We prove a two weight multi-parameter dyadic embedding theorem for the Hardy operator on multi-trees. The main result is known in the case $n = 1$ and here we prove it for $n = 2, 3$. Strikingly, the “Box” condition we impose is much weaker than the Chang-Fefferman one, given the well-known Carleson quilt counterexample. However, this is not contradictory to our work, as we impose different restrictions on the weights. An application of this is the embedding theorem of Dirichlet space of holomorphic functions on the polydisc, which appears in the work of Arcozzi, Mozolyako, Perfekt and Sarfati.

This is joint work with N. Arcozzi, P. Mozolyako, A. Volberg and P. Zorin-Kranich.