

MAKING RELEVANCE RELEVANT IN MATHEMATICS TEACHER EDUCATION

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ABSTRACT

One of the features of the reform in school mathematics is that school mathematics should be made relevant to the learners. The incorporation of mathematical modelling in school mathematics is one of the ways that is offered to realise the relevance ideal. Ostensibly the inclusion of mathematical modelling will provide school learners opportunities to develop mathematical power i.e. the ability to make sense of the world and of mathematics. A key question in this regard is: How prepared are practising mathematics teachers to incorporate mathematical modelling in their teaching? This preparedness entails that mathematics teachers be knowledgeable with mathematical modelling as content.

In this paper this mathematical modelling content is elaborated upon and reports on a study which investigated secondary mathematics teachers' knowledge of mathematical modelling as content in South Africa and Eritrea. The one major finding of this study is that these teachers deem their experience with mathematical modelling as motivational and that they do find mathematical modelling problems dealing with social issues relevant. The second major finding is that in developing mathematical models for social issues teachers utilise very low levels of mathematics which is in essence against the intention of school mathematics reform. It is argued that this disjuncture—the engagement with low level mathematics and the personal expression and experiencing of the modelling as motivational and relevant—that requires attention in mathematics teacher education programmes aimed at assisting teachers to realise the relevance ideal in their teaching. These programmes, it is suggested, should not restrict the mathematical content knowledge to concepts, facts, procedures and proofs but it should also include mathematical modelling as content and at a minimum this would mean that teachers have to experience **all** the components of the mathematical modelling process.

Keywords: Mathematical modelling; relevance of mathematics; mathematics teacher education; content knowledge

Introduction

One of the demands made on school mathematics is that it should be socially, environmentally, culturally and academically relevant. Varied meanings are attached to relevance although the most popular meaning is that learners should be able to use the mathematics that they learn in real-life situations. Notwithstanding the difficulties associated with the notion of “real-life situations” it is generally believed that the relevance ideal could be achieved through the incorporation of applications and modelling of school mathematics. The burden to implement applications and modelling of school mathematics to realise the relevance ideals is left to teachers. In many countries teachers are underprepared to effect this implementation since they themselves did not experience the applications and modelling of mathematics in a meaningful way during their preparation as mathematics teachers. The discussion that follows deals with mathematics teachers’ experiences of mathematical modelling and how, amongst other things, relevance is manifested during these experiences.

The problem of reconceptualisation and representation of content contextually

A common notion associated with relevance of mathematics is that mathematics should be represented in some context. Dowling (1996) and others have developed the notion of reconceptualisation of mathematics to index the representation of school mathematics in some contextual format. By this re-representation of school mathematics they mean that in order to make mathematics relevant designers of school texts reconceptualise mathematics and present it in a form different from what the canon is supposed to be. In this body of literature the canon of mathematics remains mostly undefined. The undertext, however, reveals that the mathematics is the content of school mathematics stripped of its designed contextual trappings. This decontextualised mathematics is in essence an elementarised version of what the French didactical school calls institutional mathematics. Notwithstanding the criticisms of the reconceptualisation movement, it is so that in order to make mathematics teachable, designers of texts and mathematical activities do develop and devise contexts, which can serve as carriers from which mathematical concepts, procedures and justifications can be developed. An unfortunate development in this movement is that the protagonists work from the assertion that this is a demonstration of mathematical modelling. The origin of this claim is well known and understandable. The consequences are more far-reaching. One such consequence is the absurdity of non-relevant context, which is the sugar-coating of normally calculational work by absurd contexts. Nevertheless, the intentions of the “concept-carrying-contexts” protagonists are laudable. No matter how fierce the critique decrying the sometimes whimsicalness of the concept-carrier-context notion, school mathematics is always a watered-downed version of institutional mathematics and is always reconceptualised to particularly make it teachable. It needs to be borne in mind, however, that whatever claims are made to embedding mathematics in context the purpose of this embeddedness is not the construction of mathematical models but rather the use of context and sometimes mathematical models as vehicles for the learning of mathematical concepts, procedures and at times justifications. Mathematical modelling should not only be a vehicle for these mathematical ideas. Remaining at this level conceals the “behind-the-scene” work and intricacies involved in the construction of a mathematical model. Julie (1992), for example, illustrates the intricacies involved in presenting a division problem in the context of a grandmother

sharing money amongst her grandchildren. Jablonka (2002) takes this demonstration further in her study of the contextual representation of school mathematics by investigating the epistemological claims behind the context in school mathematics texts. In trying to come to grips with the behind-the-scene work and intricacies involved in mathematical model construction, it is necessary that mathematical modelling should also be experienced as content. Mathematical modelling as content entails the construction of mathematical models for natural and social phenomena without the prescription that certain mathematical concepts or procedures should be the outcome of the model-building process. It also entails the scrutiny, dissection, critique, extension and adaptation of existing models with the view to come to grips with the underlying mechanisms of mathematical model construction.

Mathematical modelling as content as a way into relevance

In our quest to address the issue of relevance we inserted into our teacher education (pre- and in-service) mathematics courses a section on mathematical modelling. These courses have evolved over the past 7 years and include teachers (i) studying existing mathematical of social and economic situations in a guided way, (ii) assessing hypothetical exemplars of learner modelling work, and (iii) constructing models in an immersed way. In all these activities the teachers were exposed to the normal cycle for mathematical modelling. The underlying assumption, particularly with (iii) is that teachers should experience mathematical modelling as content. This means that teachers' experiencing of mathematical modelling should be as near as possible to the way it is done in the practice of mathematical modelling. A major characteristic of this practice is that the actual problem is initially vaguely formulated although the ultimate outcome--an artefact to realise a particular objective as specified by a client--is known to both the model-developer and requester.

Some of the situations that teachers were required to develop models for over the years were: A salary system to bring about equity based on the principle of "equal pay for equal work" taking into account years of service, promotion criteria and qualifications; the Human Development Index and other social indexes such as a community development index; school enrolment projections and garbage accumulation.

Data were systematically collected. This data comprise of observations and video-recordings of teachers at work; the rough work that was produced during the model construction process; the final reports on the models, formal and informal interview conversations and post-activity questionnaires.

The analysis proceeded by reading and rereading the data pieces of an entire work session. A description of the insights gained from the read-reread process pertaining to the broad research project—teacher behaviour when engaging mathematical modelling—was formulated and presented as a summary narrative of the session. This summary narrative was studied and commented on. The emerging comments were related to the broad question. For the analysis of subsequent sessions, the summary narrative was compared and contrasted with previous commented summary narratives in order to identify statements which could be similarly or differently commented and eventually coded.

Major Findings

Dominance of model as vehicle

One fairly consistent finding the analysis rendered is that the model-as-vehicle paradigm dominates the model-construction activity. This is seen as the search for a formula to describe the situation under investigation as illustrated in a teacher's response, figure 1 below, of her experiences with the salary scale activity.

[This is a translation of the teacher's response which was written in Afrikaans]
It was a struggle to understand the problem. The many principles, variables had your head spinning. We started by trying to get a formula from the table—excited! Oh...the equation/formula does not satisfy all the conditions. The struggle starts again from the beginning or a different strategy is sought. Decide first to work with one post level only to simplify the problem—point of departure gets a ceiling—highest position and highest number of years in post level. Build other formulae around the norm. Process is much “trial and error.” Again and again and doing things over and over. Fit and measure/Test/verify. A possibility? OK. You get some confidence because there is no right or wrong—tension of criticism is gone. It was nice to prostitute your brain and test your limits. The end is sweet.

Figure 1: Teacher's description of experiences during mathematical model construction

This notion of the existence of a formula that can be found from the data dominates most of the both initial collective and individual deliberations. This is even so when the problematic the teachers were engaged with was assumed to be of immediate relevance to the teachers. One would, for example, expect that after a teacher strike on salary increments an activity on the development of a model of salary increases that would lead to realisation of the “equal pay for equal work” principle would entice teachers to first discuss the issues related to this principle as it pertains to their own respective situations. This, after all, is one of the tenets of relevance: what does it mean to me personally. It was hoped that “what does it mean to me personally” would engage teachers in a critical discussion and analysis of the situation. However, the formula-seeking behaviour dominated over the situation-analysis, which could have led to the development of a model based on derived assumptions. We contend that this formula-seeking is related to teachers' major exposure to mathematical modelling as a vehicle in which they are in a major way required to lead learners to identify patterns based on formulae. Essentially there is nothing wrong with this behaviour. The division of approaches to modelling into empirical modelling—fitting formulae to data—and axiomatic—developing a model from a set of assumptions—requires knowledgeability of formula-seeking. However, a deeper issue is at stake. This relates to the flexibility and robustness of teacher knowledge of mathematics. There is an emerging literature corpus, which reports on the lack of flexibility of teacher school mathematics knowledge and the desirable mathematics content for teaching as it pertains to primarily the concepts and procedures found in school mathematics. The emerging finding related to school mathematical modelling extends these findings to an essentially neglected area, which is being deemed important to realise the relevance ideal to teacher knowledge about school mathematical modelling.

Immediate perceived usability

When constructing models practising teachers seem to express a preference for a kind of relevance, which is immediate to their work circumstances. Consider the excerpt, figure 2, of fieldnotes made during teachers' engagement with the school enrolment model.

The teachers had to particularise a model for planning the supply for mathematics teachers to their schools based on the number of pupils at their school and a school enrolment model provided by Gould (1993). They presented their particularisations to the class. At the end of the presentations we engaged in a conversation around their work and the experiences with the activity. Mr K started the discussion and he said: "This was one of the first pieces of work I did where I can see how I can use it in my situation. We know that the number of teachers for a year is determined by using the enrolment of the year before. Now I can actually use this model at school and we can determine the number of teachers a few years in advance."

Figure 2: Excerpt of fieldnotes on school enrolment model

This contrasts with the data on teachers' reaction to the model building activity on garbage accumulation given in figure 3 below.

PLASTIC SHOPPING BAGS

The Minister of Environmental Affairs and Tourism has recently raised the issue of plastic shopping bags contributing towards filthiness and unsightliness of township areas. The plastic bags are blown around and get attached to fences presenting a sore sight of schools as dirty environments. It has been said that "School fences appear to be constructed from plastic shopping bags."

Develop a mathematical model to describe the accumulation of plastic shopping bags against a school fence over a period of time.

Figure 3: The garbage accumulation activity

A similar discussion on their experiences with this activity produced nothing about the usability of the models that the teachers developed. Even the enticement of producing a letter to the municipal authorities about garbage collection intervals and a concomitant enticement of educating the immediate local community about the inherent health risks associated with the dumping of garbage in open spaces did not have subjective appeal to the teachers.

What engages teachers and what not is a complex issue. Immediacy in terms of what I can use in my situation as it is currently is emerging as a facet of teacher behaviour regarding relevance. One aspect of this facet is that this immediacy will be unequally distributed across teachers. Regarding the enrolment model, for example, it is so that teachers directly involved in the administrative matters of a school would find work with this model usable. Those not directly involved will have a different kind of attachment which might at times border on hostility given the status related to job security in instances when a decrease in enrolment will result in a decrease in staff leading to retrenchment. For the garbage accumulation activity, on the other hand, attachment is linked essentially to political convictions. The more teachers are convinced that they should participate in the day-to-day struggles of the communities from which their learners come, the more they will perceive model-construction of this nature as an immediate usable activity.

Settling for the simplest and the non-activation of deeper Mathematics

One of the rationales for the lobbying for the inclusion of modelling and applications in school mathematics is that it will play an activating role. Modelling and applications will, in addition to its usability features, be a catalyst for thinking about mathematics that learners (and teachers) did not think about before. This can be viewed as relevance to mathematical development. The excerpt in figure 4 is from a transcript where teachers were requested to extend the Human Development Index (HDI) after they had studied the construction of the HDI. They were to extend the HDI by adding a fourth factor, satisfaction with the government of the day, to the HDI.

T(4): I need to know what is the satisfaction of the government.
T(1): We could mention many things. To educate the people, to have fair tax on domestic production and increase the domestic production.
T(2): That is clear.
...
T(3): The basic factor for satisfaction of the government could be education. Everyone has right to education.
T(1): Right. To attain satisfaction, the government could develop clear and consistency national policy of education.
T(2): In our discussion, we agreed that there could be change with HDI. The emphasis we made was on education index. We related satisfaction of the government with education. We tried to double the education index to attain increase in HDI for the satisfaction of the government.
T(1): Okay, we need to come up to a conclusion. We could say that we will do change with education index. Therefore, we can write the revised HDI in the form of

$$\text{HDI} = \frac{\text{L. Expec.} + 2 \text{ Educ. index} + \text{GDP index}}{4}$$

Figure 4: Excerpt of discussion on the extension of the HDI

A few things can be observed from this narrative. Firstly, for extending the index the teachers were contend to simply work with the categories involved in the HDI. Although their discussion included references to fair taxation; domestic production and increase in domestic production, they were contend only to double the education component of the index and view this as equivalent to a fourth factor. The teachers remained as near as possible to the model that was studied and their extension of the model was confined to same categories contained in HDI.

Closely linked to first issue is the teachers' tendency to use only simple arithmetic. The above example is illustrative of this phenomenon. Although the HDI appears on the surface to be a simple weighted additive model, there is much deeper mathematics underlying the eventual model. As indicated the teachers tend to use fairly simple arithmetic: the doubling of one of the factors. This was not confined to the construction of indexes but was also the case for garbage accumulation and the teacher salary increments.

Lastly, there is closure on the further exploration of mathematics. Across the data the consideration of deeper level mathematics was not observable. For example, the garbage accumulation problem can lead to the mathematics involved in stocks and flows. The seeds for a development in such a direction is clearly discernible in a group of teachers' description for the construction of the plastic bag accumulation model in figure 5 below.

How can the accumulation of plastic bags be worked out? We can say the Accumulation (A_{pb}) is the proportion of learners that litter, times the proportion of the community that, multiplied by the amount of days and times the amount of plastic bags used per day gives the total accumulation. We then subtract the attempts made to clean up, found by multiplying the amount of cleaning up attempts by cleaning staff, divided by the amount of time spent on cleaning up.

Figure 5: Teachers' description of model for plastic bag accumulation

Although the above description can be faulted, the seeds for moving this description and the resultant model to the mathematics involved in stocks and flows is clearly observable if, for

example, one such formulation (Bartholomew, 1976: 162) for stocks and flows given below is considered.

$$n_j(T+1) = n_j(T) + n_{0j}(T+1) + \sum_{\substack{i=1 \\ i \neq j}}^k n_{ij}(T) - n_{j,k+1}(T) - \sum_{\substack{i=1 \\ i \neq j}}^k n_{ji}(T)$$

Where:

$n_i(T)$ —($i = 1, 2, \dots, k$) denotes the number of persons in grade i .

$n_{ij}(T)$ —number of people moved to grade j by time $T + 1$

$n_{i,k+1}(T)$ —number staff leaving the university altogether.

$n_{0i}(T+1)$ —new entrants

[Note: Bartholomew's models is for teaching staff at a university where they get promoted from one grade level to another under conditions specified by the institution.]

Nowhere in the teachers' deliberations, even during informal discussions, were anything said or done which would point that this kind of mathematics was activated. Teachers seem to be fixated on what they perceive the task at hand to be and hence resolving this perceived task is for them the point of closure.

Conclusion

It has been related above how fairly qualified teachers of mathematics deal with and experience mathematical modelling and how the notion of relevance was manifested in the work and experiences of the teachers. Relevance revealed itself as complex and variegated. In some instances it was manifested as immediatist in the sense of usable for the work situation where teachers found themselves. In other instances it is assumedly closely tied to the socio-political orientation of the teacher. And in still other instances it is linked to the teachers' awareness or not of possible deeper level mathematics embedded in the descriptions and models that they construct. This last issue is closely linked to the use of fairly elementary mathematics in the teacher's model-building activity.

It is contended that these behaviours of teachers are related to their exposure, in no small manner in South Africa at least, to mathematical modelling as a vehicle. Uncritically it is assumed that this modelling as a vehicle will satisfy the realisation of the expressed ideal of relevance of school mathematics. This is not necessarily the case particularly if the incorporation of the relevance ideal in school mathematics is ostensibly the only ideal which directly addresses the development of a mathematical temper—a spirit of dealing rationally with the desirable and undesirable effects mathematical installations in society. There is no doubt that this realisation can only be effected through mathematics teacher education programmes which, in addition to developing mathematical modelling pedagogical content knowledge, aim at developing mathematical modelling as content. After all, it is during the engagement with mathematical modelling as content that windows of opportunities are opened for dealing with relevance relevantly.

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