THE INFLUENCE OF THE "ENVIRONMENT" CREATED BY THE SOFTWARE SCILAB IN THE LEARNING OF LINEAR ALGEBRA

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ABSTRACT

This paper compares the results attained by a control group working with traditional methodology with those of an experimental group using an application software program called SCILAB. The focus is on linear algebra (matrices, determinants and linear equation systems) which forms part of "Mathematics II", one of the core subjects in the B. Sc. econ. course at the Faculty of Statistics and Economic Sciences, National University of Rosario, Argentina.

This survey comes under the "Teaching Mathematics with Computational Tools" project — P.I.D. 4-202-93-004. It is financed by Program 202 for the promotion of Scientific and Technological Investigation set up by the National University of Rosario, Argentina.

The conclusion is that the experimental group proved more successful.

Introduction

Efficient learning involves both time and effort at university level. In mathematics, students spend much time on routine calculus and fruitless operations before entering university. They will now have to concentrate more on shaping concepts and solving problems.

Administration and Economic sciences in particular, are based on advanced mathematical theories. Economic analysis, for example, includes comparative statistics, optimisation problems and dynamic control; all of which require the methodology of linear algebra, calculus and combinatorial mathematics. In engineering, mathematics does not only infuse essential academic discipline; it is also needed as a tool for overall use.

Because of the way mathematics courses are currently being developed, students have to struggle against abstractions and do not get to see any of the applications. The very nature or high level of some topics simply cannot be exemplified with graphic and numerical calculus. Inversely, the computer allows the immediate numerical verification of property as well as graphical representation in two and three dimensions. Theoretical results can therefore be used to solve concrete problems in real situations and the immediacy of the processor's answer indeed helps with the inductive exploration of knowledge.

This approach should lead to a more consistent view of the links between mathematics and its applications. But first, the students must be taught mathematic concepts. For the operative part, the computer can be used to meet that objective, as long as THE USER KNOWS WHAT HE WANTS AND CAN UNDERSTAND the result supplied to him by the computer.

Computational tools are currently restricted to fairly specific areas in many professions, e.g.; computation and numerical analysis, data processing, programming etc. Computational tools often contribute to working out problems of numerical calculus, but wider use is being hampered by the complexity of programming languages. In higher education, mathematics is generally taught without the help of any computational tool, although teaching mathematics could be made definitely more effective if suitable computational methodology and software were introduced. It is the argument that underpins the *Teaching Mathematics With Computational Tools* project, which was set up by the National University of Rosario. The experiment described in this paper is an integral part of the project.

This paper explores the achievements of two different groups. The first group, named control group, used traditional methodology, when the second experimented with a computational tool, namely an application software called SCILAB.

It can be inferred from the results obtained by the two groups that the new methodology has a positive impact on teaching linear algebra as it improves symbolic representation and manual algebraic operations skills as well as conceptualization and the mathematical representation of reality.

It has been carried out an investigation design with quasi-experimental methodology.

Objectives

The objectives of the experiment were as follows:

- assessing how far a computer tool can help with learning linear algebra at university level.

- determining the impact of computational tools on university students of linear algebra's competence in solving problems. Further effects of the new computational environment on learning linear algebra were also considered.

These objectives were completed by comparing the levels attained by both groups, control and experimental alike.

Description of *scilab*, the software system used

SCILAB is a software system developed by France's Institut National de Recherche en Informatique et en Automatique, INRIA. It has been conceived to provide experts in applied mathematics with a powerful calculus tool. It uses the syntax of the MATLAB system. This system is kept as the interpreter and offers the greatest possible similitude to ordinary mathematical writing. It allows the manipulation of mathematical objects such as vectors, matrices and polynomials. It is also an open system because it allows the user to create new functions in a simple way.

Within the framework of the above-mentioned project, a compatible version for MS-DOS IBM PC was set up. It includes on-line help and the Spanish translation for the error messages of the English original version.

SCILAB may be obtained 'anonymous' at:

ftp.inria.fr (internet # 192.93.2.54) Directory:INRIA/Projects/Meta2/Scilab

ftp.unr.edu.ar (internet # 200.3.120.67) Directory:pub/soft/scilab

Development of the experiment

The experiment was carried out in *Mathematics II*, one of the core subjects in the B. Sc. econ. course at the Faculty of Statistics and Economic Sciences, National University of Rosario, Argentina. The focus was on matrices, determinants and linear equation systems. Five hours were allotted weekly to the experiment, three hours for theory and two for practice. The working hypothesis was that, within the teaching time normally allotted to the traditional course, it should also be possible to teach how to use a computational tool together with the standard contents of the course, and yet obtain better results and rationalised knowledge.

The experiment was started in the following conditions:

- The class (two hundred students) was divided into two random groups comprising an equal number of students (statistically equivalent). -. Each group was divided into sub groups that had around thirty five students in the practical part.
- The same amount of time, i.e. two hours per week, was allocated to work practice on the course subject in both groups.
- Both groups were taught an identical level of theory in the same time slot.
- Both groups were taught by highly qualified teachers of linear algebra.
- The control group worked in a traditional classroom with the traditional practical work guide.
- The experimental group worked in the computer room of the School of Statistics; two students being set per computer. The task in the computer were performed in the classroom. No time out class was needed. The course book used was parts of *Laboratorio de Análisis Matricial — Sistema Scilab (BASILE). Módulo 1 (4)*.

Note: To determine the equivalence of the groups, it was used like pre-test the students scores in the subject Mathematics I. With these data, the chi-squared test was done with a 95% confidence interval. To evaluate both groups, a single test was given immediately after the end of the course.

Test given to the students

Evaluation in the topics: Matrices, determinants and systems of linear equations with non computational operative.

1) Solution and analysis of a system

Solve by Gaussian elimination and determine the values of k for which the system of lineal equations is:

- compatible with only solution
- compatible indeterminate
- incompatible

$$x + 2y + 3z = 1$$
$$3x - 2y + z = 2$$
$$2x - 4y - 2z = k$$

2) Symbols, matrix operative and geometric interpretation

2.1) In the following system

$$2x + 4y + z \quad = 11$$
$$6y + 12z = 24$$
$$3x + \ y \ \ - 2z = 4$$

- Solve the system by inverse matrix if it is possible
- Interpret the solution geometrically
- Verify

2.2) Write explicitly the matrix defined by:

$$A = \{a_{ij}\}_3, \quad a_{ij} = (-1)^{i+j}$$

3) Representation of reality

A builder has to make the construction of five houses rural style, seven houses Cape Cod style and twelve houses colonial style. The builder knows, of course, the materials that each house type demands. These materials are steel, wood, glass, painting and manpower. The numbers of the following matrix represent the quantities of each house type, expressed in appropriate units.

	Steel	*Wood*	*Glass*	*Painting*	*Manpower*
Rural	5	20	16	7	17
Cape Cod	7	18	12	9	21
Colonial	6	25	8	5	13

The builder has two providers that give the following prices for material unit.

	provider 1	provider 2
Steel	15	10
Wood	8	7
Glass	5	4
Painting	1	1
Manpower	10	6

The following problems are presented:

- How many units will he need of each material?
- How much does it cost each house type, according to each provider?
- Which is the total material cost for all the houses that it will build according to each provider?

Note: Respond to the three questions using matrix products.

4) Determinants.

4.1) If

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3$$

Calculate

$$\begin{vmatrix} a_1 & a_2 & a_3 + 2a_1 - 3a_2 \\ b_1 & b_2 & b_3 + 2b_1 - 3b_2 \\ c_1 & c_2 & c_3 + 2c_1 - 3c_2 \end{vmatrix}$$

4.2) Complete

i) If $A = \{a_{ij}\}$ is a triangular matrix then $D(A) =$

ii) If $A = \{a_{ij}\}$ is a diagonal matrix then $D(A) =$

iii) $D(I_n) =$

Results

1) Although they were in no way bribed with any extra incentives, e.g., partial exemption from exams or even easier exam passes and special grade awards; most students reacted both actively and co-operatively to the new methodology. They were not unduly concerned about the 200% increase in the workload induced by their need of becoming proficient in linear algebra and the SCILAB system.

2) The assessment of both groups consisted in a comparative analysis of their skills in the following variables:
- algebraic routine operations
- conceptualization
- matrix use of symbols and routine operations
- modelization, i.e., mathematical representation of reality.

When selecting these assessment criteria, both the students' required background knowledge and the specific objectives of the work set were taken into account.

The following items were included in assessing the results:
- solution and analysis of a linear equations system.
- basic operations by row of a matrix
- determination of the rank of a matrix.
- Gaussian elimination.
- interpretation of systems with three variables.

So that a figure could be assigned to the variables, both groups were simultaneously submitted to a test in which students had to operate without the calculus help of a computer.

The results are expressed as a percentage of the number of correct answers given by the students:

	CONTROL GROUP	EXPERIMENTAL GROUP
ALGEBRAIC ROUTINE OPERATIONS	73.9	73.3
CONCEPTUALIZATION	62.5	66.7
MATRIX USE OF SYMBOLS AND ROUTINE OPERATIONS	56.5	70.8
MODELIZATION	26.7	55.6
OVERALL ASSESSMENT	63.5	70.3

When assessing each item, the skills each group's students acquired for a given item was averaged out separately. The figure is expressed as a percentage of the number of correct answers given by each student.

For "algebraic routine operations", both groups scored almost equally well, even though it could have been assumed that the computer-assisted student would lose the skill of solving problems manually.

For "conceptualisation", the experimental group achieved a slightly higher score.

For "matrix use of symbols and routine operations" and "modelisation", the experimental group obtained markedly better results. As far as "matrix use of symbols and routine operations" is concerned, the fact that the computer cannot accept the writing of mistakes may explain the significant difference between both sets of results. In addition, as the software makes use of symbols with a strong resemblance to those used in mathematics, the student benefits from additional training in that variable.

ACKNOWLEDGMENT: The present work was financed by the project "Enseñanza de matemática con herramienta computacional" Secyt UNR and the "Consejo de Investigación de la Universidad Nacional de Rosario". The authors address grateful thanks to these institutions.

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