THE HISTORY OF MATHEMATICS IN THE EDUCATION OF MATHEMATICS TEACHERS: AN INNOVATIVE APPROACH.[1]

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ABSTRACT

It is a widespread view that mathematics teachers should have a working knowledge of the History of Mathematics, for three reasons. First, so they can make lessons more lively and interesting by adding to the lessons stories about mathematicians of the past; second, to help students develop a sense of Mathematics as a human production, always evolving; and, third, to develop a better understanding of the foundations of Mathematics. We understand there is a fourth reason: through that study the teacher can develop an understanding of the process of meaning production for Mathematics that would allow her/him a much finer reading of the learning processes in the classroom, as well as an understanding of the possibility of different meanings being produced for the 'same' mathematical object, for instance, 'linear equation', function' or 'dimension'. We have developed and conducted a course on the History of Mathematics for undergraduate students in which we read and discussed, over 30 two-hour sessions, four texts: (1) C. Wessel's paper on the analytical representation of directions; (2) G. G. Granger's text on the philosophy of style (a section related to Euclid's 'Elements'; (3) a section of A. Aaboe's 'Episodes from the Early History of Mathematics' (part of chapter 2); and, (4) a section from R. Hersh and P. Davis' 'The Mathematical Experience' (on the Chinese Remainder Theorem). We went from a primary source (difficult reading for them) to texts which discussed 'style', 'interpretation' and 'different presentations', aiming at helping them develop an awareness of the processes involved in meaning production for Mathematics; as much as possible we related the current experience with the experiences they had as undergraduate students taking Mathematics courses. Data from the course will be presented and discussed.

KEYWORDS: History of mathematics, meaning production, education of mathematics teachers.

Introduction

It is generally agreed by mathematics educators around the world that a working knowledge of the history of mathematics is important for mathematics teachers. The reasons given for that vary. Given the existence of a whole ICMI Study on the relation between history and mathematics education (Fauvel & van Maanen, 2000; see particularly chapters 4 and 5) it is not necessary to go any further.

To the current views, we want to add another. We will argue that a course on the history of mathematics should also be a place where students will discuss processes of meaning production, for both historical and present-day texts.

At our department

In this section we present our department's official view on why the history of mathematics (HM) should be in the curriculum and at which point. This will set the background on which our approached was developed.

In our curriculum a course on the HM is mandatory for both future teachers and future researchers; it is offered for seniors.

The course description says:

> OBJECTIVES (at the end of the course the student will be able to): have a general view of the historical development of mathematics up to the 18th century, as well as its relations to the social development. To establish relations between the development of mathematical concepts in ancient times and their theoretical development from the European Renaissance on. To identify the key mathematicians of the past and to link them to their work.

These objectives are clearly related to specific contents, not to formative processes. We think that the underlying idea is that "…as much as the mathematical content, the mathematics teacher needs to know its history, that is: the history of the content of mathematics" (Baroni & Nobre, 2000; our translation).

We searched the pedagogical project of our undergraduate program (Dept of Mathematics, 1992), looking for further explanations on the choices made; there we found:

> "Many of the difficulties and dissatisfactions regarding [the undergraduate program] are related to the lack of connection between different courses and to the lack of an organised development of learning." (p. 10)

And in this case one could think of the HM as offering some kind of 'stitching' of the different courses and contents, by working with their developments and relations along history. But a different solution was adopted,

> "[…] to group the contents in well defined paths, which we will call 'areas', with structure and extension such that they allow the student a more global and deeper understanding". (p. 10)

and the course on the HM was left to the last year.

The view that what you learn in a course includes the way, in which you have learned it, is becoming more widely accepted (Cooney at al., 1999). For instance, what you learn in mathematical courses that include a parallel discussion of historical and mathematical aspects is

different from what you learn in traditional courses that do not do it. And the way in which history is approached would also make a difference.

In the case of teacher education, his/her mathematical education must, in our view, include a discussion of processes, which the future teacher will face in his/her professional life, and by that we mean the courses on Analysis and Algebra, for instance. In other words, if the future teacher is to leave the university better prepared to teach than when she/he entered it, it is not enough to get him to review/practice the content she/he is going to be teaching plus offering 'foundations' for those school topics.

As the authors of a recent and detailed survey on 'teacher preparation research' point out (Wilson et al., 2001), there is no sound evidence "[evaluating] the relationship between teacher subject matter preparation and student learning." (p. 6), and also that,

> "The conclusions of these few studies [that deal with that question] are provocative because they undermine the certainty often expressed about the strong link between college study of a subject matter area and teacher quality" (p. 6)

With that in mind we designed and conducted a course on the MH for undergraduates. The core of the course would be directed towards helping students to develop an awareness of meaning production processes, such as when one is 'interpreting' or trying to understand a primary source text, but also when studying mathematics in present-day ('live') textbooks or attending lectures.

On Meaning Production

Probably the most repeated phrase in situations like the ones indicated above is "what is he talking about?" Similarly, teachers at all levels could ask about their students, "what do they think I am talking about?" Unfortunately, our guess is that this question is much less frequently asked than the other one.

Our central objective in the design of the course was that students could develop an awareness of those processes in his/her own thinking but also, as we are primarily interested in teacher education, that they developed an awareness of being on both sides of the meaning production process, that is, an awareness that not all that is natural or familiar to him or her is natural or familiar to the students, a fact which has important implications for the classroom activities.

What was needed to guide our work was a model, which dealt with meaning and knowledge production. But that model would have to deal primarily with processes, as the students' thinking has always to be read "during the flight".

We decided to adopt the Theoretical Model of Semantic Fields (TMSF), developed by one of us as a tool to support teaching and research in mathematics education (Lins, 1992, 2001).

Its central notions are those of 'knowledge' and 'meaning'. 'Knowledge' is characterised as a statement in which a person believes (a statement-belief), together with a justification she/he has for making that statement. 'Meaning' is characterised as what a person actually says about an object, in a given situation. It is not everything that a person could eventually say about that object. Meaning production and knowledge production always happen together, and objects are constituted through meaning production.

A third notion on the TMSF is relevant here, that of 'interlocutors'. It has to do with why a person thinks she/he can make a given statement in a given activity. We understand interlocutors as modes of meaning production that a person internalises as legitimate during his or her life; they are cognitive elements, not real people. In other words, to believe we can say something we must also believe that 'someone else' would say the same thing with the same justification.

The course

We decided that the starting point would be an excerpt of a text by Caspar Wessel, a Norwegian surveyor who read, in 1797, the paper "On the analytical representation of direction; an attempt. Applied chiefly to the solution of plane and spherical polygons" (Wessel, 1959). The text was translated to Portuguese, from the English version available to us.

Wessel says about his paper that,

> "This present attempt deals with the question, how may we represent direction analytically; that is, how shall we express right lines so that in a single equation involving one unknown line and others known, both the length and the direction of the unknown line may be expressed."

Wessel was a surveyor, interested in solving the problems of his profession, and what he does is simply to develop a representation of lines that allows him, using complex numbers in the calculations, to "solve polygons" (that is, given some of the elements of a polygon to determine all the others, a most common problem in surveying).

To anyone this is not a simple text and trying to read it with present-day eyes made some passages completely obscure or meaningless. That was what made the text seem appropriate: we would be able to discuss meaning production for that text as the production of a plausible account of what Wessel was talking about, which were the objects he was dealing with.

The main question that came up from them during the reading of Wessel concerned the kind of reading we were asking them to do: were we producing an interpretation in the sense of a particular (more, or less, correct) reading of the text, or were we in fact simply producing a plausible account for it? That led us to choose as the second text a section of G. G. Granger's book, "The philosophy of style" (Granger, 1974), in which he approaches the differences in the thinking of several mathematicians from the point of view of different styles:

> "[Our purpose is] to distinguish the plurality of modes of expression and of construction of a concept, and to produce an understanding of how this plurality is linked to distinct ways of practicing and even, if one wants to adopt this expression, to live the symbolism." (our translation) (p. 35)

We proposed the reading and discussion of a section on what Granger calls the Euclidean style. It is particularly interesting because there is plenty of discussion about the relationship between number and magnitude in Euclid; at some points Granger seems to suggest that Euclid could not associate both notions, while at other points he suggests that Euclid did not do so because it would be against his style.

The main question that emerged from that reading was a great difficulty, for the students, to conceive an actual separation between geometrical magnitudes and numbers. So we proposed the reading of a section of A. Aaboe's "Episodes from the early history of mathematics" (Aaboe, 1964), where he presents some of the work of ancient Greek mathematicians.

The fourth and last text was a section of "The mathematical Experience", by P. Davis and R. Hersh, on the Chinese Remainder Theorem. There they give seven presentations of the theorem, ranging from an old Chinese one, to one from computer science and a generalisation to structures other than $\mathbf{Z}$, found in a book on algebraic number theory. They analyse the differences among them, but without assuming each refer to different objects. We used this text as a model for their final assignment.

The students during the course

In this section we illustrate the types of questions and comments coming from the students.

During the presentation of the course it became clear that the students' expectation was that the course would consist of lectures on chronologically organised 'stories', involving mathematicians of the past, their mathematical work and their personal lives, a kind of "the rich and famous in the history of mathematics".

Difficulties appeared soon. The students were struggling because the words were being used in ways unusual to them. For a phrase like,

> "[For as we pass from arithmetic to geometrical analysis, or from operations with ordinary numbers to those with right lines, we meet with quantities that have the same relations to one another as numbers, surely; but they also have many more."
> (Wessel, op. cit., p. 56)

we wanted the students to produce their own understanding; we tried to help them with questions like "what could 'relations between numbers' be?"

On the second day of the course, however, one of the students asked us "whether the rest of the course would be like 'that'". When we asked what the 'like that' meant, she said, "well, you know, like tripping [the slang]…" and our answer was "yes", but with the addition that in respect to their intellectual capacity we would not 'water down' the course, despite being aware of their difficulties. This was a highly relevant passage to us, because it made us aware that at least some of the students were actually frustrated by having to think instead of sitting passively at a lecture; it also gave us the opportunity to introduce a discussion about how we saw the process they were going through as completely similar to studying, say, Analysis from Rudin's textbook.

Two exchanges can be seen as typical here. The first happened as we were discussing § 3, where Wessel says,

> "If the sum of several lengths, breadths and heights is equal to zero, then is the sum of the lengths, the sum of the breadths, and the sum of the heights each equal to zero." (Wessel, op. cit., p. 59)

This is a most intriguing passage of the text, because it does not relate directly to anything said before it in the text. When we asked the typical "what is he talking about?" there was a deep silence and then one student said,

> "Well, he is saying that if the sum of the length, the breadth and the height is equal to zero, then the length, the breadth and the height are each equal to zero."

This is a kind of situation in meaning production that in most cases goes unnoticed. It is quite common that the teacher will simply say that repeating the phrase is not enough that the student has to 'explain it'. What could happen in a teacher's mind that would make him or her see that student's statement as a repetition of the original one? We argue that the teacher could complete the student's statement to make it identical to the original one. Why? Possibly because the teacher was not aware of the possibility that the student could not, in that specific situation, produce any meaning for the original statement, and that means that cognitively he could not see the plurals, and that is why his statement is all in the singular.

We asked the student to read aloud, for the whole class, § 3, and he did so, reading it in the plurals. When asked again to explain what Wessel was talking about he said,

> "Uh? He is saying that if the sum of the length, the breadth and the height is equal to zero, then the length, the breadth and the height are each equal to zero."

Exactly the same statement, only this time spoken slowly and with a different intonation, as teachers many times do when a student says she/he did not understand something… The crucial point here is that the student was clearly convinced that what he was saying was right and that he was not even aware of the removal of the plurals. But what about the 'objective' text in front of him?

Some of his colleagues said they understood his 'explanation' and that they agreed with him. We were actually puzzled, as we could not produce any plausible meaning for what had happened, and we told them so. That led to more exchanges and we finally understood what was happening. For those students, lengths, breadths and heights were measurements of line segments and at best (or worst, because it is a pretty weird situation) they could be zero. To talk about a sum of lengths being equal to zero is already talking about all lengths being zero and that is what the student was talking about, using, instead of, say, three lengths, a length, a breadth and a height.

But Wessel was, in our understanding, talking about completely different objects. For him lengths, breadths and heights were directed line segments in three orthogonal directions (he does not say this, though). We came to this interpretation precisely because we wanted to produce a plausible meaning for the original statement, without having to change it.

Best of all, this 'incident' gave us a great opportunity to deepen our discussion of meaning production: how many times in their lives, studying 'present-day' mathematics, similar situations might have happened? How many times they (and us) might have read what was not written, simply because we could not produce meaning for what was actually printed?

A second, very brief, exchange, illustrates a different point. Struggling with the text, a student asks the professor:

"Would this guy have ever spoken to a mathematician while writing this?"

and explained her question:

"Because it seems he doesn't know what he is talking about…"

She transforms her own difficulty in producing meaning for the text into Wessel's ignorance of mathematics. But she knew from the beginning he was a surveyor, not a mathematician. Would she say something similar if reading a mathematics textbook, written, supposedly, by a mathematician? We think most likely she would not; she would place the difficulty on herself. But in Wessel's case she felt it was legitimate to question his mathematical understanding.

On the section "On meaning production" we spoke of interlocutors we internalise during our lives and which are the sources of legitimacy for what we anticipate we can say. There is, in the situation above, an explanation using the notion of interlocutor: the student had internalised, along her life and experiences that only mathematicians know how to talk about mathematics. Maybe that included remarks by mathematics professors about physicists and engineers.

The crucial process, however, that we want to emphasise was that she transformed her own difficulty in producing meaning for the text into Wessel's ignorance of mathematics. Had she done that before or while studying mathematics, perhaps saying that such and such author does not 'write well'? (but not that the author does not know mathematics…). Also, the discussion generated by her question, about legitimacy and meaning was quite useful, in again, establishing a link between 'interpreting historical texts' and 'studying present day texts', and the link was meaning production.

The reading of the second and third texts followed the pattern of the first, although they were different in the kind of content. We did not stay too long on the fourth text, as it was to be used mainly as a model for the final assignment.

We shall now discuss some general aspects of the final coursework produced by the students. The assignment was to choose a mathematical concept/notion/idea, to find several different presentations (as in Davis and Hersh) and to comment on how each presentation would constitute different objects.

The themes chosen were: the plane; square root; 2nd and 3rd degree polynomial equations; logarithms; irrational numbers; Pythagoras' theorem; fractions; parabola; ellipsis; integer numbers; parallel lines; rule of l'Hôpital; the fundamental theorem of Calculus; differentiation; complex numbers; the number ð.

The criteria for marking was to give full marks (10) if both different presentation and comments were given and correct, to give half of full marks (5) if only the presentations were given and correct; intermediate marks could be given in accordance to this.

Four papers were given full marks: the plane; the fundamental theorem of Calculus; complex numbers; parabola. One was given near full marks, differentiation (9.0), and the remaining eleven were given 5.0's or 5.5's.

These results seem to suggest that there is a clear cut between those who accept that different presentations do constitute (or suggest the constitution of) actually distinct objects, and those who could only see the same objects through different presentations (and for that reason could not say that different objects were there).

Final remarks

This was a first attempt at producing a course on the HM for undergraduate students (particularly future teachers).

It was clear that these students' understanding of history and of the role of studying history was stereotyped and superficial; we think this is an issue to be addressed at future versions of this course.

Two objectives were achieved: to get students to participate reflectively in processes of meaning production and to establish a first link between interpreting a historical text and studying present-day mathematics.

Having only a third of the students reached a sufficient understanding of the process of meaning production, in this particular course, might be taken as too low. But the course was indeed ambitious from the beginning and we knew it was a difficult task. For that reason the one-third success achieved was, we think, very encouraging, and the level of interaction of the students in the discussions is another source of support for the further development of this approach to HM in the education of future teachers.

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