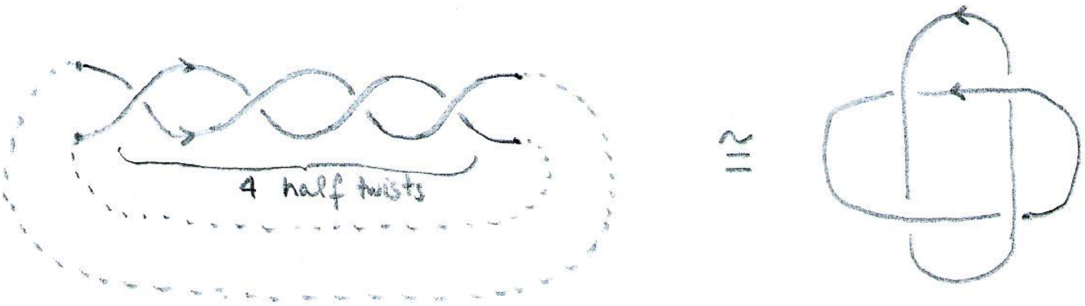
Γεωμετρική Τοπολογία, Φυλλάδιο 6.

6.1. The $(2,n)$ torus link $T_{2,n}$ is constructed by taking two parallel strings, introducing n (negative) half-twists and then joining the ends, as in the Figure.



1. Show that $T_{2,n}$ is a knot if n is odd, and a 2 component link if n is even.

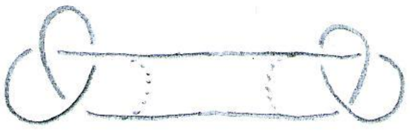
2. Find a recursive relation for the bracket and for the Jones polynomial of a $(2,q)$ torus knot for q odd, $q \geqslant 3$.

3. Verify your relation by calculating $T_{2,3}$, $T_{2,5}$ and $T_{2,7}$ and comparing with the Table on page 71 of the handwritten notes.

6.2. Show that the Jones polynomial of any oriented link L_c, with c components, satisfies, for $t=1$,

$$V_{L_c}(1) = (-2)^{c-1}.$$

6.3. The <u>sum</u> $K_1 \# K_2$ of two knots K_1 and K_2 is the knot obtained by tying first the knot K_1 and then the knot K_2 on the same string, and then joining the ends, as in the Figure.



$T_- \# T_+$

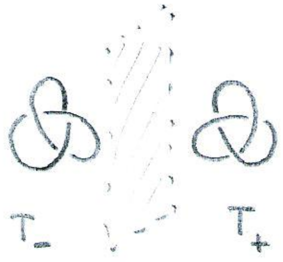
Sum of the trefoil knot and its reflection.

1) Show that $w(K_1 \# K_2) = w(K_1) + w(K_2)$

2) Show that $V_{K_1 \# K_2} = V_{K_1} \cdot V_{K_2}$.

(Consider the calculation of $\langle K_1 \# K_2 \rangle$ where you split first the crossings of K_1 and then the crossings of K_2.)

6.4. The <u>disjoint union</u> $K_1 \sqcup K_2$ of two knots K_1 and K_2 is the link consisting of two knots K_1 and K_2 which can be separated by a plane in $\mathbb{R}^3$.



Disjoint union of the trefoil knot and its reflection.

Show that

$$V_{K_1 \sqcup K_2} = \left(-t^{-1/2} - t^{1/2}\right) V_{K_1} \cdot V_{K_2} .$$