

12.1.

1. Define $F: \mathbb{R}^n \times I \rightarrow \mathbb{R}^n$ by $F(x, t) = (1-t)x$.

Then $F(x, 0) = x$, $F(x, 1) = 0$ and $F(0, t) = 0$ for all t .

2. Define $F: (\mathbb{C} \setminus \{0\}) \times I \rightarrow \mathbb{C} \setminus \{0\}$ by

$$F(z, t) = z(1-t) + \frac{z}{|z|} t.$$

Then $F(z, 0) = z$, $F(z, 1) = \frac{z}{|z|} \in S^1$ and

if $|z|=1$, $F(z, t) = z$ for all t .

12.2. For $n=2$, the sets U and V are homeomorphic to

open discs. For the open disc $D^2 = \{x \in \mathbb{R}^2 : |x| < 1\}$

there is a deformation retraction to $\{0\} \subseteq D^2$, and therefore

$\pi(D^2, 0) = 1$. Hence, if $x_0 = (1, 0, 0) \in S^2$,

$$\pi(U, x_0) = 1 = \pi(V, x_0).$$

Now, U and V are open subsets of S^2 , and $U \cap V$ is homeomorphic to an annulus, hence it is path connected.

By the Seifert-van Kampen theorem (2), since $\pi(V, x_0) = 1$,

$$\pi(S^2, x_0) \cong \pi(U, x_0) / N = 1.$$

The above argument can be adapted for any sphere S^n , for $n \geq 2$. Why not for $n=1$?