which has been randomly generated from databases of questions and which is unique to each student; (3) students complete the quiz and submit their responses; then (4) the computer corrects the quiz and generates a report for the student which contains (i) percentage score (ii) electronic links and/or references to the text where they may review topics that they did not pass, and (iii) the questions which were incorrectly answered. If they pass the quiz with a grade predetermined by the professor, they are cleared to take a real quiz that is administered in the Testing Annex to the Mathematics Tutoring Laboratory or any other specially designated place.

In the quiz system, after students take practice quizzes, they are directed to a tutorial. This will generally be in the form of an Internet link. However, as always the system allows professors to substitute any other link or reference of their choosing.

3. The Text

A textbook has been written and was pilot tested during the past academic year. Pilot testing will continue for two more years before commercial publication. The book offers a mix of traditional and what might be referred to as "reform" elements. Perhaps this is the major strength of the textbook. It bridges two visions of what a precalculus textbook should be. Topics are presented using simple common sense examples to build on the intuition of students. Multiple representations and the corresponding interconnections are explored throughout the book. The presentation is informal in style. Each section starts with a note on the prerequisite topics that students should review using the Internet-based quiz system. Practice problems are embedded in the topics and examples being presented so that students will have the opportunity to test their understanding while reading the main text. Science and Engineering oriented examples and problems are found throughout the book. To get a clear idea of the nature of the textbook one would need to examine the presentation in more detail than is reasonable to include in a short article. The content and order of the textbook is quite traditional except in the part of the book where a group of topics is presented in a three-dimensional setting.

4. Introduction to Three Dimensions in Precalculus

The need for geometric visualization of three dimensions is very pronounced in engineering. All engineering majors must take graphics, physics, and statics courses immediately following or concurrently with precalculus where the material frequently occurs in three dimensions. However the formal presentation of three dimensions does not occur until the third semester of the calculus sequence. The primary impediment to presenting three dimensions earlier in the students' academic program is that it requires that they think abstractly. The traditional presentation typically does not include means of concretely visualizing coordinates in three dimensions. Hence, when students contemplate lengths of vectors, slopes of lines, and other simple topics in three dimensions, they cannot actually see the vector or the line with which they are working. Our pilot testing experience shows that students are comfortable with concepts in three dimensions when points, lines, vectors, and curves can be represented with concrete objects in three dimensions. To do this a set of manipulatives, resembling something like a Lego for three dimensional explorations in precalculus, was developed. We call this tool, the 3-D Kit. The most fundamental element in the 3-D Kit is a suitcase, which opens to form three dimensions. A rough sketch is provided in Figure 3. The pole, which forms the z-axis, fits inside of the suitcase and when the suitcase is opened, there is a hole within which it can be placed to form three-dimensional axes. The exterior part of the suitcase which forms the x and y axes are covered with an erasable board which allows the user to write on the xy plane. There is also an attachment which provides raised x and y axes as shown in Figure 4. Also within the suitcase are multicolored balls, which serve as points; antennas, which serve as vectors; foam covered strips of wire which serve as curves and as contours; and the necessary props to sustain points, vectors, and curves in place.

By incorporating three dimensions into the precalculus program, engineering students are being prepared for topics they will shortly need in other science and engineering courses. However, this approach also strongly reinforces two-dimensional concepts presented earlier in the course. For example, the concept of a rate of change, or slope is presented early in the course in a two-dimensional setting. The concept is presented again in three dimensions when dealing with the slope of a plane in the x direction and in the y direction. This provides multiple situations and visualizations to reinforce the basic concept of a slope. All of the presentations in three dimensions support parallel themes in two dimensions. Hence, presentations in three dimensions serve both to expose students to three dimensions which is important for engineering students and to reinforce the basic concepts of precalculus as learned in two dimensions. The textbook includes an introduction to the Cartesian three-dimensional space, vectors in three dimensions, functions of two variables, linear functions of two variables and planes. Some of these ideas are useful later in the course when treating linear systems of equations in three variables, and in the presentation of conic sections.

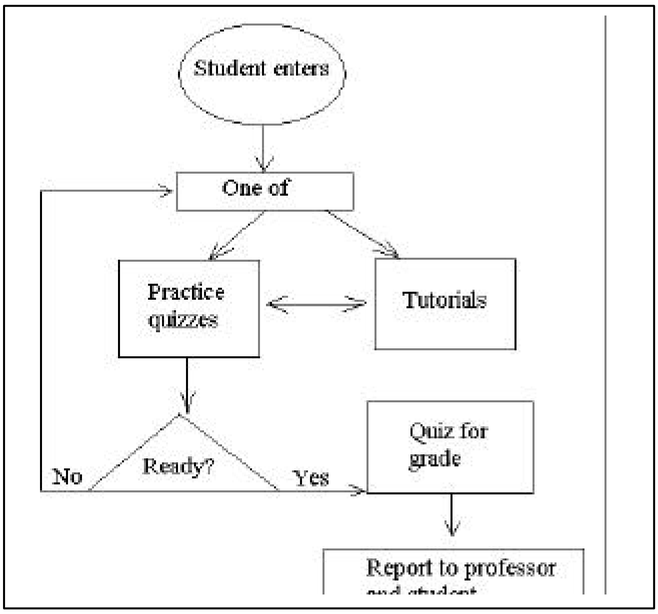

Figure 1

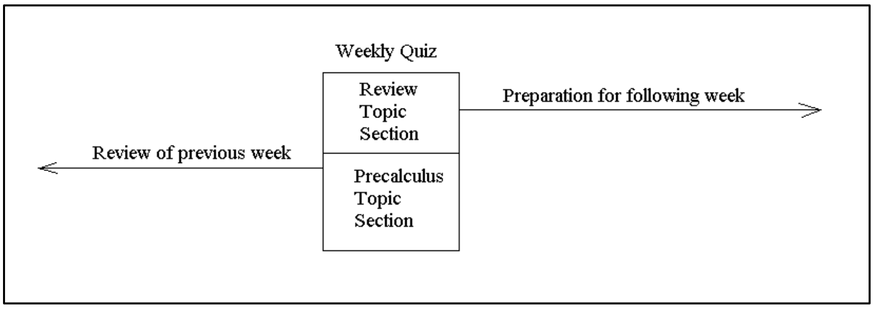

Figure 2

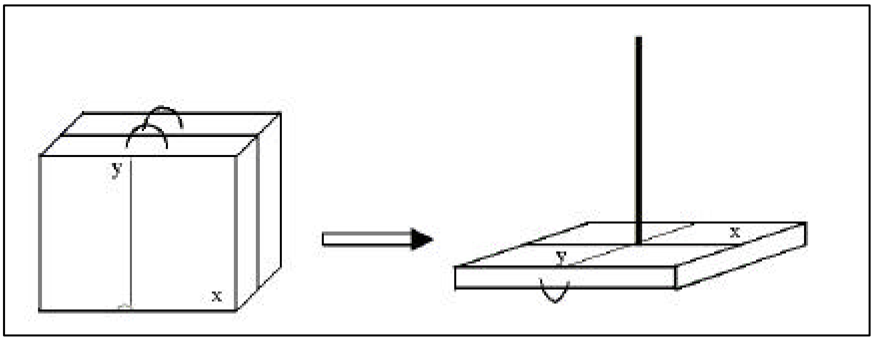

Figure 3

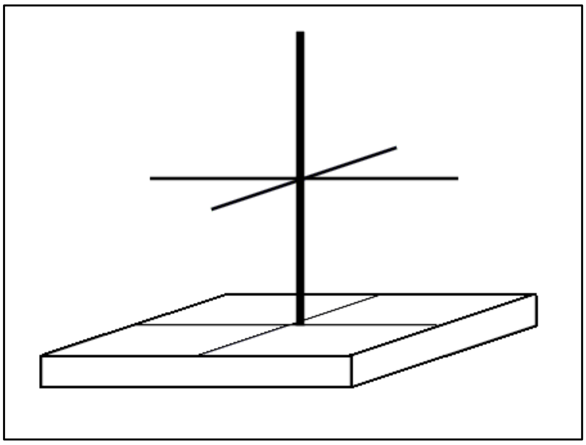

Figure 4