WRITING IN A REFORMED DIFFERENTIAL EQUATIONS CLASS

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ABSTRACT

In an attempt to promote the development of understanding over rote memorization, writing in mathematics has received increased attention in recent years. In Calculus, the Rule of Three (based on communicating ideas through *algebraic*, *graphical* and *numerical* means) has been replaced by the Rule of Four in which *writing* plays a central role. Educators agree that the benefits of writing include the promotion of understanding, and the initiation of the posing of questions. Writing also helps generate meaning, and helps in the retentio n of content. In this paper, I evaluate the use of writing for analyzing a problem and its solution. The setting is a reformed differential equations class offered at the Lebanese American University. Unlike a traditional ode course where students are provided with a cookbook of methods for solving differential equations, the emphasis in a reformed ode course is placed on the geometry of the solutions and on an analysis of the outcomes. In many instances, students are asked to solve a differential equation by plotting its solution curves without identifying them analytically, and the sketch is to be supplemented by an argument justifying it. In addition, various real life problems are modeled and essay questions are asked to analyze the graphs describing these models. Results show that students first reject the idea, but later rate writing as essential. Furthermore, an improvement in the style and content of the writing exercises is usually noticeable at the end of each semester.

Keywords: Reformed differential equations curriculum; writing in mathematics.

1. Introduction

In recent years, the curriculum of ordinary differential equations has undergone fundamental changes in favor of the visual aspect of the field. Traditionally, differential equations were taught in a very mechanical way: Equations are usually classified, and for each class a method of solution is presented. Since differential equations are widely used in engineering and the physical sciences, this mechanical approach has defeated the purpose of the course as an aid to understanding real life problems (such as the harmonic oscillator, predator-prey models, competing species models, and others.) The traditional approach to teaching differential equations has its roots in the way Calculus has been taught throughout the past centuries. Even though the ideas of Calculus were inspired by problems in astronomy, and even though Calculus later showed to be very useful for answering questions in various sciences, this mathematical field has been taught traditionally as a set of rules and procedures with very little reference to its uses in the real world. More than a decade ago, educators and researchers begun questioning this approach for teaching Calculus, and many discovered that teachers and students alike are "losing sight of both the mathematics and of its practical value" (Hughes-Hallett, Gleason, 1998, p. v). Following the first program announcement for Calculus reform of the National Science Foundation in the United States, many math instructors begun re-designing their classes, and many of them emphasize now the algebraic, the visual, and the numerical aspects of the field (the Rule of Three). Clearly, the development of advanced graphing calculators and of dynamical computer programs was a contributing factor to the adoption of this approach. More recently, *writing* was added to the Rule of Three. According to Hallett, Gleason, et al., students need to learn "to reason with the intuitive ideas and explain the reasoning clearly in plain English" (p. vi). In general, researchers agree that the benefits of writing include the promotion of understanding, and the initiation of the posing of questions; writing also helps generate meaning, and helps in the retention of content (Rose, 1989, 1990).

Differential equations are a beautiful application of the ideas and techniques of calculus to solve various real life problems. Consequently, the new approach for teaching calculus lead to a similar approach for teaching differential equations. In the article "Teaching Differential Equations with a Dynamical Systems Viewpoint", P. Blanchard (1994, p. 385) suggests that teachers do not give any more equations for which explicit solutions exist, but rather use computers and graphing calculators to graph the approximate solutions of a differential equation and require students to interpret and justify what they see. In the book Differential Equations by Blanchard, Devaney and Hall (1998), the authors write (p. v), " the traditional emphasis on specialized tricks and techniques for solving differential equations is no longer appropriate given the technology that is readily available…. Many of the most important differential equations are nonlinear, and numerical and qualitative techniques are more effective than analytic techniques in this setting." Addressing the students, the authors add that many exercises of the book ask to analyze models and to explain verbally the conclusions. Thus, in the new ode curriculum, writing is as essential as the solution process itself.

Research on writing in mathematics is not very extensive yet. In the literature, some papers and books have emphasized the skills required to write a good mathematical proof. (e.g. MAA Notes 14 (1989)); others, such as J. Meier & T. Rishel (1998), M. Porter & O. Joanna (1995), A. Schurle (1991), have discussed the effects of writing on the learning itself. In particular, Schurle discusses whether writing helps students learn about differential equations. However, the curriculum adopted by the author is the traditional one. In this paper, I will assess primarily writing as a tool for analyzing and understanding results obtained mostly geometrically in a reformed ode course. Writing to understand concepts is in addition evaluated.

2. The New ODE Curriculum

An ordinary differential equation of order n is an equation of the form:

$$\frac{d^n y}{dt^n} = f(t, y, \frac{dy}{dt}, \frac{d^2 y}{dt^2}, ..., \frac{d^{n-1} y}{dt^{n-1}})$$

Finding a solution to this equation means finding a function $y(t)$ satisfying that equation. Analytically, this requires expressing $y(t)$ implicitly or explicitly in terms of t. In a traditional differential equations course, analytical methods of solution are described for very specific types of equations. In a reformed course however, more emphasis is placed on the geometry of the solutions. In many instances, solutions are drawn without a slight knowledge of their analytic representations, and students are expected to read information from these graphs. For instance, in studying the logistic population model $\frac{dP}{dt} = kP(1 - \frac{P}{N})$ (a first order differential equation), students are expected to read from the slope field the growth of the population given any initial condition (See Figure 1). In studying harmonic oscillators, second-order equations $\frac{d^2 y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m} y = 0$ are transformed into systems of the form $\begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} = -\frac{k}{m} y - \frac{b}{m} v \end{cases}$, and students are expected to read from its vector field the change in the position as well as in the velocity of the motion of a mass attached to a spring (see Figure 2). Clearly, this qualitative approach for solving differential equations gives a new dimension to the field of differential equations since in the traditional setting, rarely were students asked to interpret solutions that were obtained analytically.

3. The Setting

The course, Ordinary Differential Equations, as is offered at the Lebanese American University in Beirut, is a 3-credit course aimed at engineering students who have taken prior to it the calculus sequence. Before enrolling in a school of engineering at any university, students of Lebanon have to pass the official baccalaureate exam (mathematics section) offered at the end of their secondary school years. Teaching in Lebanon is still traditional. Only in few private schools are graphing calculators and computers in use. Yet, the teaching of 3rd and 4th semester calculus at the Lebanese American University incorporates the use of *Mathematica* in the form of projects combining the geometric and the analytic sides of mathematics. The class meets three times a week (50-minute sessions) in a regular classroom. The book adopted for the past three years has been Differential Equations by P. Blanchard, R. Devaney & G. Hall, a reformed text that emphasizes the geometric approach and analyses of outcomes. Furthermore, two computer software programs are used regularly: ODE Architect, a multimedia tool with enormous visual capabilities and generally used for classroom presentations; and

Interactive Differential Equations (IDE), a collection of labs designed to build a complete understanding of a particular concept. Computer homework are usually assigned from IDE and they generally require a great deal of visual observations that can only be communicated though writing.

4. Sample Writing Exercises and the Students' Reactions

As mentioned above, the book of Blanchard, Devaney and Hall emphasizes the geometrical approach to differential equations and requires analyses of outcomes. The authors for instance introduce the idea of a differential equation by modeling a population growth problem. According to them, how the differential equation is written is not of much importance; the importance lies in "what the equation tells us about the situation being modeled" (p5). Throughout the section, various models are solved geometrically and discussed primarily in a verbal manner. Exercises fall also in the same line of thought. For instance, in one exercise (p. 15), learning is modeled by the differential equation $\frac{dL}{dt} = 2(1 - L)$, where $0 \le L(t) \le 1$ is the fraction of a list learned at time t. One question asks students to analyze whether a person who starts up knowing none of the list can ever catch up with another who starts up knowing half of the list. Similarly, many problems that I give on exams always require some verbal discussions. Some questions for instance ask to analyze results obtained geometrically such as: Given a slope field of a first order differential equation, draw a representative collection of solutions and describe verbally the main similarities and differences of solutions to various initial value problems; or: Associate differential equations with slope fields and justify the answer with a short paragraph; or: Identify systems as being Predator-Prey or Competing Species systems, and write a small paragraph justifying the identification; in particular discuss what happens when one of the species is extinct.

Other exam questions require writing essays to examine the level of theoretical understanding. For instance, one might ask students to discuss the existence and uniqueness of solutions to initial value problems. Another question that I add frequently to my tests is about the general linear system:

$$\frac{dx}{dt} = ax + by$$
$$\frac{dy}{dt} = cx + dy.$$

Students are asked to discuss in an essay the condition(s) that a, b, c, and d have to satisfy in order to obtain for instance two distinct real eigenvalues. Then they are asked to discuss the different kinds of phase portraits that can occur in this case

Assignments from the workbook Interactive Differential Equations also encourage students to explore mathematical concepts through writing. In one favorite exercise, the love affair between Romeo and Juliet is modeled by a linear system of differential equations (see system above). The values of a, b, c, and d are changed to reflect new factors affecting the relationship. Questions posed require in most cases an analysis of feelings. Here is a sample: What are Juliet's feelings for Romeo when he is most attracted to her? What do you expect to happen to the relationship? Suppose the two lovers had exactly the same emotional profile in terms of their response to each other and their responses to their feelings ($\frac{dx}{dt} = ax + by, \frac{dy}{dt} = bx + ay$), investigate some situations and write a small paragraph.

Do students accept the idea of writing essays in mathematics? And how do they react to questions of this sort?

In the beginning, most students reject somehow the idea of writing in mathematics. As one student puts it: This is a Math, not an English class! Students have been trained in schools to solve any mathematical problem in a mechanical way; a discussion of the problem and a justification of its outcome have rarely been considered important. Consequently, the idea of writing in mathematics is alien to them. In fact, students of a reformed ode class have to adapt first to the idea of solving a mathematical problem geometrically rather than analytically. In Habre (2000), I investigated strategies for solving a differential equation as adopted by students of a reformed ode course offered in the United States. Results showed for instance that most students think primarily of analytic solution techniques; only few showed approval of the qualitative approach, while all the others had serious reservations about it. Since writing is a consequence of the geometric approach, it is not surprising therefore that most students initially reject this idea. For instance in the exercise modeling learning, only 36% of the students investigated discussed it verbally. It was unfortunate that by the time it was due, the analytic technique for solving a separable differential equation had been discussed in class. Consequently, 43% of the students tried to solve the problem analytically. The remaining students combined both approaches perhaps in an attempt to justify analytically what they had discussed verbally. By the time the first exam is usually given, students are in general more adapted to the idea of writing, yet their problem lies in not knowing how much writing is required. In many cases, their verbal discussions become lengthier as time goes on. As for the content, many discussions include the right amount of information needed and some may be even lengthy; but there are always students who do not write enough and students who cannot accept the idea of writing in Mathematics. Figures 3, 4, and 5 show a sample of students' answers to the Predator -Prey essay question:

$$\frac{dx}{dt} = 10x(1 - \frac{x}{5}) - 8xy$$
$$\frac{dy}{dt} = -4y + \frac{1}{10}xy$$

In Figure 3, the analysis of the student is almost complete with a detailed description of the behavior of the predator and prey population. This student writes: " [This] is a predator –prey system. When $x > 0, \frac{1}{10}xy > 0$ this has a positive effect on $\frac{dy}{dt}$ which makes the predator y grows because they are eating preys. When $y > 0, -8xy < 0$ this has a negative effect on $\frac{dx}{dt}$ which make the prey x decay because they are eaten by the predators. When x = 0, preys are extinct; $\frac{dy}{dt} = -4y$ [then] the predators will decay because they don't have food to eat $(-4y < 0)$. When y = 0, [then] predators are extinct, [then], the preys will grow according to a logistic model since there are no predators to eat them." Figure 4 on the other hand is the work of a student who seems to have understood the system but did not write enough. The student writes: " Here we have a predator-prey model because if the predator decreases or decays, [then] the prey will grow in logistic model; however if the prey decays, the predator will also decay exponentially…" In this writing exercise, this student was not specific as to which variable represents the prey and which represents the predator. The student also did not discuss explicitly the effect of the positive and negative signs of the mixed terms. In my opinion, this student may be one of those who do not know how much writing is required in problems of this kind. As for Figure 5, it shows the work of a student who simply writes very little. For many students however, writing is seen at the end of the semester as an essential component in the learning process. In one questionnaire distributed at the end of one semester, students were asked to answer to the following question: In many instances, writing was essential to communicate an idea/concept. What is your opinion on writing in mathematics (differential equations in particular)? Your opinion should be independent of your English capabilities. All the students who responded to this question agreed that writing was essential in the course. Some reasoned (rightfully) that writing complements the geometrical approach adopted in the solution process:

> "Since the course stresses on the geometrical way of solving DE's, this makes the writing very essential for the student to be able to express and tell the way or the steps followed in solving and drawing the solution."
>
> "Writing is very important in this course especially when no analytical solution is attainable. It is useful to describe the behavior of solutions where we have geometrical approach. Even when we have a quantitative solution, we need to explain it to let others understand what the equations we have written express."

Others argued that writing was also necessary for enhancing the learning and for showing that concepts have indeed been understood:

> "I found absolutely no problem in the "essay questions" on exams and in homework. In fact, I think that they were very useful because they clarify concepts in our mind. Once we write to explain an idea in our own words (often with the aid of sketches), we make sure we fully understand it."
>
> "Generally, writing in mathematics is very important. Personally I think solving an equation by only using the mathematical symbols without explaining the procedures followed isn't that good because it may become a procedure done by heart, while with writing and explaining the professors can make sure if the students understand the material."
>
> "Writing is an essential and useful process in mathematics…. For example, when solving a system analytically, a student may solve it either by chance or by cheating in some instances! So writing provides the instructor about the student's understanding of the subject taken."

In conclusion, the eventual positive reaction of the students concerning writing is extremely encouraging. It is comforting to know that students do consider writing as a tool to enhance learning as well as a tool to clarify ideas presented geometrically. However, I think that students will always ask questions such as: How much is enough? How detailed should I be? Or: Is this what you want? It has proven difficult to answer these questions and consequently to grade essay questions. However, time and practice will certainly improve the style in which essay questions are asked. This in turn should help students know what exactly to write, and help instructors in the grading process.

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Figure 1. The slope field of the population model & some solutions.

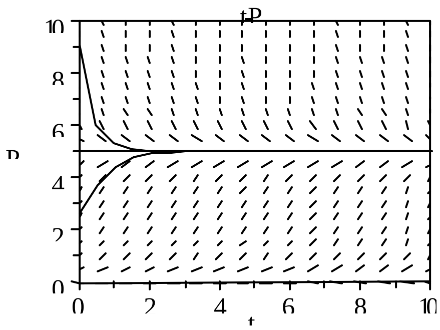


Figure 2. The vector filed of a simple harmonic oscillator, one solution curve, and its tx and ty time series.

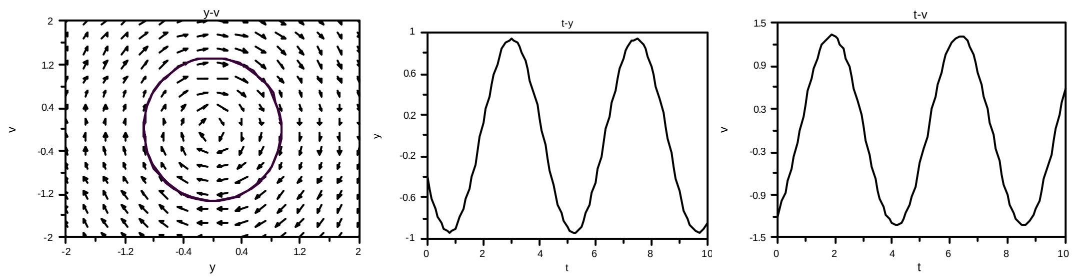


Figure 3. An almost complete analysis of the predator-prey system.

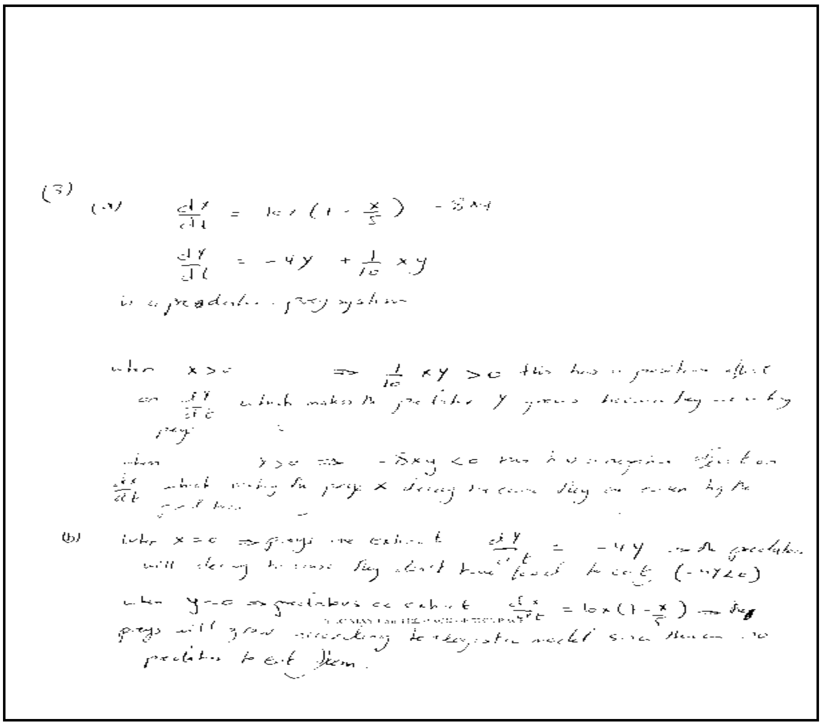


Figure 4. A student who does not seem to know how much and what to write in analyzing the predator-prey system.

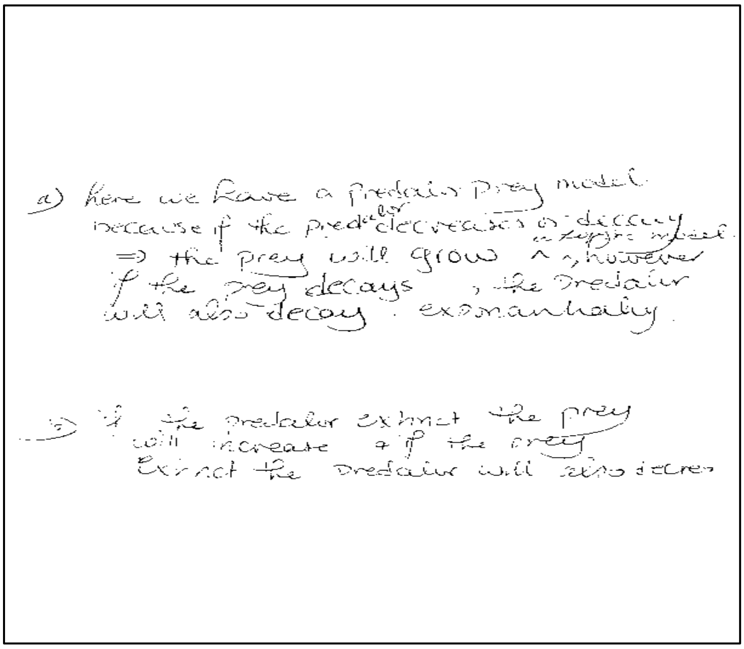
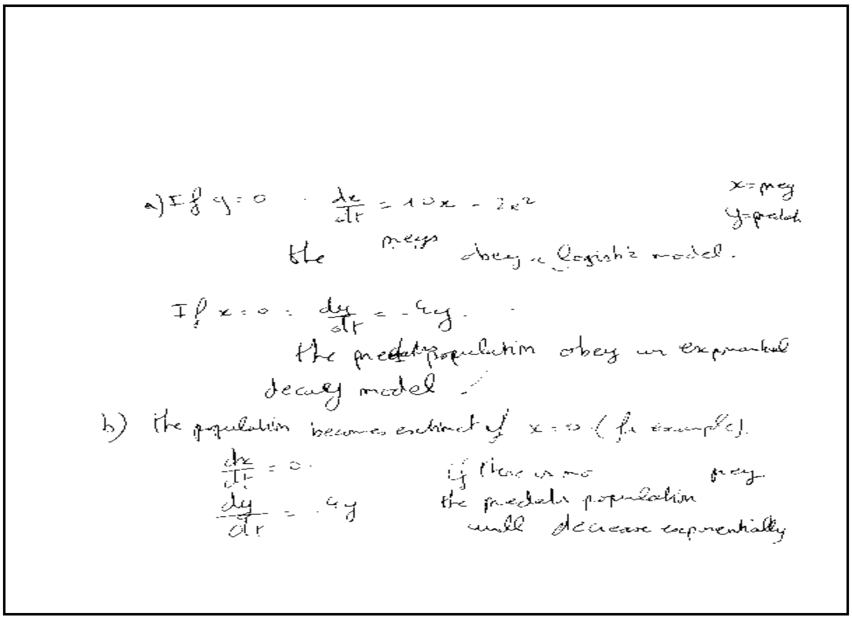


Figure 5. A student who writes very little.



a) If $y = 0$. $\frac{dx}{dt} = 10x - 2x^2$

x = prey
y = predator

the prey obey a logistic model.

If $x = 0$: $\frac{dy}{dt} = -4y$.

the predator population obey an exponential decay model.

b) the population becomes extinct if $x = 0$ (for example).

$\frac{dx}{dt} = 0$. if there is no prey

$\frac{dy}{dt} = -4y$ the predator population will decrease exponentially