THE BLK SCHOOL PROJECT IN BADEN-WUERTTEMBERG

Hans-Wolfgang HENN
University of Dortmund
Fachbereich Mathematik, IEEM
Vogelpothsweg 87
D 44221 Dortmund, Germany
e-mail: Wolfgang.Henn@math.uni-dortmund.de

ABSTRACT

The project to be described is entitled "Promoting classroom culture in mathematics" and is the contribution of the state of Baden-Wuerttemberg to a programme of the Bund-Länder-Kommission (BLK), a commission of the German Federal Government and the governments of the Federal states. The title of the programme is "Furthering the efficiency of mathematics and science teaching". This four-year programme for the lower secondary level is intended as a meaningful response to the less than satisfactory German TIMSS results. The approach of this project focuses on changing the teaching style, the major objective being to develop a holistic concept for mathematics teaching, integrating comprehension, active participation and long-term productive learning. A report will be given on initial experience obtained with the project in Baden-Wuerttemberg.

1. TIMSS and the BLK Project

The not very flattering results of Germany in TIMSS, the Third International Mathematics and Science Study, turned out to have the effect of a catalyst inducing a nation-wide debate about educational goals and the content of mathematics teaching. The most perceptible signal of response is the programme "Furthering the efficiency of mathematics and science teaching", initiated by the Bund-Länder-Kommission (BLK), the Federation-Laender Commission for Educational Planning and Research Promotion. This four-year programme for the lower secondary level was started with the school year of 1998/99. In preparation of the BLK programme (BLK 1997), a report was made pointing out the basic assumptions and principles of future educational policy as well as the various problems encountered in mathematics and science teaching at school. Based on this report, the Germany-wide school experiment with accompanying measures was put into place. A major approach adopted for this experiment was that schools and teachers were not to be confronted with ready-made teaching concepts. Instead, 11 so-called modules were developed which offer promising starting points for the teachers who are free to develop their own approaches for promoting mathematics and science teaching.

The Federal states of Germany participate in this BLK-programme with altogether 30 experiments. Six schools each forming a so-called school set participate in an experiment. The Federal states have chosen the modules they wished to apply in the participating schools, and have organised their respective experiments in their own manner.

2. The school project in Baden-Wuerttemberg

The project "Promoting Classroom Culture in Mathematics" is the contribution of Baden-Wuerttemberg to the BLK programme (Blum&Neubrand 1998, Henn 1999). There are three different school sets: one representing six "Hauptschulen" (lower level education), one representing six "Realschulen" (intermediate level education), and one representing six "Gymnasien" (higher level education). The schools are working together closely and there is a lively exchange of experience between the school sets. For our model schools, we have focused on the following four out of the 11 modules:

Module 1: Developing a problem culture in mathematics and science teaching. Emphasis is on open problems, appealing to all students. Individual problem solving abilities are challenged. Problems are given in varied contexts, to permit development of various, qualitatively different solutions and to provide systematic and productive exercises.

Module 3: Learning from mistakes. Psychological and pedagogical theories describing the conditions which foster learning from mistakes form the basis and are applied in practical teaching in the class room. "Mistake-friendly" lessons can improve the students' mental activities.

Module 5: Experiencing the growth of competence: cumulative learning. Possibilities to make a vertical net and prepare the ground for cumulative learning are explored. Long-term orientation and guidance in building learning history should help students to accumulate a bigger "learning possession".

Module 10: Assessment: Comprehension and feedback on growing competence. The development of challenging examination problems and examination types suitable to measure comprehension and the versatile application of knowledge are highly important to the task of improving teaching.

The modules are applied in all year groups of the lower secondary level. The approach underly-

ing our project aims not so much at changing the mathematical content, but rather focuses on a change of teaching style. The aim is to develop a holistic design of teaching which leads to understanding, active involvement and long-term fruitful learning. Central deficiencies of the current teaching style are short-term learning for the next test, restrictions resulting from a rigidly guided "questioning-developing" teaching style, and a strong bias towards calculation in mathematics teaching, whereby calculations are used without insight and understanding.

The central question is not "what is to be learned", but "how should the learning process proceed", "how can mathematical literacy be promoted", and also "how can learning processes be measured". The willingness to question and to rethink current teaching, to change one's own reception and to realize opportunities brought about by new practice and teaching methods are important. We do not intend to reject everything from the past for being bad, or to follow new fashionable slogans such as "tasks as open as possible" or "application at all cost". Our objective rather is to bring about a reasonable shift of emphasis, balancing the importance of instruction (by the teacher) and construction (by the students themselves), teaching and discovery, convergent routine problems and divergent open problems, different modes of testing and achievement measurement.

Of course this reorientation has to develop in a natural way in the course of the years spent at school from elementary to upper secondary level. Our approach to the BLK-project was highly influenced by the concepts of the Dortmund project *mathe 2000* of the group around E.Ch. Wittmann and G.N. Mueller (Mueller et al. 1997). Inherent in the concepts is the concentration on fundamental ideas of mathematics and a long-term development following the spiral principle, across the years spent at school. An aspect of major importance is student-centred mathematics teaching, which means taking seriously the answers, ideas and products of the students. This means a conscious change in teachers' as well as students' attitudes, especially towards attributing more importance to learning processes than to results. The special merit of a "good teacher" is not his "good explanations", but the ability to promote thinking processes and active discovery learning in a productive learning environment.

Two main aspects which are not self-evident, at least in teaching at upper secondary level, have emerged: to take children and their products seriously, and to construct productive learning environments.

3. Taking children seriously

In "How children compute", a book worth reading, the authors plead that teachers should think and argue with children, listen to them, take their products seriously, not ignore their mistakes, but rather discuss them productively (Selter&Spiegel 1997). We usually expect that children think in the same way as we do, the mathematicians (whereby we often have remarkable blinkers …). However, as was clearly pointed out by Selter and Spiegel, children use to calculate in a different way, which may differ from the way we do it, the way we assume, the way other children do it, and then again from the way they have used themselves before in dealing with the "same" problem.

Consequently it is not enough to question children, but rather to take their questions seriously, discuss and try to understand them. Thus teachers are able to recognize and understand their students' cognitive structures. *One* way is to gather all solutions without any comment on the blackbord in a first step and then to discuss them. Often learners then point out the mistakes they made themselves. The following examples illustrate this approach.

Example 1: Geometry (grade 6):
The problem is (relating to Fig. 1): *Complete the drawing in such a way that there are two adjacent*
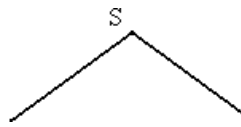
supplementary angles.

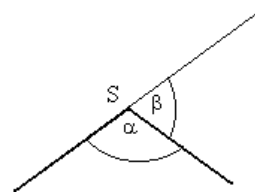**Fig. 1**   **Fig. 2**

In a normal lesson, students would be guided to a solution by making one line at the side of the angle longer (cf. Fig. 2). Here, the teacher's students were able to accomplish the task by themselves, without any hints and guidance. Single solutions were then presented at the blackboard. Questions were only allowed after the drawing was finished. Then the student had to explain his or her solution, mistakes had to be realized. Solutions turned out to be much more general than the narrow schoolbook solution. Often there where several pairs of adjacent supplementary angles. Fig. 3 shows some of the solutions:
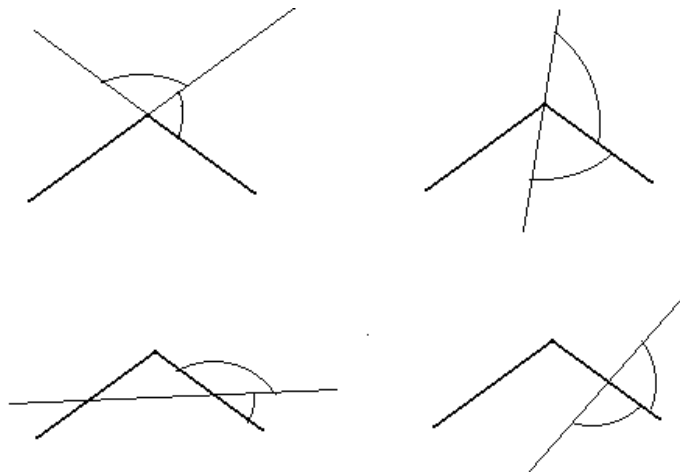

Fig. 3

Interestingly enough, the schoolbook solution of Fig. 2 was not mentioned (and not forced upon the students by the teacher). Taking the children seriously created a productive working atmosphere. "I could easily see the disappointment of students whose solution was already presented by others. A positive disappointment", reports the teacher.

Example 2: Expressions (grade 7):

In a grade 7 project class the three solides given in Fig. 4 were presented.
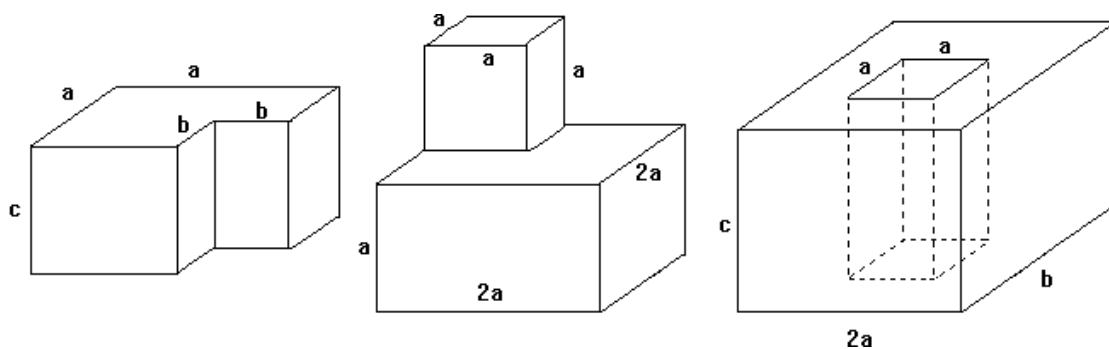
Fig. 4

The class agreed to tackle the task to determine surface areas and volumes of the solids individually or in groups. Students started to work, results were written (without comment) onto the blackboard. After some time there were up to eight different expressions for the sought quantities. Now a discussion started about who had found the correct solution, obviously the teacher did not intervene. Some expressions, for example including a term a^6, could be sorted out and found incorrect with respect to the unit. For the remaining terms it was not obvious whether they were correct or incorrect. The students tried intensively to compare the expressions, which gave an excellent motivation for finding strategies for the manipulation of expressions. To calculate with the children was of great value also for the teacher: "I was very content about how the lesson developed. I gained insight into the cognitive structure of students".

4. Productive practise

E.Ch. Wittmann describes the didactics of mathematics as a *design science* which delevops and researches "productive learning evironments". Problems with rich content are worked on holistically. This is presented exemplarily in both *Handbooks of Productive Arithmetic Practise* for the four elementary school years (Wittmann&Mueller 1990/1992). The single learning sections create meaningful relations and propose problems of different degrees of difficulties, leading to a natural differentiation. In contrast to the usual step-by-step teaching not all obstacles are removed. Students gain experience in using "common sense" and are challenged to think about problems on their own, to judge their own considerations and to test whether they make sense.

One example for a productive learning environment is the following sequence of problems on number walls, developed by a collegue for his grade 5 class:

Example 3: For the empty number wall given in Fig. 5 the following questions were asked:
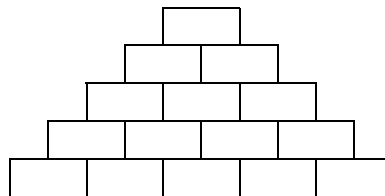

Fig. 5

Can you build number walls in each of the following cases?
- *In the first line write down five numbers you like.*
- *Write down only four numbers in line one.*
- *There are only odd numbers in line two.*
- *At the top is a number close to 500.*
- *At the top is exactly 500.*
- *Can you find several walls with 500 at the top?*
- *Is it possible that there are only numbers divisible by three in line three?*
- *Can you write down five numbers in any space and still complete the number wall?*

Number walls are a tried and tested exercise format which is introduced in elementary education. The sum of two adjacent spaces will appear in the space above. This exercise format has been successfully used also for other number spaces and operations, as well as for variables. More complex

questions can be posed easily. For example, write down the unit fractions $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...$ from top to bottom on the left side of the wall. Then all other places can be filled unambigiously. What do you observe?

Standard problems, too, can often be improved in a "productive way", by taking away a restriction, by opening up a too rigorous problem statement, or by posing the problem rather vaguely. Some examples from our project:

Example 4: In the standard problem

Arrange the following numbers according to their size

$$-1; \quad \frac{2}{3}; \quad 3.\overline{9}; \quad -\frac{143}{6}; \quad -184.76; \quad \frac{14}{7}; \quad 8.23; \quad -5\frac{2}{9}.$$

the words "according to their size" were omitted in a grade 7 project class. This resulted at first in a creative restlessness among the children who wondered what might be meant by "arrange". They were used to managing convergent, unambiguous questions. Only after the teacher pointed out that there may be more than one solution (without, of course, mentioning them), the children started to work and arranged the numbers according to positive/negative, number space, or size.

Example 5: *Choose four fractions. Use them to make expressions as large as possible (as near as possible to 1, ...).*

Example 6: *Calculate some powers that you like.*

It is typical of such problems that pupils work intensively and offer many ideas, but naturally make mistakes, too. But they realize and correct their mistakes. It is time well spent because children are involved quite differently. Emotional AND cognitive aspects are addressed.

Often only standard knowledge and skills are applied to standarized question types. If practical teaching continues in such a way, thinking and computing are separated. A problem setting should be looked at from different perspectives. Inverse problems often appear to be easier at first sight, but then lay bare missing basic concepts. Here some tried and tested examples from our project:

Example 7: *State two different fractions between* $\frac{6}{17}$ *and* $\frac{7}{17}$ *or explain why there are none.*

Example 8: *Find all solutions or explain why there are none:*
$$6 \times (20 - ?) = 144; \quad 7 \times (12 + ?) = 100.$$

Example 9: Better than the convergent question *"7 + 5 = ?"* which asks for the synthesis of twelve is the divergent question *"Which is the most beautiful twelve?"* asking for the analysis of twelve and resulting in many correct and important answers, i.e. $12 = 11 + 1$ or $12 = 1 + 2 + 3 + 3 + 2 + 1$.

These illustrative examples show that only by choosing another formulation in standard problems new aims can be addressed to further creative ideas, to differentiate the possiblities, to order, and to classify.

7. Assessment: Measuring and feedback of gain in competence

It is important to differentiate between learning and assessment situations. Understandably so, students try to avoid failure in assessment situations. Nevertheless, problems that are open and ask for own decisions have to be included in tests. In our experience students did not see these problems as something new because firstly they were used to such problems from their lessons and secondly these questions were included sensitively in test situations. In particular, one cannot force creativity

in relatively little time and under stress. However, some test problems from our project classes (grades 5) show what is possible.

Example 10: *Armin buys a rubber and two pencils from Beate. The rubber costs 2 €, and each pencil 1 €. Beate asks for 6 €, and Armin protests. Then Beate writes down an expression and explains her calculation to Armin. Then Armin understands. He tells Beate that the term is correct but that she has broken a rule in her calculation. Armin pays the correct price. What is the expression Beate wrote down and what rule has she used incorrectly?*

Example 11: *Form expressions with the numbers 24, 9, 8 and 5 and calculate them. For at least three of the expressions the results should be between 0 and 10. For at least three of them the results should be between 100 and 110.*

Naturally, with this type of questions, basic arithmetic knowledge is checked, but in addition, algebraic competence on expressions is necessary. The important computation rules are used independently rather than merely checked out of context.

Example 12: Supplement:

$$a) \ 7 \times (50 + [\ ]) = 350 + 28; \quad b) \ 7 \times 14 + [\ ] = [\ ] \times (14 + 6).$$

For a) the solution $7 \times (50+4) = 350+28$ is unique. For b) mostly, as expected, the solution $7 \times 14 + 7 \times 6 = 7 \times (14 + 6)$ was given triggered by the distribution law. In reality, the equation $7 \times 14 + a = b \times (14 + 6)$ has the positive integer solution $a = 2 \times (10b-49)$ for every positive integer $b \geq 5$. In a few cases some of those solutions where found by trying out different numbers, which, of course, is a creative, original achievement.

8. Experience resulting from the project

In posing more open problems one has to take starting difficulties into account. Students tend to ask for a recipe, and are unsure in the beginning. They often are afraid of failure and do not start at all. But their attitude changes after some time. "During the school year unsureness was lessening, and with growing confidence the students would develop more problem solutions on their own and accepted that there is more than one way to reach the solution", reported two of the colleagues involved. One could observe a growing familiarity with more complex, more open problem statements. "The development of a wide range of solutions was only possible when I myself as a teacher retreated in the decisive moment – left the problem completely to the class and did not break down the problem into bite-sized pieces by questioning-answering techniques until they were convinced that the problem could be solved only with a linear equation – is that not what often goes completely wrong in mathematics teaching?"

The increase in creative and heuristic abilities is difficult to measure. But in the feedback we received it was reported that problems were increasingly dealt with as a matter of routine, with perserverance instead of resignation. All involved were convinced that students gained metaknowledge rather than a collection of easily assessible but quickly forgotten information. The deliberate change in the teacher's rôle (to stay back in working and solution phases, to challenge and to accept solutions, to encourage alternatives) was not restricted to project classes only!

9. The WUM inservice teacher education

The experience gained up to now in the 18 project schools resulted in the development of a new regional inservice teacher education named "*W*eiterentwicklung der *U*nterrichtskultur im Fach *M*athematik" for all school types (abbreviated to WUM, that is to say, further development of the

teaching culture in mathematics). Teachers themselves request that an inservice teacher education team comes to their school. In courses lasting for one whole and three half days, generation of problem awareness is the first item on the agenda, followed by a number of short presentations introducing the new methods (productive practise, opening and variation of problems, open-ended approach, non-routine examination problems). The main task of the teachers then is independent preparation of teaching material for their own lessons. The tremendous demand shows the great interest of our teachers.

10. Conclusion

Obviously, our experience accumulated in the BLK project for about three years now is not yet reliable enough to serve as a source for deriving reliable final conclusions. Certainly however the teaching climate and the active participation of most learners have improved decidedly. We have reason to believe that through open, more challenging work and problem style improved basic concepts will be developed.

REFERENCES

-BLK, 1997, *Gutachten zur Vorbereitung des Programms "Steigerung der Effizienz des mathematisch-naturwissenschaftlichen Unterrichts*", Heft 60 der Materialien der Bund-Länder-Kommission für Bildungsplanung und Forschungsförderung, Bonn.

-Blum, W., Neubrand, M. (eds.), 1998, *TIMSS und der Mathematikunterricht*, Hannover: Schroedel.

-Henn, H.-W. (ed.), 1999, *Mathematikunterricht im Aufbruch*, Hannover: Schroedel.

-Mueller, G.N., Steinbring, H., Wittmann, E.Ch., 1997, *10 Jahre "mathe 2000"*, Leipzig: Klett.

-Selter, Ch., Spiegel, H., 1997, *Wie Kinder rechnen*, Leipzig: Klett.

-Wittmann, E.Ch., Müller, G.N., 1990/1992, *Handbuch produktiver Rechenübungen*, Stuttgart: Klett, Band 1 und 2.