**SELECTED TOPICS FROM GEOMETRY/TOPOLOGY**
**(CHARACTERISTIC CLASSES)**

## I. De Rham cohomology

1. Differential forms
2. The exterior algebra of a smooth manifold
3. Orientable smooth manifolds
4. Integration of differential forms
5. Smooth homotopy and the Poincaré lemma
6. The Mayer-Vietoris sequence
7. Poincaré duality and applications
8. The degree of a smooth map and applications
9. The Künneth formula for de Rham cohomology with compact supports
10. Elements of smooth intersection theory
11. The Lefschetz fixed point theorem and applications
12. Generalised Mayer-Vietoris exact sequences
13. Presheaves and the Čech-de Rham theorem

## II. Vector bundles

1. Basic notions and examples
2. Direct sums and inner products
3. The functors $K$ and $KO$
4. Products
5. Constructions with vector bundles and their sections

## III. Geometry of characteristic classes

1. Connections on vector bundles
2. Induced connections
3. Invariant polynomials
4. Characteristic classes of complex vector bundles
5. The Pfaffian
6. The Euler class
7. The splitting principle for complex vector bundles
8. Pontryagin classes and applications

## IV. Topological theory of characteristic classes

1. Stiefel-Whitney classes and applications
2. Grassmann manifolds and universal bundles
3. The cohomology ring of Grassmann manifolds
4. The Thom isomorphism mod 2
5. Construction of Stiefel-Whitney classes
6. Orientable bundles and the topological Euler class
6. Applications to smooth manifolds
7. Elements of obstruction theory
8. Topological theory of Chern classes
9. Topological Pontryagin classes and cobordism theory
10. The signature theorem of Hirzebruch

## Bibliography


1. R. Bott and L.W. Tu, Differential Forms in Algebraic Topology, Springer, 1982.
2. I. Madsen and J. Tornehave, From Calculus to Cohomology, Cambridge University Press, 1997.
3. J. Milnor and J. Stasheff, Characteristic Classes, Princeton University Press, 1974.
4. Handwritten notes (in Greek).