8.1. To show $C = \{ z \in \mathbb{C} : 1 \leq |z| \leq 2 \} \cong S^1 \times [0,1]$.

Consider the mapping $\varphi : C \to S^1 \times I$, $\varphi(z) = \left(\frac{z}{|z|}, |z| - 1 \right)$.

By the Proposition on page 3, φ is cts.

We check that φ is a bijection (it has inverse $(w,t) \mapsto (1+t)w$).

C is compact (it is closed and bounded in $\mathbb{C}$),

and $S^1 \times I$ is Hdff (Prop, page 9). Hence,

(by Corollary page 12) φ is a homeomorphism.

8.3. Consider the mapping $f : \mathbb{R} \to S^1 : t \mapsto e^{2\pi i t}$.

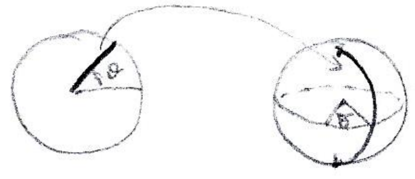
Then $s - t \in \mathbb{Z}$ iff $f(s) = f(t)$. Hence the equivalence relation determined by the surjection f is the same as $\sim$.

f is clearly cts. If we show that f is open, it follows that S^1 has the quotient topology (Proposition, page 6) and hence it is homeomophic to $\mathbb{R}/\sim$.

Any open set in $\mathbb{R}$ can be represented as a union of open intervals U_i of length < 1. Consider such an interval. It is mapped by f bijectively to an arc in S^1, which is the intersection of S^1 with an open disc in $\mathbb{C}$, hence it is open in the subspace topology. Since $f(\bigcup U_i) = \bigcup f(U_i)$, it follows that f is an open mapping.

8.5. Consider the mapping $\tilde{f} : \Delta \to S^2$ which maps each ray in Δ to a meridian in S^2.

The point $re^{i\vartheta} \in \Delta$, $0 \le r \le 1$, $0 \le \vartheta < 2\pi$, is mapped to the point with longitude ϑ and latitude φ such that $\sin\varphi = 2r-1$.



(Show that $\tilde{f}(re^{i\vartheta}) = (2\sqrt{r-r^2}\cos\vartheta,\ 2\sqrt{r-r^2}\sin\vartheta,\ 2r-1)$.)

Then $\tilde{f}(z) = \tilde{f}(w)$ iff $z \sim w$ in Δ. So there is a well defined bijection $f : \Delta/_\sim \to S^1$ and f is cts (Proposition, page 5).

$$\begin{tikzcd} \Delta \arrow[d, "p"'] \arrow[dr, "\tilde{f}"] & \\ \Delta/_\sim \arrow[r, "f"] & S^2 \arrow[l, bend left, "g"] \end{tikzcd}$$

Let $g = f^{-1}$. We show that g is cts by showing that for any point $x \in S^2$ and any open nbd V of $g(x)$, there is an open nbd U of x s.t. $g(U) \subseteq V$.

Let $x \in S^2 \setminus \{(0,0,1)\}$. Then $g(x) = [z]$, for $|z| < 1$.

Let V be an open disc in Δ, centre z, radius $\varepsilon < 1 - |z|$. Then V is mapped bijectively by $\tilde{f}$ to an open nbd U of x in S^2, and by p to an open nbd $p(V)$ of $[z] \in \Delta/_\sim$, such that $g(U) \subseteq p(V)$.

Let $x = (0,0,1)$, then $g(x) = [z]$ for $|z| = 1$. Let $V \subset \Delta$ be an annulus $\{w \in \Delta : 1 - |w| < \varepsilon\}$. Then V is mapped by $\tilde{f}$ to an open nbd $\mathcal{U}$ of x in S^2, and by p to an open nbd $p(V)$ of $[z]$ in $\Delta/\sim$, so that $g(\mathcal{U}) \subseteq p(V)$.

To show that $\tilde{f}$ is not an open mapping, consider the open subset $A = \{ z \in \Delta : \operatorname{Re} z > 0 \}$. Then $\tilde{f}(A)$ contains the point $(0,0,1)$, but does not contain any open nbd of $(0,0,1)$.