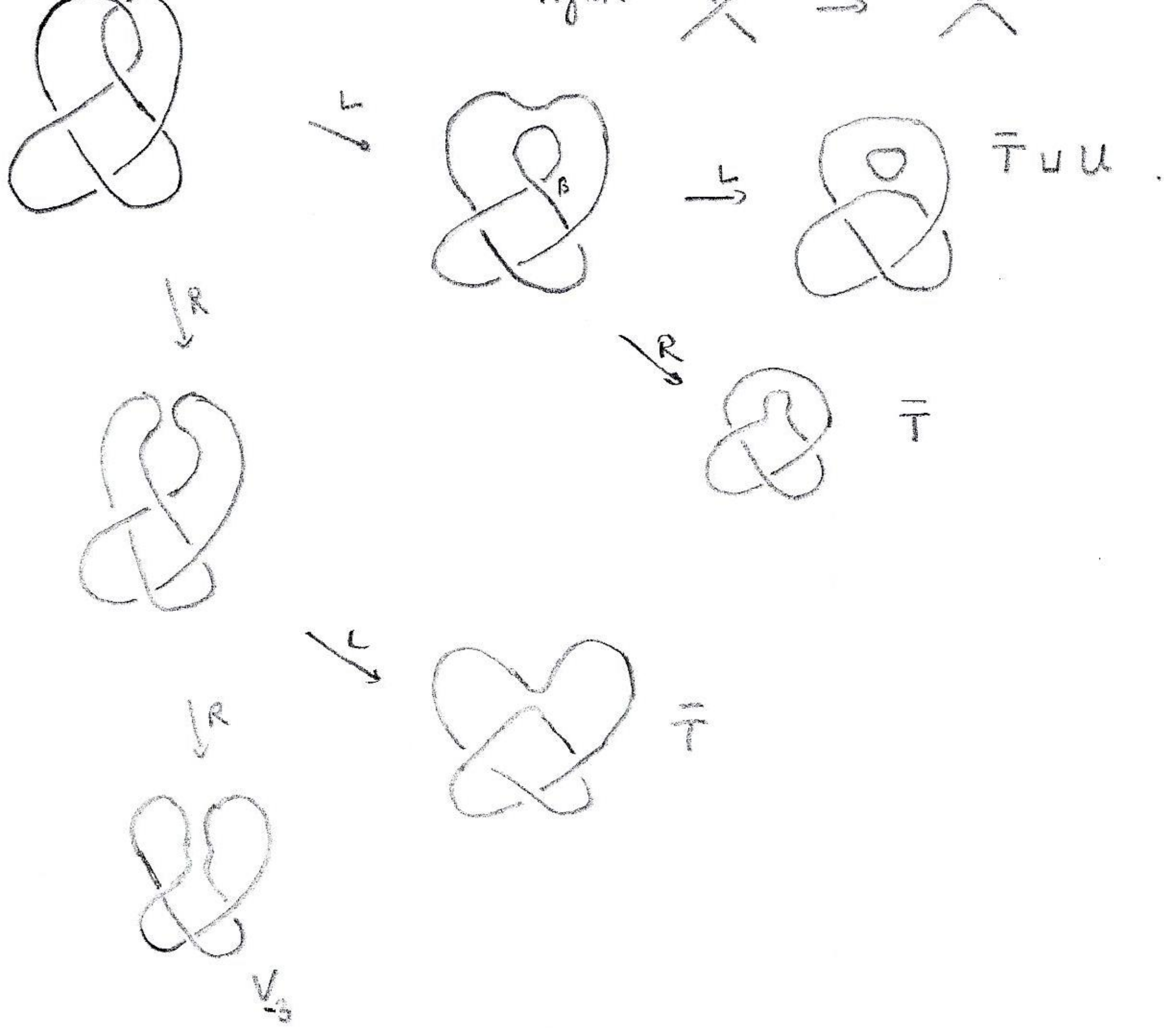
Θέμα 1.

Splitting to the left: 

right: 



$$\langle D \rangle = A\left(A\langle \bar{T} \sqcup U\rangle + A^{-1}\langle \bar{T}\rangle\right) + A^{-1}\left(A\langle \bar{T}\rangle + A^{-1}\langle V_{-3}\rangle\right).$$

$$= A\left(A\left(-(A^2+A^{-2})\right)\left(A^{-7}-A^{-3}-A^5\right) + A^{-1}\left(A^{-7}-A^{-3}-A^5\right)\right)$$

$$+ A^{-1}\left(A\left(A^{-7}-A^{-3}-A^5\right) + A^{-1}\left(-A^{-3}\right)^3\right)$$

$$= \left(-A^4+1\right)\left(A^{-7}-A^{-3}-A^5\right) - A^{-11}$$

$$= -A^{-3}+A+A^9+A^{-7}-A^{-3}-A^5-A^{-11}$$

To see if it is chiral, compute the Kaufman polynomial.

$w\Big($  $\Big) = -5.$

$$f[D] = (-A)^{-3(-5)}\langle D\rangle = -A^{15}\langle D\rangle$$

$$= +A^{12} - A^{16} - A^{24} - A^{8} + A^{12} + A^{20} + A^{4}.$$

$$\left(V_D(t) = t^{-1} - t^{-2} + 2t^{-3} - t^{-4} + t^{-5} - t^{-6}.\right)$$

Hence $D \not\simeq \overline{D}$.

Θέμα 2.

Let $c_1 \in CX$ be the point $[(x,1)]$ for any $x \in X$, and $p: X \times I \to CX$ the projection.

α) Let $[(x,t)]$ be any point in CX, for $t \neq 1$.

We define path $\sigma: I \to X \times I$: $\sigma(s) = (x, (1-s)t + s)$.

Then $\sigma(0) = (x,t)$, $\sigma(1) = (x,1)$, and $p \circ \sigma$ is a path in CX from $[(x,t)]$ to c_1.

Hence CX is path connected.

β) Consider two different points $[(x,t)]$ and $[(y,s)]$ in CX.

i) $t=1$, $s \neq 1$. Let $\varepsilon < \frac{1}{2}(1-s)$ and $\varepsilon < s$.

Then $U = p(X \times (s-\varepsilon, s+\varepsilon))$ is open in CX, since $p^{-1}(U) = X \times (s-\varepsilon, s+\varepsilon)$ which is open in $X \times I$,

$V = p(X \times (1-\varepsilon, 1])$ is open in CX, since $p^{-1}(V) = X \times (1-\varepsilon, 1]$ which is open in $X \times I$.

$[(y,s)] \in U$, $[(x,1)] \in V$ and $U \cap V = \phi$.

ii) If $t \neq s$, similarly we choose $\varepsilon < \frac{1}{2}|t-s|$.

iii) If $t = s \neq 1$, then $x \neq y$ and there exist U_1, V_1 open in X s.t. $x \in U_1$, $y \in V_1$ and $U_1 \cap V_1 = \phi$. Let $U = p(U_1 \times [0,1))$, $V = p(V_1 \times [0,1))$.

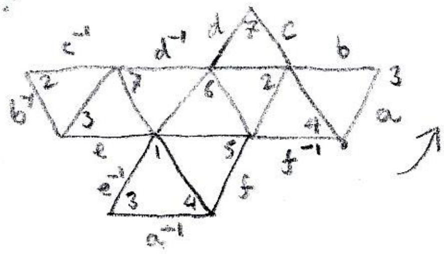
Then U is open in CX because $p^{-1}(U) = U_1 \times [0,1)$ is open in $X \times I$. Similarly for V.

$[(x,t)] \in U$, $[(y,t)] \in V$ and $U \cap V = \phi$.

Hence CX is Hdff.

γ) Since CX is Hdff, $X \times I$ is compact and $p : X \times I \to CX$ is cts, CX is compact.

Θέμα 3.



$abcdd^{-1}c^{-1}b^{-1}ee^{-1}a^{-1}ff^{-1}$

$\sim aa^{-1}ff^{-1}$.

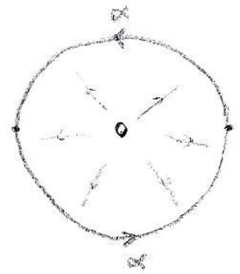
sphere. (See also problem 10.4)

Θέμα 4 α) See solution to 11.4.

β) Consider $\mathbb{R}P^2$ as the closed disc Δ, with opposite points on the boundary identified: $\mathbb{R}P^2 = \Delta/\sim$ where $\sim$ is the equiv. relation generated by

$$z \sim w \quad \text{if} \quad |z| = |w| = 1 \text{ and } z = -w.$$

Let $p : \Delta \to \mathbb{R}P^2$ be the projection, and let $x_0 = p(0)$, $A = p(S^1)$.



We define $\tilde{F} : (\Delta \setminus \{0\}) \times I \to \Delta \setminus \{0\}$

$$\tilde{F}(z, t) = (1-t)z + t\frac{z}{|z|}.$$

If $|z| = 1$, $\tilde{F}(z,t) = z$ for all $t \in I$.

Hence $\tilde{F}(z,t) \sim \tilde{F}(w,t)$ iff $z \sim w$, and there is a well defined mapping $F : (\mathbb{R}P^2 \setminus \{x_0\}) \times I \to \mathbb{R}P^2 \setminus \{x_0\}$ such that $F(p(z), t) = p(\tilde{F}(z,t))$.

$$\begin{tikzcd}
(\Delta \setminus \{0\}) \times I \arrow[r, "\tilde{F}"] \arrow[d, "p \times id"'] & \Delta \setminus \{0\} \arrow[d, "p"] \\
(\mathbb{R}P^2 \setminus \{x_0\}) \times I \arrow[r, "F"'] & \mathbb{R}P^2 \setminus \{x_0\}
\end{tikzcd}$$

F is cts, since $p \circ \tilde{F}$ is cts, and F is a deformation retraction of $\mathbb{R}P^2 \setminus \{x_0\}$ to A:

$$F(p(z), 0) = p(\tilde{F}(z,0)) = p(z)$$

$$F(p(z), 1) = p\left(\frac{z}{|z|}\right) \in A$$

and if $p(z) \in A$, $|z| = 1$ and $F(p(z), t) = p(z)$ for all $t \in I$.

It remains to show that $A \cong S^1$.

Let $f : A \to S^1$ be the mapping $f(p(z)) = z^2$.

f is well defined, since $p(z) = p(w)$ iff $z^2 = w^2$,

and f is a bijection.

$$\begin{tikzcd} & S^1 \arrow[dl, "p"'] \arrow[dr, "z^2"] & \\ A \arrow[rr, "f"'] & & S^1 \end{tikzcd}$$

f is cts, since $f \circ p$ is cts.

f^{-1} is cts, since $f^{-1}(z^2) = p(z)$ and

p is cts.

Hence $A \cong S^1$.