University of Crete
Departments of Mathematics and Applied Mathematics

Analysis Seminar

1:15pm, Tuesday, 27 September, 2016
Room A-303

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An analogue of Hilbert's Tenth Problem for the ring of holomorphic functions

Consider polynomials $f(x)$ of an array x of variables, with coefficients in $\mathbb{Z}[x]$, where $\mathbb{Z}$: integers, z: a variable. We ask: Question: Is there an algorithm (in the Caves of Plato) which, given any $f(x)$ as above, decides (with certainty) whether the (functional) equation $f(x) = 0$ has solutions x, each of which is a holomorphic function of the variable z? Another way to ask the question is "can one solve algorithmically algebraic differential equations of order zero over the ring of global analytic functions?" Put that way the implications of the answer for the physical sciences should be obvious. The answer to the question is unknown. But we have evidence that it might be "NO". This evidence will be presented. It involves meromorphic parameterisations of elliptic curves and "existentially definable subsets" of the ring of holomorphic functions (i.e. properties of elements of that ring which can be defined by using only existential quantifiers and equations - no negations). Connections with other areas - especially Diophantine Geometry - will be mentioned. The talk will be dedicated to the memory of Lee Rubel, who was the first to ask these and other similar questions.