9.2. $\Rightarrow$. We assume X Hdff, and show that $X \times X \setminus \Delta$ is open, hence Δ is closed.

Let $(x, y) \in X \times X \setminus \Delta$. Then $x \neq y$, and since X is Hdff, there are open sets U and V in X s.t. $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Since $U \cap V = \emptyset$, $U \times V$ does not contain any pair (u, v) with $u = v$. Hence $(U \times V) \cap \Delta = \emptyset$ and $U \times V \subseteq X \times X \setminus \Delta$. But $U \times V$ is open in the product topology. Hence $X \times X \setminus \Delta$ is open.

$\Leftarrow$ Let $x, y \in X$, with $x \neq y$. Then $(x, y) \in X \times X \setminus \Delta$. If Δ is closed, then $X \times X \setminus \Delta$ is open, and there is a nbd W of (x, y) contained in $X \times X \setminus \Delta$. Hence there are open sets $U, V \subseteq X$ s.t. $x \in U$, $y \in V$ and $U \times V \subseteq W$. But then $(U \times V) \cap \Delta = \emptyset$ and hence $U \cap V = \emptyset$.

9.4. S^1 is compact and $\mathbb{R}$ is Hdff. If $f : S^1 \to \mathbb{R}$ is cts, then $f(S^1)$ is compact (Theorem, page 12). But $\mathbb{R}$ is not compact, hence $f(S^1) \neq \mathbb{R}$ and f cannot be surjective.

9.6. Let U and V be open subsets of A, such that $U \cap V = \emptyset$ and $U \cup V = A$.

Consider the sets $A_1 \cap U$ and $A_1 \cap V$. They are open subsets of A_1, their union is A_1 and their intersection is empty. Since $A_1 \neq \emptyset$, at least one of $A_1 \cap U$, $A_1 \cap V$ is non empty. Assume $A_1 \cap U \neq \emptyset$. Then, since A_1 is connected, $A_1 \cap V = \emptyset$. Show by induction that $A_n \subset U$ for all $n \in \mathbb{N}$, and hence that $V = \emptyset$.

9.8. Assume C is convex subset of $\mathbb{R}^n$. Then, for any $x, y \in C$, and $t \in [0,1]$, $\sigma(t) = (1-t)x + ty$ belongs to C.

Assume C is not connected. Then there exists $f: C \to \{0,1\}$ cts and surjective. Then there exist $x, y \in C$ s.t. $f(x) = 0$ and $f(y) = 1$. But then $f \circ \sigma$ is cts, $f \circ \sigma(0) = f(x) = 0$, $f \circ \sigma(1) = f(y) = 1$. Contradiction, since $[0,1]$ is connected (Theorem, page 13).