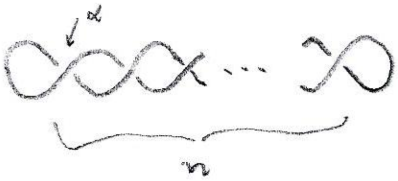
Preliminaries.

$U_n =$  $\qquad \langle U_n \rangle = (-1)^{n-1} \left(A^2 + A^{-2} \right)^{n-1}$

$V_1 =$  $\qquad \langle V_1 \rangle = A \langle U_2 \rangle + A^{-1} \langle U \rangle = -A^3$

$V_{-1} =$  $\qquad \langle V_{-1} \rangle = A \langle U \rangle + A^{-1} \langle U_2 \rangle = -A^{-3}$

$V_n =$ 

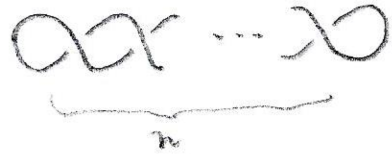
split α to the left: 

split α to the right 

$$\langle V_n \rangle = A \langle U \sqcup V_{n-1} \rangle + A^{-1} \langle V_{n-1} \rangle$$

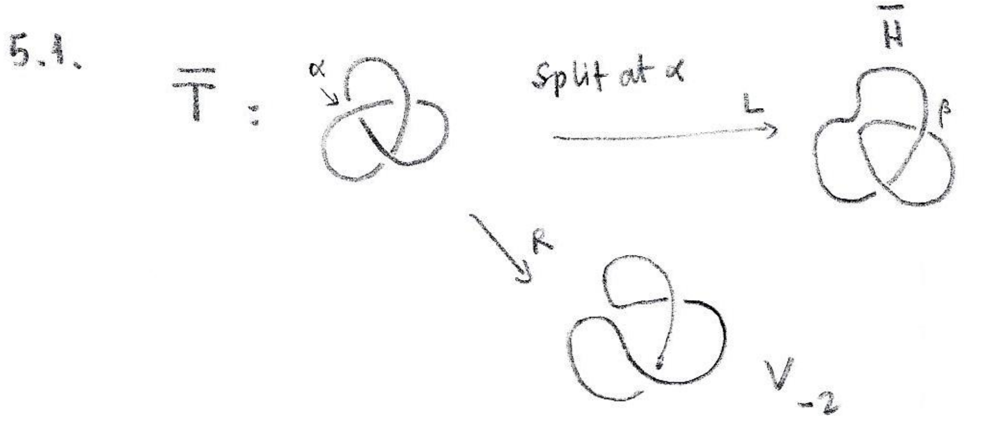
$$= \left(-A \left(A^2 + A^{-2}\right) + A^{-1}\right) \langle V_{n-1} \rangle = -A^3 \langle V_{n-1} \rangle .$$

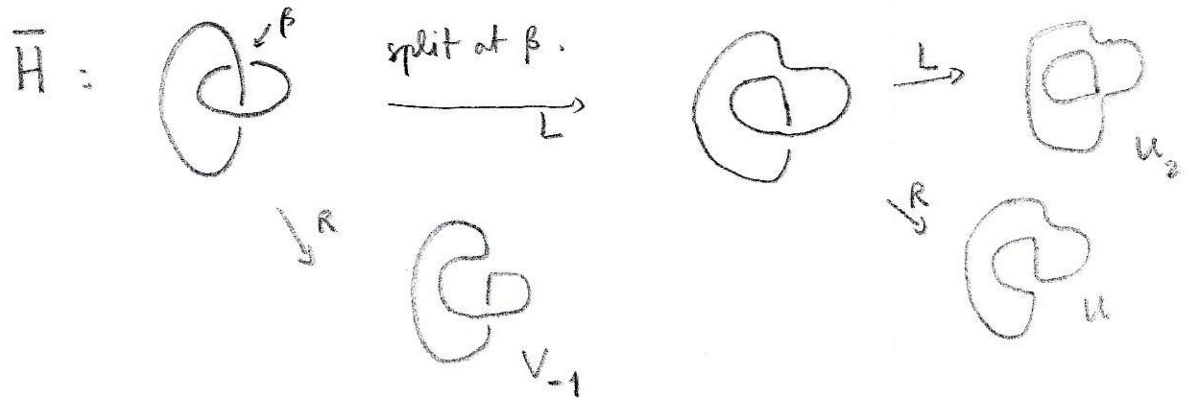
By induction $\langle V_n \rangle = (-A^3)^n$.

$V_{-n} =$ 

$$\langle V_{-n} \rangle = (-A^{-3})^n .$$

5.1.





Hence

$$\langle \overline{H} \rangle = A \left(A \langle U_2 \rangle + A^{-1} \langle U \rangle \right) + A^{-1} \langle V_{-1} \rangle)$$

$$= A^2 \left(-A^2 - A^{-2} \right) + 1 + A^{-1} \left(-A^{-3} \right)$$

$$= -A^4 - 1 + 1 - A^{-4} = -\left(A^4 + A^{-4} \right) = \langle H \rangle .$$

Finally

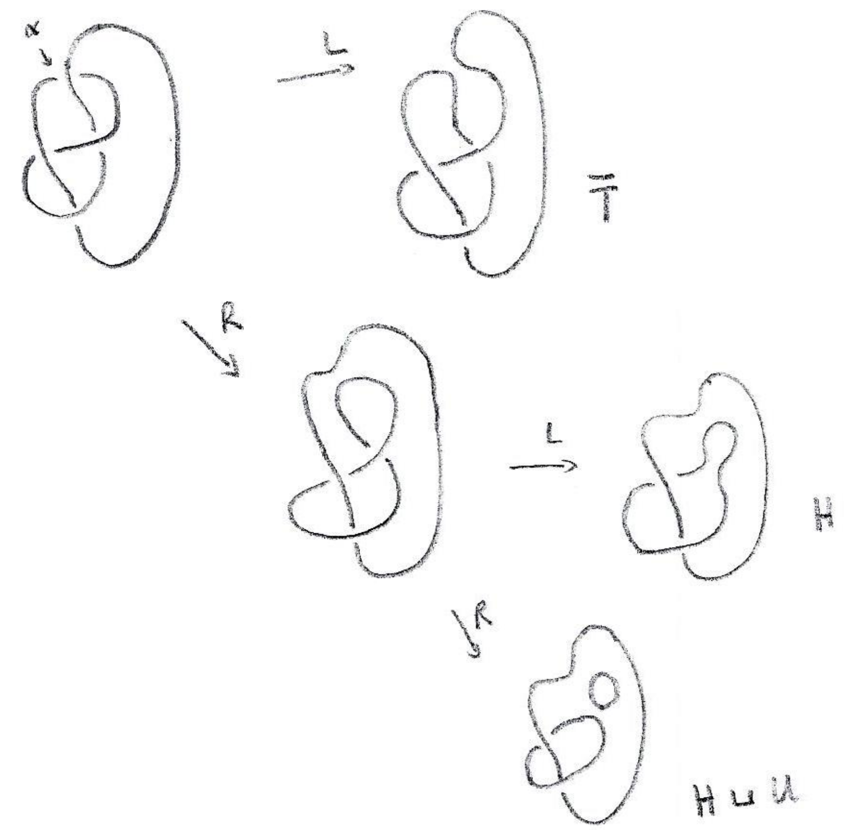
$$\overline{T} = A \langle \overline{H} \rangle + A^{-1} \langle V_{-2} \rangle$$

$$= -A \left(A^4 + A^{-4} \right) + A^{-1} A^{-6}$$

$$= A^{-7} - A^{-3} - A^5 .$$

5.3.

E :



$$\langle E \rangle = A\langle \bar{T} \rangle + A^{-1}\left(A\langle H \rangle + A^{-1}\left(-A^{2}-A^{-2}\right)\langle H \rangle \right)$$

$$= A\langle \bar{T} \rangle + \langle H \rangle - \langle H \rangle - A^{-4}\langle H \rangle = A\langle \bar{T} \rangle - A^{-4}\langle H \rangle .$$

5.4. Similarly, $\langle \bar{E} \rangle = A\left(A\left(-A^{2}-A^{-2}\right)\langle H \rangle + A^{-1}\langle H \rangle \right)$

$$+ A^{-1}\langle T \rangle .$$

$$= -A^{4}\langle H \rangle + A^{-1}\langle T \rangle .$$

Substitute $\langle H \rangle$, $\langle T \rangle$ and $\langle \bar{T} \rangle$ to show $\langle \bar{E} \rangle = \langle E \rangle$.

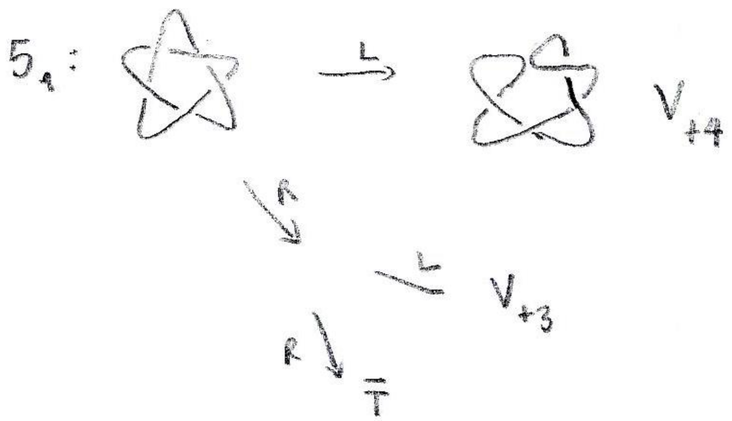
5.5. Choose orientation and compute $w(E) = 0$.

Hence $f[E] = (-A)^{-3w(E)}\langle E \rangle = \langle E \rangle$.

5.6.

$$\langle B \rangle = -\left(A^{12} + A^{-12}\right) + 3\left(A^{8} + A^{-8}\right) - 2\left(A^{4} + A^{-4}\right) + 4 .$$

5.7. a.



Hence $\langle 5_1 \rangle = A\,\langle V_{+4} \rangle + A^{-1}\left(A\,\langle V_{+3} \rangle + A^{-1}\,\langle \overline{T} \rangle\right)$.

$$= A^{13} - A^{9} + A^{5} - A - A^{-7}$$

$$w(5_1) = -5$$

$$f[5_1] = (-A)^{15}\,\langle 5_1 \rangle$$

$$= -A^{28} + A^{24} - A^{20} + A^{16} + A^{8} .$$

From the table on page 71, check that this agrees with the Jones polynomial

$$V_{5_1}(t) = t^{-2} + t^{-4} - t^{-5} + t^{-6} - t^{-7} .$$