MATHEMATICAL TECHNOLOGY TRANSFER - INDUSTRIAL APPLICATIONS AND EDUCATIONAL PROGRAMMES IN MATHEMATICS

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ABSTRACT

Mathematical technology is a term referring to the interdisciplinary area combining applied mathematics, engineering and computer science. Computational technology has made sophisticated mathematical methods viable for practical applications. There is a window of opportunity for mutually beneficial two-way knowledge transfer between academia and industry. This also means a challenge for the university education. The modern and dynamic view of mathematics should be reflected in educational practices. New kinds of expertise are called for.

The area of applications for mathematical technology is wide and diverse. Models are used to

-replace or enhance experiments or laboratory trials.
-create virtual and/or visualized images of objects and systems
-forecast system behaviour and analyse what-if situations
-optimise certain values of design parameters
-analyse risk factors and failure mechanisms
-create imaginary materials and artificial conditions
-gain understanding of intricate mechanisms and phenomena
-perform intelligent analyses on measurement data
-manage and control large information systems, networks, data-bases.

The education should bring the flavour of this fascinating art to the classroom. We should shape the image of an emerging profession, industrial mathematician, computational engineer or symbonumeric analyst? The education should convey the vision about mathematics at work, to display the diversity of application areas, to demonstrate the practical benefits. A number of groups worldwide are working towards fresh solutions in applied mathematics education. The goal is to combine mathematical knowledge with modelling skills, project work and a touch with real world applications. Possible tools for improvement include

-revision of curricula
-educational software environments
-problem seminars and project work
-teachers (re)training

We point to the challenge of mathematics education to find a way to communicate to the students the end-user perspective of mathematical knowledge. In this paper we describe the growing sectors of real life applications, industrial processes and R&D-questions where mathematical methods have a significant role. These examples are meant to emphasize the nature of mathematics as a versatile environment of problem solving. We discuss the educational challenge, curriculum development, the contents and viewpoints that could be used in undergraduate education and teachers training. The relevant didactic point is the search for and presentation of illustrative and interesting case examples on a right level of abstraction and technical sophistication.

1. Computational Technology

The development of technology has modified in many ways the expectations facing the mathematics education and practices of applied research. Today's industry is typically high tech production. Sophisticated methods are involved at all levels. Computationally intensive methods are also used in ordinary production chains, from timber industry and brick factories to bakeries and laundry machines. The increased supply of computing power has made it possible to implement and apply computational methods. Mathematics is emerging as a vital component of R&D and an essential development factor. The increasing demand and sphere of applications and the evolving computational possibilities have created what may be called *mathematical technology* or *industrial mathematics*. Terms like *computational modelling* or *mathematical simulation* are also used to describe this active contact zone between technology, computing and mathematics.

Mathematics as a resource for development Modeling means an imitation of a real system or process. The model is assumed to represent the structure and the laws governing the time evolution of the system or phenomenon that it was set out to mimic. Once we are able to produce a satisfactory model, we have a powerful tool to study the behavior and hence to understand the nature of the system. The models can be used to

-gain understanding of intricate mechanisms by testing assumptions about the systems nature.

-carry out structural analysis tasks

-replace or enhance experiments or laboratory trials.

-evaluate the systems performance capabilities

-forecast system behaviour and analyse what-if situations, to evaluate the effects of modifications, consequences of changes to systems parameters.

-perform sensitivity analyses and study the system behaviour at exceptional circumstances.

-analyse risk factors and failure mechanisms

-create virtual and/or visualized images of objects and systems in design processes

-create imaginary materials and artificial conditions prior to the possible synthesis or construction.

-optimise certain values of design parameters or the whole shape of a system component.

-perform intelligent analyses on the measurement data which is produced by the process monitoring, experimenting etc.

-manage and control large information systems, networks, data-bases

The model can describe situations that are impossible to be realized as physical models or are too extreme for making observations (one can't repeat the Big Bang or observe at close distance the explosion of a mine, but one can numerically simulate both).

2. Increasing Sphere of Applications

Economics and management. The daily functioning of our modern society is based on numerous large-scale systems. Examples are transportation, communication, energy distribution and community service systems. The planning, monitoring and management of these systems offers a lot of opportunities for mathematical approach. System models, methods of operations analysis, simulation etc. can be used to gain understanding on the behaviour of these mechanisms.

Corporate management uses methods in which mathematical knowledge is embedded in different levels. Econometric models are used especially at the banking sector to describe the macro level changes and mechanisms in the national economy. Risk analysis, game theory, decision analysis etc are used to back up strategic decisions, to design a balanced financial strategy, to optimise a stock portfolio. The mathematics of the financial derivatives (options, securities) has been a sector of rapid mathematical development in recent years.

Traffic and transportation. Roads, railway networks and air traffic contain many challenges for modelling. In railway industry one is interested on the mechanical models about the rail-wheel contact (Fig 8), explaining the phenomena of wear, slippage, braking functions etc. The train itself is a dynamical system with a lot of vibrations and other phenomena. Analysis of traffic flow, scheduling, congestion effects, planning of timetables, derivation of operational characteristics etc. (Fig 6) need sophisticated models. In air traffic guidance systems and the flight control of an aircraft represent sophisticated mathematical control theory.

Maritime industry. The maritime and offshore industries use advanced mathematical methods in the design of ships and mechanical analysis of offshore structures. An example is the dynamical behaviour of floating structures under wave force effects and wind conditions. Individual technical tasks like the optimal design of an anchor cable or the laying of communication cables at sea-bottom lead to interesting mathematical problems. One particular challenge is the modelling of the sea and the wave conditions itself for the sake of simulation purposes.

Space technology. Modelling of the mechanical properties of the manmade structures in the spatial orbit lead to advanced mathematical questions. An example could be the stability study of a large extremely light antenna structures in the weak gravity field. Each individual space mission represents a massive task for dynamical modelling and optimal control.

Product design and geometry. The modern toolbox of analytic and numerical method has made mathematics a real power tool for design engineers, production engineers, architects etc. One can bypass costly trial and error prototyping phases by resorting to symbolic analysis and numerical models. Mathematics is a natural tool to handle geometrical shapes (Fig 9), like the surfaces of car bodies and in the visualization techniques in CAD and virtual prototyping. In fact entertainment industry is one of the great clients for mathematical software nowadays. Visualization and animation is the basis of computer games and the vivid special effects in movies etc. These tricks are made possible by mathematical models.

Performance analysis, manufacturing systems, reliability. The major source of economic added value in using mathematical methods comes from the possibility of simulate devices, mechanisms, systems including complex large scale systems prior to their physical existence. A whole new system - like an elevator system in a high rise building, a microelectronic circuit containing millions of elements, or a high tech manufacturing system – can be designed and tested for its performance and reliability.

Chemical reactions and processes. Chemical processes are being modelled on various scales. In the study of molecular level phenomena mathematical models are used to describe the spatial structures and dynamical properties of individual molecules, to understand the chemical bonding mechanisms etc. The chemical reactions are modelled using probabilistic and combinatorial methods, the reaction kinetics take the form of differential equations etc. An example is the biochemical response in the design of a laboratory test (Fig 1). Chemical factories use large models to monitor the full-scale production process (Fig 3). The increasingly important area of environmental monitoring benefits from models that describe and explain biochemical processes.

Materials behaviour. Materials science is one of the really active fields where the mathematics based methods have proved their necessity and power. The aim is to understand the microlevel molecular and subatomic effects, subtle engineering of special compounds etc. The behaviour of non-typical materials (Fig 7) or new materials like semiconductors, polymer crystals, composite materials, piezoelectric materials, optically active compounds, optical fibres (Fig 5) etc. create a multitude of research questions, some of which can be approached with mathematical models. The models can further be used to design and control the manufacturing processes.

Metal industry The whole production chain of metals starting from mining industry, enrichment processes, furnace, casting, hot rolling, sheet forming, profiling etc. contains a lot of challenge to mathematical models. Quite modern and sophisticated methods are employed, like optimal control theory, free boundary problems, optimisation methods and advanced probabilistic methods. There are delicate questions like modelling of the material deformation during manufacturing processes, the phase change phenomena in the heat treatment of steel (Fig 11) and the study on the fatigue mechanisms (Fig 10).

Food and brewing industry Mathematics has to do with butter packages, lollipop ice-cream, beer cans and freezing of meat balls (Fig 2). The food and brewing industry means biochemical processes, mechanical handling of special sorts of fluids and raw materials. These less typical constituents lead into non-trivial mathematical questions. The control of microbial processes is quite crucial and adds to the complexity. Some of the questions deal with simple aesthetics, like the problem of proper filling of lollipop moulds in an automatic production chain.

Flow phenomena The ability to model sophisticated phenomena, including non-linear effects, the possibility to solve the equations with advanced numerical methods, combined with the latest visualization tools have created a luxury environment for mathematical engineering. The computational simulation can be used to support the design of systems from tooth paste tubes, regional heating networks and aircraft fuselage design to ink-bubble printers and making the fascinating flow phenomena visually observable. One of the important fields of application is diffusion phenomena, like the spreading of pollutants in air, soil, rivers etc.

Semiconductor industry The tiny devices are so small that it takes a microscope to see the details. The modelling of the single transistor has generated a lot of research. The industry wants accurate device models describing the performance characteristics of a chip prior to its production. To find the optimal architecture for an integrated circuit demands heavy calculations. The procedure of etching or electron beam lithography that is used during the manufacturing of the integrated circuit leads to interesting problems for mathematical modelling.

Systems design and control The design engineers and systems engineers have always been active users of mathematics in their profession. The possibility to set up realistic large-scale system models (Fig 3) and the development of modern control theory have made the computational platform a powerful tool with new dimensions.

Measurement technology, signals and image analysis The computer and the advanced technologies for measurement, monitoring devices, camera, microphones etc. produce a flood of digital information. The processing, transfer and analysis of multivariate digital process data (Fig 4) has created a need for a considerable amount of mathematical theory and new techniques. The area of signal processing is one of the hot areas for applied mathematics. Examples of advanced measurement technologies are mathematical imaging applications (Fig 12).

Experiments and data analysis The ample output of process data means a demand of mathematics. Intelligent methods are needed for the utilization of experimental data. The process control and monitoring systems, the sampling procedures etc have to be designed carefully. The quality inspection at different parts of the production chain and the testing procedures for the finished products all involve the questions for intelligent techniques for the handling of data. An example is the area of accelerated testing of mechanical components (Fig 10). There has been a speedy development of methods for data-analysis and the novel techniques for processing data.

3. Types of Models and Mathematical Projects

Mathematical models represent many different forms and types. *Continuous* models deal with quantities (like time, distance, force, electric potential) that vary smoothly over space and time. The models typically take the form of a set of algebraic or differential equations, integral equations, PDEs. Discrete models deal with quantities that vary in a stepwise manner, they take values from a discrete set. Examples of discrete models are recurrence relations, difference equations, Markov chain, digital coding and signals, autoregressive – moving average time series models (ARMA), graphs, integer LP-models.

A model which is based on the understanding of the internal mechanisms (physics, chemistry, biology, economics etc) is called a *mechanistic* models. When a mechanical model and analytic solution is not available we may resort to *simulation* model. *Empirical* models and *model fitting* are terms that describe the efforts to deduce the model equations from measurement data.

Deterministic models describe the phenomenon by predicting the actual values of the dependent variables. Known input values lead to unique output values. *Stochastic* models incorporate different random effects into the model structure and they are aimed to describe random behaviour and predict the probability distribution of the output values.

Models may represent different scales, conceptual levels and the model may contain inner structures, *partial models* or *submodels*. *Macroscopic* model tries to catch the big picture, *microscopic model* zooms at more minutiae details. Take for example the dynamics of weather phenomena where different version of models are needed to describe the formation of rain drops or local air pressure variations, to explain the creation of tornadoes or to understand the greenhouse effect.

Models can also be categorized due to the purpose for which they were created. Models may be designed to understand the mechanism of change, a *transition* like growth, decay, saturation or switching from one state to another. Other models are aimed at describing permanence, *equilibrium* and balanced condition. An example of this sort would be a set of equations describing the operating status of a chemical reactor.

Often the models are used for the purposes for control. *Optimisation models* are geared to a specific purpose, to help to find the best operating conditions, to find an optimal design for a product, etc. *Control models* are the devices for control engineering, process control and different mechanisms of guidance. Examples of this sort are the models for steering the operation of power stations and the guidance systems for air traffic.

4. Educational challenge

As the previous material shows, the computer age has generated a need and a window of opportunity for a new kind of expertise. This field could be called industrial mathematics, mathematical technology, computational engineering. This presents a challenge to the educational programmes and curriculum development. Some universities already offer specialized MS-programs oriented towards the professional use of mathematics. There are excellent programs that equip the students with the skills that are needed in the mathematical projects in the R&D-sections in industry. In general there is still a lot of room for improvement. Some mathematics departments have stayed too long in the pasture of isolated abstract mathematics and failed to face the challenge coming from the changing world.

A good educational package would contain a selection of mathematics, computing skills and basic knowledge of physics, engineering or other professional sector. The job title in industry is very seldom that of a mathematician. It can be a researcher, a research engineer, systems specialist, development manager. Industrial mathematics is teamwork. Success stories are born when a group of specialists can join their expertise and visions together in a synergic manner. The team-work makes communications skills a necessary matter. It would be very important to train oneself to work in a project team, where the interpersonal communication is continuously present. To become a good applied mathematician one should be curious about other areas as well, to be interested and learn basic facts from a few neighbouring areas outside mathematics.

To tackle the fascinating tasks and challenges, development questions in modern industry, the student need a solid and sufficiently broad theoretical education and operational skills in the methods of applied mathematics. However, the most important single skill is the experience in modelling projects. The lectures, books and laboratory exercises are necessary, but the actual maturing into an expert can only be achieved by “treating real patients”.

From the point of view of successful transfer of mathematical knowledge to client disciplines a crucial and current educational challenge is the theme of mathematical modelling. Many departments have introduced modelling courses in the curriculum in recent years. The active development is reflected in a boom of literature. A variety of books of different flavour are available on the subject. A course in modelling may contain study of case examples, reading texts and solving exercises from literature. The actual challenge and fascination is the students’ exposure to open problems, addressing questions arising from real context. The real world questions may be found from the student’s own fields of activity, hobbies, summer jobs, from the profession of their parents etc. Reading newspapers and professional magazines with a mathematically curious eye may produce an idea for a modelling exercise. A good modelling course should

(a) contain an interesting collection of case examples which is able to stir students’ curiosity
(b) give an indication of the diversity of model types and purposes
(b) show the development from simple models to more sophisticated ones.
(c) stress the interdisciplinary nature, teamwork aspect, communication skills
(d) tell about the open nature of the problems and non-existence of “right” solutions
(e) bring home the understanding of practical benefits, the usage of the model
(f) tie together mathematical ideas from different earlier courses

The modelling courses have been run in different forms. Traditional lecture course with weekly exercise session is a possibility. It would be important to implement group work mode and PC-lab activities in the course. The most rewarding form of activity might be projects and weekly session where the student report and discuss about their work and progress on the problems. A very successful form and educational innovation is a modelling week, and intensive problem-solving workshop that has been implemented in Europe and US since late 80ties.

The supply of good classroom examples and case studies from different application areas is a key factor for the development of attractive and inspiring educational modules in applied mathematics. Especially in the courses on mathematical modelling we would need a flow of fresh problems to maintain an intellectual urge. It would be important to have ongoing contacts to different special sectors, professions, diverse pockets of innovative processes.

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Illustrative examples

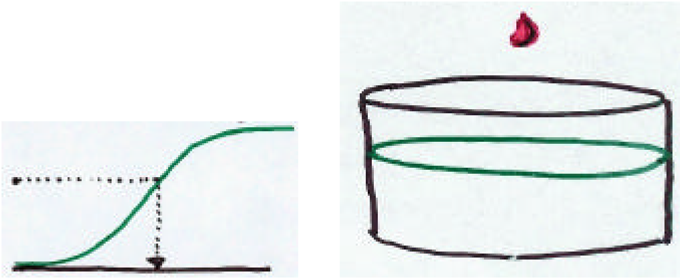

Fig 1. Design of a blood test for clinical use

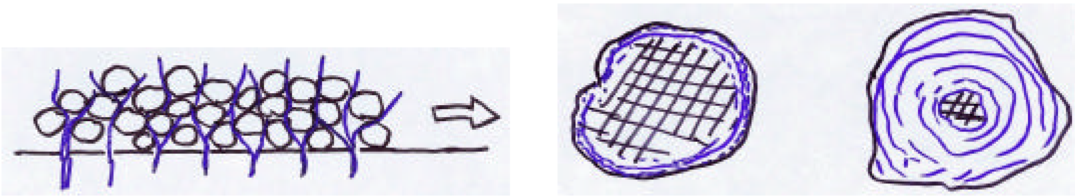

Fig 2. Quality control for the freezing of meat balls

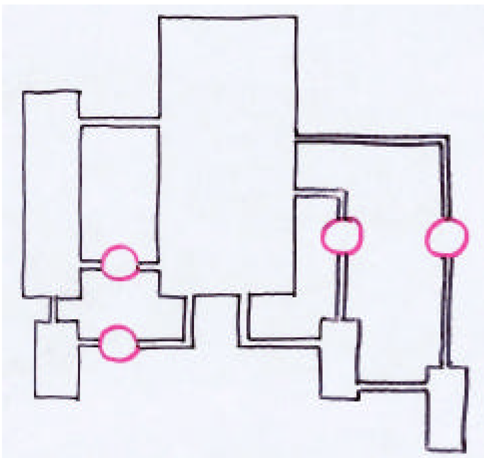

Fig 3. Chemical process modelling

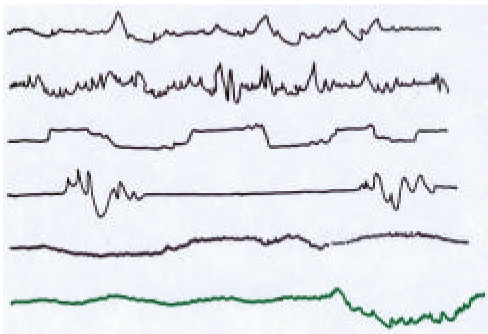

Fig 4. Process monitoring and diagnostics

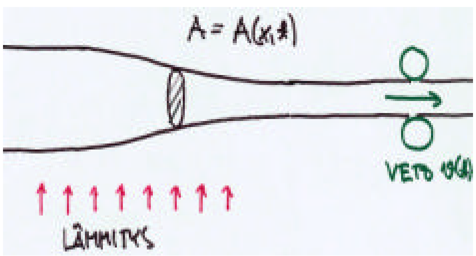

Fig 5. Tapering process for optic fiber

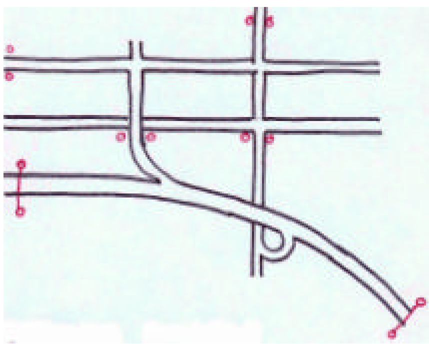

Fig 6. Dynamic traffic guidance

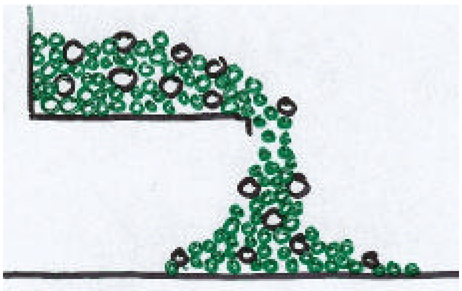

Fig 7. Models for granular materials

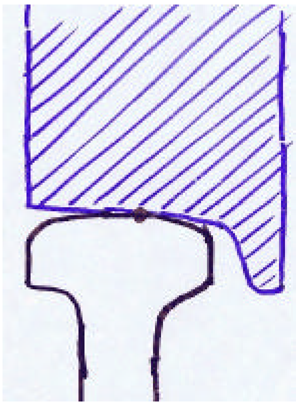

Fig 8. Modelling the rail-wheel contact

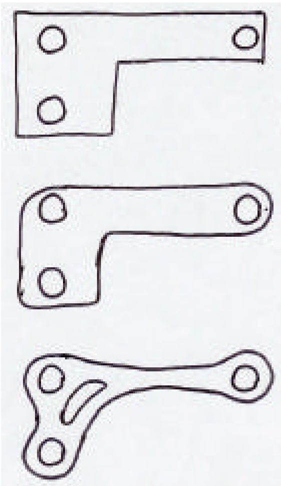

Fig 9. Optimal shape design

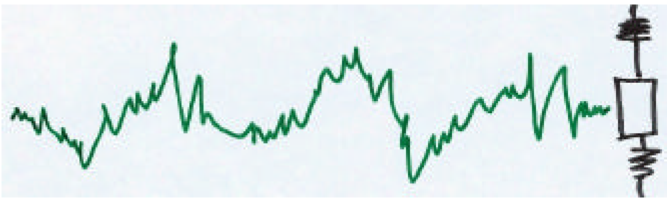

Fig 10. Analysis and design of accelerated fatigue testing

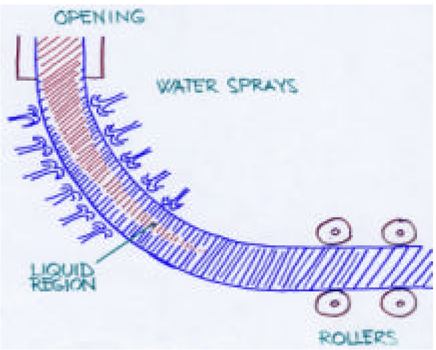

Fig 11. Continuous casting of steel

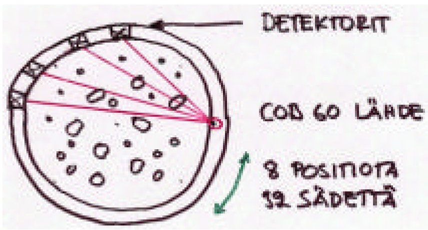

Fig 12. On-line process tomography