7.4

(4) $x = (R+\cos\varphi)\cos\theta,\ y = (R+\cos\varphi)\sin\theta,\ z = \sin\varphi$
$\theta \in [0, 2\pi],\ \varphi \in [0, 2\pi]$

$$T\varphi = (-\sin\varphi\cos\theta, -\sin\varphi\sin\theta, \cos\varphi)$$
$$T\theta = (-(R+\cos\varphi)\sin\theta, (R+\cos\varphi)\cos\theta, 0)$$

$$T\varphi \times T\theta = (-\cos\varphi\cos\theta(R+\cos\varphi), -(R+\cos\varphi)\sin\theta\cos\varphi, -\sin\varphi(R+\cos\varphi))$$

$$\|T\varphi \times T\theta\| = \sqrt{\cos^2\varphi\cos^2\theta(R+\cos\varphi)^2 + \cos^2\varphi\sin^2\theta(R+\cos\varphi)^2 + \sin^2\varphi(R+\cos\varphi)^2} =$$

$$= \sqrt{(R+\cos\varphi)^2} = R+\cos\varphi$$

$$A(S) = \int_0^{2\pi}\int_0^{2\pi} R+\cos\varphi \, d\varphi\, d\theta = \int_0^{2\pi} [R\varphi + \sin\varphi]_0^{2\pi} d\theta =$$
$$= \int_0^{2\pi} 2\pi R \, d\theta = 4\pi^2 R$$

(3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$x = a\cos\theta\sin\varphi$, $y = b\sin\theta\sin\varphi$, $z = c\cos\varphi$

$\theta \in [0, 2\pi)$, $\varphi \in [0, \pi]$

$T\theta = (-a\sin\theta\sin\varphi, b\cos\theta\sin\varphi, 0)$

$T\varphi = (a\cos\theta\cos\varphi, b\sin\theta\cos\varphi, -c\sin\varphi)$

$T\theta \times T\varphi = (-bc\cos\theta\sin^2\varphi, -ac\sin\theta\sin^2\varphi, -ab\cos\varphi\sin\varphi)$

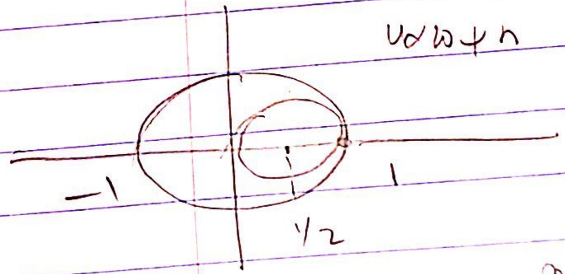
$\|T\theta \times T\varphi\| = \sqrt{b^2c^2\cos^2\theta\sin^4\varphi + a^2c^2\sin^2\theta\sin^4\varphi + a^2b^2\cos^2\varphi\sin^2\varphi}$

$A(S) = \int_0^{\pi}\int_0^{2\pi} \|T\theta \times T\varphi\| \, d\theta \, d\varphi$

(3, -2, 1)

(17) $x^2+y^2=x \iff \left(x-\frac{1}{2}\right)^2+y^2=\left(\frac{1}{2}\right)^2$ κύλινδρος

κάτοψη



S_1: παίρνω άνω ημισφαίριο: $z=\sqrt{1-x^2-y^2}$

$(x,y,\sqrt{1-x^2-y^2})$ $\quad T_x \times T_y = \left(\frac{-x}{\sqrt{1-x^2-y^2}}, \frac{-y}{\sqrt{1-x^2-y^2}}, 1\right)$

$$S_1 = 2\iint_{(x-\frac{1}{2})^2+y^2=(\frac{1}{2})^2} \frac{1}{\sqrt{1-x^2-y^2}}\,dx\,dy$$

$T_x \times T_y$

Πολικές : $S_1 = 2\int_{-\pi/2}^{\pi/2} \left(\int_0^{\cos\theta} \frac{r\,dr}{\sqrt{1-r^2}} \right) d\theta =$

$= 2\int_{-\pi/2}^{\pi/2} \left[-\sqrt{1-r^2}\right]_0^{\cos\theta} d\theta =$

$= 2\int_{-\pi/2}^{\pi/2} 1 - \sqrt{1-\cos^2\theta}\; d\theta = 2\pi - 2\int_{-\pi/2}^{\pi/2} |\sin\theta|\, d\theta$

$= 2\pi - 2\left[\int_{-\pi/2}^{0} -\sin\theta\, d\theta + \int_0^{\pi/2} \sin\theta\, d\theta\right] =$

$= 2\pi - 2\left[[\cos\theta]_{-\pi/2}^{0} - [\sin\theta]_0^{\pi/2}\right] =$

$= 2\pi - 4$

$S_2 = 4\pi - (2\pi - 4) = 2\pi + 4$

$\frac{S_2}{S_1} = \frac{2\pi + 4}{2\pi - 4}$

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$$f(x,y) = \frac{2}{3}\left(x^{3/2} + y^{3/2}\right)$$

$[0,1] \times [0,1]$

$$T(x,y) = \left(x, y, \frac{2}{3}\left(x^{3/2} + y^{3/2}\right)\right)$$

$$T_x = (1, 0, \sqrt{x}), \quad T_y = (0, 1, \sqrt{y})$$

$$T_x \times T_y = (-\sqrt{x}, -\sqrt{y}, 1), \quad \|T_x \times T_y\| = \sqrt{x+y+1}$$

$$A(S) = \int_0^1 \int_0^1 \sqrt{x+y+1}\, dx\, dy =$$

$$= \int_0^1 \frac{2}{3}\left[(y+1+x)^{3/2}\right]_0^1 dy =$$

$$= \frac{2}{3}\int_0^1 (y+2)^{3/2} - (y+1)^{3/2}\, dy =$$

$$= \frac{2}{3}\left[\frac{2}{5}(y+2)^{5/2} - \frac{2}{5}(y+1)^{5/2}\right]_0^1 =$$

$$= \frac{2}{3}\left(\frac{2}{5}\cdot 3^{5/2} - \frac{2}{5}\cdot 2^{5/2} - \frac{2}{5}\cdot 2^{5/2} + \frac{2}{5}\right) =$$

$$= \frac{4}{15}\left(3^{5/2} - 2\cdot 2^{5/2} + 1\right) = \frac{4}{15}\left(\sqrt{3^5} - 2\sqrt{2^5} + 1\right) =$$

$$= \frac{4}{15}\left(9\sqrt{3} - 8\sqrt{2} + 1\right)$$