On the axioms of operator space theory

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Abstract

In their recent book on operator spaces Effros and Ruan define an operator space to be a linear space together with a sequence of norms $\{\|.\|_n\}_{n\in\mathbb{N}}$ where, for each $n\in\mathbb{N}$, $\|.\|_n$ is a norm on $\mathbb{M}_n(V)$ satisfying the following two axioms:

O1. $\|[\alpha_{ij}][v_{ij}][\beta_{ij}]\|_m \leq \|[\alpha_{ij}]\|_{m,n}\|[v_{ij}]\|_n\|[\beta_{ij}]\|_{n,m}$
for all $n, m\in\mathbb{N}$, all $[v_{ij}]\in\mathbb{M}_n(V)$ and all $[\alpha_{ij}]\in\mathbb{M}_{m,n}$, $[\beta_{ij}]\in\mathbb{M}_{n,m}$

O2. $\|[v_{ij}]\oplus[v'_{kl}]\|_{n+m} = \max\{\|[v_{ij}]\|_n, \|[v'_{kl}]\|_m\}$
for all $n, m\in\mathbb{N}$ and all $[v_{ij}]\in\mathbb{M}_n(V)$, $[v'_{kl}]\in\mathbb{M}_m(V)$

We study the way those two axioms are interrelated as well as their relation to other simple properties of operator spaces.