

TEACHING AND LEARNING MATHEMATICS WITH VIRTUAL WORLDS

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ABSTRACT

Much of mathematics and statistics is taught using textbook examples and exercises. It is difficult through these to give students a feel for the broad issues involved. Ideally students could carry out experiments in the real world to then model and analyse mathematically. However, this is not practical for large classes and the logistics may even detract from the learning. We propose using virtual worlds instead, allowing students to manipulate the parameters of a simulated experiment and record the results. These simulations should be messy, requiring the students to think about measurement issues and noise and how these impact on the mathematics. The results can then be used as a starting point for teaching, in place of the traditional exercise settings. We give experiences and feedback from several virtual worlds for statistics and discuss current work on virtual worlds for calculus.

1. Introduction

Teaching large service courses in calculus and statistics is a difficult challenge, requiring the motivation of students whose main interest is typically not mathematics and whose mathematical backgrounds can vary greatly. On the other side, student learning is affected by similar issues. Additionally a current concern for many students is time pressure and the need for efficiency. Lectures and laboratories based on **virtual worlds** aim to address these issues. They motivate theories and methods by providing a concrete setting, which relates to nonmathematical interests. They emphasise qualitative aspects of the course, allowing students with weaker mathematical backgrounds to gain confidence. And they achieve these aims without adding to student workloads, an important efficiency for large classes. The use of virtual worlds is also suitable for smaller and more specialised settings, and for secondary schools.

In this paper we describe some virtual world activities used in teaching an introductory statistics course. Section 2 describes an exercise using virtual rats, a simple enrichment of a textbook exercise. This is extended in Section 3 to a virtual world involving a more complex interface, with animated plants growing under various conditions, in which students have a more active role. Section 4 then looks at similar settings under development for teaching a comparable service course in calculus. We conclude with general discussion in Section 5 on the effectiveness of the approach.

The focus of the paper is on student experiences with the virtual worlds. Technological approaches to teaching such as this are often exciting for the developer but are ultimately worthless unless students enjoy them and, more importantly, can see that they will help their learning. At the end of the final laboratory session the students were surveyed to obtain feedback on their use of the various virtual worlds. The survey questions were very general, such as “Please comment on the use of virtual rats in Practical 2”. Of particular interest is whether the students would identify the motivating principles behind the use of the virtual worlds. The comments received, both positive and negative, are given along with descriptions below.

2. Virtual Rats

Most introductory statistics textbooks are rich with real data sets, allowing students to relate the results of their explorations and analyses back to real scenarios. This is certainly desirable; Cobb and Moore (1997) suggest that “statistics requires a different *kind* of thinking [to mathematics], because data are not just numbers, they are numbers with a context.” However, it is still a somewhat passive experience because the students have to take the context for granted. They have not been involved in obtaining the data and so lack ownership of the setting. Mackisack (1994) gives an overview of the other benefits of experimental work. For instance, the students also get an appreciation of the practical issues involved in carrying out experiments and collecting data, an outcome encouraged by Higgins (1999). The first aim of using virtual worlds is to engage the students in thinking about the design of the experiment and the origin of the variability in the data, while not allowing this to be so time consuming that the rest of their learning suffers. Additional emphasis on the practical issues mentioned above is provided by the virtual plants in Section 3.

As an example of a virtual experiment, consider a setting described by Moore and McCabe (1998) of a two-way analysis of variance, which involves an experiment for exploring the effects of calcium

and magnesium on blood pressure. Three levels of each mineral in the diet of rats were considered, giving nine possible treatments to try. A standard textbook exercise would give the resulting data, or simply the summary statistics from the nine treatment groups, and then use questions to have the students visualise the data or test for main effects and interactions using ANOVA.

A simple interface for a virtual version of this experiment is given in Figure 1. Here the user can specify the calcium and magnesium levels (each Low, Medium, or High) for a particular rat and then click the measure button to find out its blood pressure after the treatment. Clicking again returns the blood pressure for another rat. An appreciation for the effect of the treatments and of the variability of results can be obtained easily in this simulation; carrying out this actual experiment would not be practical in a statistics course, especially with a large first-level class. A student commented that “it was an interesting way of collecting the data rather than simply being given data to work with.” Another noted that “it was a good way to investigate the effects on rats without using actual rats”, suggesting that they really identified with the virtual setting, despite the simplicity of the interface. Along similar lines, a further student appreciated the difference between the virtual world and the real world by saying that it was “better than using live ones as it cuts outside variables.”

Diet Settings	Measurements
Calcium level: <input type="text" value="Low"/>	Blood pressure: <input type="text"/>
Magnesium level: <input type="text" value="Low"/>	<input type="button" value="Measure"/>

Figure 1. Blood pressure experiment

Using this in a computer laboratory exercise, the students are not told to try all nine possible treatments. Instead they are told that they have a budget of 30 rats and have to use these to explore the effects of the two minerals on blood pressure. They have to think about how best to do this, including such standard questions as determining the number of possible treatments. Indeed some will just look at the four treatments using the Low and High levels of each mineral, giving more observations for each treatment, while some will work with the full nine. One mineral can also be left fixed, allowing exercises in one-way ANOVA or in two-sample comparisons. One student complained that “the number of combinations we had to do was a bit tedious”. However, this is the whole point of the exercise, to make the activity of data collection more concrete. In fact most students highlighted the speed of the process: “very easy to use, and being able to repeat the treatment so quickly makes it very efficient and time saving.”

The virtual world is set up so that there is an identifiable interaction between the two minerals. This exercise is used at the beginning of the course, introducing main effects plots and interaction plots from which the students can detect and understand the interaction present. The lecture course itself does not say much about two-way analysis of variance, yet this simple exercise allows students to appreciate the ideas involved and the types of effects that can result.

3. Virtual Plants

The virtual rats have proved successful in getting students to think about statistical issues beyond the mechanical techniques that are typically emphasised. However, they leave out one important step and that is the measurement process. The way the measurements are carried out, with possible errors and biases, can have profound effects on the statistical analyses that are carried out. It is not clear, for example, whether the variability that the students observe in their virtual blood pressures is coming from differences in the rats or from errors in the measurements.

Simulating the measuring of blood pressure on rats is a difficult activity to capture in a concrete manner. An alternative setting was created using virtual plants (Bulmer, 2001). The interface, shown in Figure 2, is similar to that for the rats with two factors that can be controlled. Four levels of nitrogen fertilizer can be specified (None, Low, Moderate, and High) along with three levels of irrigation (None, Some, and Lots). However, rather than being able to obtain a series of measurements by clicking a button, the user instead gets a movie which shows the growth of 12 plants, 6 with one treatment and 6 with another. Figure 3 shows the last frame of one such movie; all plants received some water, but the plants at the rear had moderate fertilizer while the plants at the front had low fertilizer. A student noted that this was “better than [the rats practical] – could actually see the results in picture form which gave a better idea of what happened.” The virtual plants were generated using L-systems (Prusinkiewicz and Lindenmayer, 1990), which has the side benefit of introducing curious students to some contemporary mathematics.

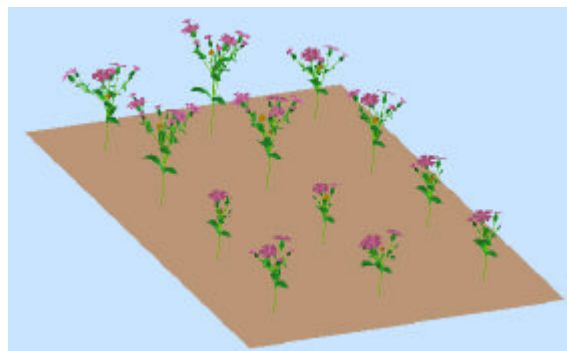
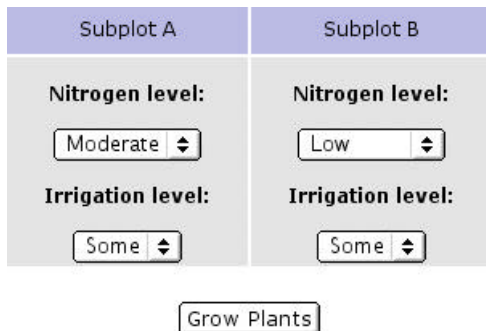


Figure 2. Plant growth experiment

Figure 3. Final frame of plant growth movie

Instead of being given a measure of growth for each plant, the students now have to deal with task. Students need to start by thinking about why they might be measuring plant growth. They could measure the heights of plants if they wanted to see which treatment gave taller plants, or they could count branches, leaves, or flowers if they wanted to see which gave higher yields.

Measurements of height can be made from the screen using a plastic ruler. It is quite a satisfying experience to walk into a computer laboratory and see a room full of students with rulers up to the screens. They are physically engaging with the setting, rather than passively taking a set of mysterious numbers from a textbook exercise, or even from a real study.

Measuring height with a ruler is difficult because it is hard to know where the top of a plant is; they branch outwards after an initial vertical growth. Counting flowers or leaves is difficult because it is hard to know you've seen them all, just as in real life. The measurement process should be difficult

and students should have think about what simplifications or estimates they are making. One student complained that “it would have been more helpful and easier to use if some kind of variable was presented with the movie as a result (rather than making student measure it off the screen)” but most saw the purpose of the exercise in that it “made it more interesting than copying information out of a data set.”

The main complaint was that the graphics were not clear enough to make accurate measurements, such as counting the number of leaves. Again this is partly useful, keeping the measurement process difficult, but it does not reflect reality very well. If students were working from a photograph of the field then they would not have the pixelation problem. A new set of virtual plants has been developed for 2002 using ray traced graphics to give much clearer images of the plants. One disadvantage of this is that it uses a perspective projection, rather than the parallel projection seen in Figure 3, which will make measurements of height more difficult. This may be offset by the inclusion of shadows and other visual cues.

4. Calculus and Dynamics

Service teaching in statistics is not the only area where students need to appreciate the relationship between the mathematical ideas and the underlying reality that it models. Much of calculus teaching is directed towards students in other disciplines, such as engineering and physical and biological sciences, for whom similar motivational issues are present.

Exercises are currently being developed based around such settings as a virtual pendulum and a virtual planetary system. The pendulum world is simply a series of videos of real pendulums with different string lengths and weights. This is a simple setting that could in fact be done quite easily by students, but again it may be more efficient to have the virtual world preprepared on the computer. Different students can collaborate on their analysis of the same physical setting. As with the plants, no measurements are given to the students. It is up to them to make measurements of the string length and the weight (which is an interesting visual problem) and then of periods and other dynamic quantities. The planetary system is computer generated, showing a fictitious system and allowing students to make measurements about radii (a non-trivial task for elliptical orbits) and motions.

These settings aim to work on two levels. Firstly, the students can graphically explore the relationships between the quantities they measure, such as the period and string length of the pendulums. This is appropriate for students at the secondary level, and leads on to the idea of summarising relationships using function curves. Secondly, students at the upper secondary and tertiary levels can look at mathematical models for these systems, use their measurements or external information to estimate the parameters in these models, and then compare their models with the observations. They can then look at discussing why there might be discrepancies between the model and the measurements.

5. Discussion

The virtual rats experiment was very simplistic, lacking any graphical output to immerse the user, but in the past four semesters it has consistently received positive feedback. Almost any statistical textbook exercise could be converted into such an experiment by first modelling the data and then

using that model and random number generation to recreate the data “live”. It gives students ownership of their work by putting the data into a context. It also means that students each have different data sets, which helps encourage cooperative learning (Magel, 1998) and broadens their learning experiences.

For the course in question, the students are also given projects in which they are asked to design and carry out their own experiments, followed by statistical analysis and the writing of a mock journal article. This is a rich form of assessment, but in a class of 500 it is difficult to have one-on-one discussions about each student’s intended experiment and the issues they may face. The use of the virtual worlds in the laboratory setting allows students to discuss experimentation aspects in an immediate way, using the simulation to highlight important points.

In all the use of virtual worlds has been very effective, both in terms of time and in terms of engaging student interest. “It was efficient (non time consuming) and allowed for an understanding (better) through experiencing it (working it out)”, as one student wrote. The virtual plants emphasise the “nonmathematical statistics” proposed by Higgins (1999) and along with other visual models, like the pendulum, are thus suitable for a range of mathematical abilities, particularly in secondary schools and in service courses. Fearnley-Sander (2001) has suggested a similar approach for teaching and learning algebra, and it is likely that virtual worlds have many other applications in mathematics education.

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