1.3. Let U_u, U_r, U_f be the sets

$U_u = \{(x,y,z) \in S^2 : z > 0\}$, $U_r = \{(x,y,z) \in S^2 : y > 0\}$,

$U_f = \{(x,y,z) \in S^2 : x > 0\}$, and $V_u = p(U_u)$, $V_r = p(U_r)$, $V_f = p(U_f)$.

We note that p is injective on each of U_u, U_r, U_f:

if $\pm(x,y,z) \in V_u$, then $z \neq 0$ and exactly one of (x,y,z), $-(x,y,z)$ belongs to U_u.

The sets U_u, U_r, U_f are homeomorphic to the disc D^2:

$q_u : U_u \to D^2 : (x,y,z) \mapsto (x,y)$, $q_r : U_r \to D^2 : (x,y,z) \mapsto (x,z)$

$q_f : U_f \to D^2 : (x,y,z) \mapsto (y,z)$ are bijective, continuous, and have continuous inverse, for example $q_r^{-1}(s,t) = (s, \sqrt{1-s^2-t^2}, t)$.

We define the maps $\varphi_u, \varphi_r, \varphi_f$:

If $\pm(x,y,z) \in V_u$ and $z > 0$, $\varphi_u(\pm(x,y,z)) = (x,y) \in D^2$.

If $\pm(x,y,z) \in V_r$ and $y > 0$, $\varphi_r(\pm(x,y,z)) = (x,z) \in D^2$

If $\pm(x,y,z) \in V_f$ and $x > 0$, $\varphi_f(\pm(x,y,z)) = (y,z) \in D^2$.

The sets V_u, V_r, V_f cover $\mathbb{R}P^2$: each $(x,y,z) \in S^2$, has at least one non-zero coordinate. If $z \neq 0$, then $\pm(x,y,z)$ belongs to V_u, if $y \neq 0$, $\pm(x,y,z) \in V_r$, if $x \neq 0$, $\pm(x,y,z) \in V_f$.

To show that $\varphi_u, \varphi_r, \varphi_f$ define a 2-manifold structure on $\mathbb{R}P^2$, we have to check that the transition functions are continuous: assume $\pm(x,y,z) \in V_u \cap V_r$. Then $yz > 0$, and we can choose the sign of x so that $y > 0$ and $z > 0$. Then $\varphi_u(\pm(x,y,z)) = (x,y)$, and $\varphi_u(V_u \cap V_r) = \{(s,t) \in D^2 : t > 0\}$. For $(s,t) \in D^2$, $t > 0$,

$$\varphi_u^{-1}(s,t) = \pm\left(s, t, \sqrt{1-s^2-t^2}\right) \in V_u \cap V_r .$$

Then

$$\varphi_r \circ \varphi_u^{-1}(s,t) = \left(s, \sqrt{1-s^2-t^2}\right)$$

which is continuous. //.