UNDERGRADUATE MATHEMATICS FOR PRIMARY SCHOOL TEACHERS:

The Situation in Portugal

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ABSTRACT

At the beginning of the 1990s a national reform of the mathematics curriculum took place in Portugal. This was not accompanied by a corresponding reform in the training of primary school teachers.

In Portugal teachers are trained in higher education institutions that are officially free to do whatever they believe is appropriate. This leads to wide variation in the training programmes, with some exhibiting a considerable degree of irrelevance (Gomes, Ralha & Hirst, 2001).

This is a worrying scenario, because the curricular reform that took place presents new ways of understanding the teaching of mathematics, imposing new challenges on teachers.

In Portugal it hasn't been until recently that the scientific community has begun to show an interest in the mathematical training of primary school teachers (APM, 1998). There are very few studies in this area, and they mostly deal with the pedagogical knowledge component of teaching, minimizing the importance of teachers' subject knowledge.

In this study we undertake a brief analysis of the pre-service mathematical training for primary teachers currently offered in Portuguese institutions. We shall consider some studies in this area and discuss the possible consequences for the reform of pre-service mathematical education. In particular we pay attention to teachers' subject knowledge of basic mathematics, following the research of Liping Ma (1999)

The process of introducing mass schooling seems to have started, in Portugal, a bit later than in most European countries: it dates from the mid 1970s and it gradually implied considerable changes in both teacher recruitment and training models.

By 1986, an Education Act lists as *specialised functions* within teacher education the following: special needs education, school management, student teaching supervision, curriculum coordination, in-service teacher training, etc. Those specific dimensions came, as expected, to be implemented into the educational system and, in relation to infant and primary teacher education, one also moved from not considering this education a university matter to the creation, in the mid 1980s, in all Portuguese regions of the so called ESE(s) (Higher Education Schools) within the University system. The Government also decided, in 1998, that both infant and primary teachers would have the same academic qualifications as, for example, secondary school teachers; this is called a "licenciatura" degree and it takes from 4 to 5 years to accomplish (Formosinho, 2000). All these important changes within the Portuguese Educational System brought, as can be acknowledged from national assessment reports on the university degrees (CNAES, 2000), considerable reflection on methods of teaching and organisational aspects but did not bring any reflection on the contents of training courses for these "new" teachers supposedly better prepared to deal with modern educational challenges than "old" ones.

In fact, it hasn't been until recently that the scientific community in Portugal has begun to show some interest in the mathematical training of primary school teachers (APM, 1998). Evidence of this neglect can be found, for example, in searches conducted through periodicals such as *Gazeta da Matemática*, which was first issued in 1940 with the specific goals of helping the A-level students and support the A-level teachers (G.M., N. º 1). There were, then as well as nowadays, no references to the mathematical training of primary teachers or to the problems related to ~~the~~ mathematics teaching at primary schools. On the other hand, a specific search through the magazine *Escola Democrática* reveals some discussion about the mathematics curriculum, particularly at the time of the introduction of the so called "Modern Mathematics". More recently we find several articles concerning primary school mathematics in *Educação e Matemática*, a periodical published by the Portuguese Mathematics Teacher Association. However, no matter whether or not these are specialized mathematical magazines, we have reasons to believe that these articles are not as widely known as one might expect.

The situation appears to be quite different in other countries; in summary

- Using *L'Enseigment Mathématique* as a reference, we can picture the way the so called "elementary mathematics" was treated and the importance given to the mathematical training of primary school teachers, through several articles published for more than a century reporting on the situation worldwide.
- Comparing Portuguese and some British infant and primary teachers' education one identifies

	Students	**Entry requirements**	**Structure**
Portugal	Almost all women. "Regular" students (average age 18 years old).	Upper-secondary; No special requirements for any subject.	4 years degree: 3 years + 1 year in-service training.
England	Majority of women. Three different age groups identified: "regular" students (21 years old average), mature students in their 30s and mature students in their 40s.	Academic requirement for admission to 1st degree studies; To achieve at least grade C in the GCSE examination in both Mathematics and English.	4 years degree: first degree + 1 year PGCE or 4 year Bachelor + QTS

TABLE 1: Comparing Portuguese and some British infant and primary teachers' education.

One can clearly identify a worrying scenario if one adds to the lack of research the fact that a national reform of the mathematics primary curriculum also took place in Portugal at the beginning of the 1990s. This reform, which presents new ways of understanding the teaching of mathematics, imposing new challenges on teachers, was definitely not accompanied by a corresponding reform in the training of primary school teachers. We still have infant and primary school teachers trained in three different kinds of higher education institutions: universities, polytechnics and private ones, that are officially free to do whatever they believe is most appropriate. This leads to a wide range of training programmes with some exhibiting a considerable degree of irrelevance. In an analysis of the mathematics programmes of the different institutions several questions were raised (Gomes, Ralha & Hirst, 2001), namely:

- About the coherence exhibited by the mathematical curriculum.
- About the relevance of some topics such as Topology, Matrices or Algebraic Structures.
- About the number of hours dedicated to the study of mathematics.

Questions	**Mathematical content**	**Coherence**	**Relevance of topics**	**Time dedicated to mathematics**
Analysis	It ranges from a condensed type Mathematics degree (for secondary school teachers) to a condensed type Education degree	The same contents repeatedly appear in different disciplines but the similarities are not explicitly identified. Disconnected topics.	Topics such as Matrices, Topology or Algebraic Structures are often questioned as relevant by most students.	It ranges from less than 6% to 17% of the total training time.

TABLE 2: Analysis of mathematics curriculum in different Portuguese infant and primary teachers' education.

In Portugal, to a large extent, the undergraduate students arrive at the training institutions with a mathematical training equivalent to nine years of mathematics. In an inquiry to the 1st year students of the Initial Teachers Training Course (Gomes & Ralha, 1999), it was verified that 28% of the students had more than 9 years of mathematics. Although almost all students considered mathematics to be interesting and useful, they find it hard to study (66%). Paradoxically, the majority of those asked believe that teaching mathematics to primary school children will be an easy task (72%).

Assuming that elementary mathematics is fundamental mathematics in the sense defended by Ma (1999), that is, even though it is presented in an elementary format it constitutes the foundations of the future mathematical learning and contains the rudiments of many important concepts in more advanced branches of the discipline, then the only sensible path to take seems to be to guarantee solid and efficient mathematical knowledge in the future teachers.

As a starting point to the study of the kind of mathematical knowledge Portuguese primary school teachers should have, we decided to analyse the mathematical primary school curriculum. It was also decided that we should do a pilot study, doing some observations of trainee teachers' classes, in order to gain a clearer picture of the real situation. We focused our attention on the teacher rather than on the children.

Pilot Study

In Portugal there is an official curriculum that results from the reforms mentioned above. Even though the curriculum does not exhibit great changes in respect to mathematical content (in accordance with international trends), it reflects a significant change concerning the main goals and the guiding principles for teaching mathematics.

A total of 6 groups of trainee teachers were observed (18 teachers), one lesson each, over a period of 2 months. The lessons were all video taped. It was our intention to pick up general information to be synthesised and reflected upon in further studies.

Our main focus was on the teacher and his/her approach to mathematical concepts but we also took into account the following hierarchical list of items: language (as used by both teachers and their pupils), approaches to problem solving activities, organisation and planning of mathematical activities, manipulative aids considered (by teachers) to be useful, etc.

A. Mathematics Teaching Goals

The three main goals for the teaching of primary school mathematics are stated as (DGEBS, 1989):

- Development of the ability for reasoning;
- Development of the ability to communicate;
- Development of the ability to solve problems.

This clearly reflects the influence of the NCTM Standards on the Portuguese primary mathematics curriculum even though initial teacher training in Portugal seems to be quite different from that in the U.S.A.

We believe it is crucial to have an explicit understanding of the essence of these purposes in order to avoid the error of only changing some aesthetic aspects of the mathematics lessons. As we observed, even when the classes were organized in groups, the predominant type of work was individual and traditional.

B. Teachers' Role

The main task imposed upon the teachers is to develop children's positive attitude towards mathematics (DGEBS, 1989). The affective component is regarded as a crucial one. There are several studies that relate love/hate for mathematics with success/ failure in the discipline (Renga & Dalla, 1993, McLeod, 1992). Dehaene (1997) also claims that "*children of equal initial abilities may become excellent or hopeless at mathematics depending on their love or hatred of the subject*" (p.8).

In what form does the affective relation between the teacher and Mathematics influence the relation of the pupil with Mathematics? Is it possible for a teacher who does not like mathematics to make students like it? From our observations it appears that when the teacher doesn't like mathematics or feels uncomfortable with the subject he/she tries to spend the least time possible on the subject. However he/she makes a considerable effort not to pass on the negative feelings to the students and also in preparing the mathematical lessons.

According to the curriculum, it is the teacher's responsibility to organize the means and create the proper environment for the fulfilment of the program.

However this responsibility raises some concerns:

- On the quality of the mathematical training of teachers; already it has been said and evidence from international studies proves that nobody can teach what they do not know and it is not enough to have a superficial knowledge of elementary mathematics. In fact, how can one expect that a teacher can create a proper environment for learning if he/she is not confident of his/her knowledge? If he/she repeatedly fails to give satisfactory answers to the questions that the pupils ask him/her? This way, not only will the environment be inadequate but probably it will also generate an atmosphere of unhappiness and frustration among the pupils.

- On the autonomy of the teacher; expecting the teacher to organize the means and create the proper environment for the fulfilment of the program seems to indicate that the teacher is supposed to re-create the curriculum. This attitude seems to be, like many others, imported from the United States, where a good teacher is one who constructs his own curriculum. In accordance with Ball and Cohen, cited in Ma (p.150),
  "this idealization of professional autonomy leads to the view that good teachers do not follow textbooks but instead make their own curriculum"

C. Problem Solving and Manipulatives

The core of the Portuguese curriculum is stated as being problem solving. It appears as if the only goal of mathematical activity is to be able to solve problems. Apparently problems are replacing content, becoming the contents themselves.

This educational approach, while exhibiting some short-term advantages, as for example improving self-confidence and motivation, raises several concerns, namely:

- Concerning the definition of problem. There are several different definitions of problem by different authors. Do teachers know exactly what we mean when we talk about a problem? What kind of problems do teachers use in their classes? The trainee teachers who were observed revealed incapacity to formulate problems. They believe that a problem is something that has a specific context, already exists in textbooks; it is motivating and different from the usual activities. They don't think it is their job to formulate problems and when facing a problematic situation they were unable to explore it.
- Concerning different approaches to mathematics teaching. According to Schroeder & Lester (1989), we can distinguish three different approaches: (1) teaching about problem solving, (2) teaching for problem solving, and (3) teaching via problem solving. What we found was that teachers use only the second approach. Are they aware of the other approaches?
- Concerning the teachers' ability to solve problems. Most of the teachers are not used to solving problems on their own. They look for solutions and just copy them.

The use of manipulatives is strongly recommended in the curriculum. The trainee teachers in the study always took materials for the class. This attitude seems to be justified for two reasons:

- The teachers believe that the use of manipulatives facilitates learning, motivates the students and makes learning more fun.
- Teachers involved in supervision expect trainee teachers to propose different activities, using manipulatives that are not typically used.

However, the use of manipulatives appears sometimes to be unnatural. In fact there were cases in which the teachers imposed the use of manipulatives even when the students didn't seem to

need them. Strangely, there were other times when the teacher did not allow students to use the manipulatives.

D. Shape and Space (Introduction to Geometry)

The teaching goals presented in the curriculum for Geometry are:

- Development of the aesthetic sense and creativity;
- Development of the ability to compare, classify and transform;
- Understanding the world of shapes;
- Acquiring vocabulary and elementary geometric notions.

In the observed classes, the contents related to Geometry were less treated than those related to Number and Operations.

We may conjecture some reasons for that:

- Teachers' insufficient geometrical knowledge. Teachers don't feel confident in dealing with geometrical questions so they tend to avoid them.
- Teachers attribute little importance to geometry. It looks as if teachers consider the questions related to number and operations much more important than those related to geometry. Besides, at this level, they think that geometry "*concerns the formation of concepts about space and the mere observation of geometrical entities in space.*…[Geometry] *tends at primary level to be all observation and no problems."* (Fielker, p.16).

The main focus of the geometry lessons was on the so-called "arbitrary" contents (Hewitt, 1999) which include names, definitions, notations and things alike, and where pupils can't come to acquire them by themselves and so, they explicitly need to be informed about. It looks as if the only important goal for the teaching of geometry is the recognition and naming of shapes. This attitude seems consistent with the one observed by Clements & Battista (1992) of some American teachers. However, even though the teachers emphasized the knowledge of the "arbitrary contents", we found situations when they were not confident of their own knowledge. For instance they used "right triangle" instead of "right-angled triangle" or the term "diagonal" to mean "oblique". It is significant to report that the teachers consider such incidents unimportant. They assume that students pay no attention to them as if students had some sort of filter that separates the things that are important to understand and memorize from the ones that are not. This is only one example of the lack of importance attributed to rigorous treatment of geometry (and of mathematics in general). This attitude is contrary to the recommendation of the *Geometry Conference* which says that "*the degree of rigor in the teaching of mathematics may vary according to circumstances, but that should never be an excuse to misinform or to mislead the student"* (p.286).

One of the topics in the mathematics program refers to the relative position of two lines in the plane and also in space. Students should be able to recognize parallel and perpendicular lines from the observation of solids. These terms parallel and perpendicular seem to appear in the program as opposites. But two lines that are not parallel don't have to be perpendicular. However the teachers in the pilot-study don't explore the possible relations between two lines. Moreover, they don't even consider the difference between these relations in the plane and in space. The only definition they used for parallel lines was "two lines that never meet". They seem to be unaware of the limitations of this definition.

As for perpendicular lines they defined them as being two lines that meet each other and make a right angle. But at this stage the students lack the notion of angle. So they just memorize the definition without any sort of understanding. This way, although the teachers defend meaningful learning, they are promoting meaningless learning, based on memory, which seems to reveal an insufficient content knowledge on their part (Ma, 1999).

Conclusion

Analysis of the results came to convince us that the so-called "elementary mathematics" is neither easy nor easy to teach. The role played by primary school teachers is crucial in what concerns the introduction of mathematical contents and therefore the mathematical training of these teachers deserves a deep analysis and the achievement of clear evidence.

We are looking for some kind of mathematical training, eventually with some cultural influence, clearly justified that makes the future teachers able to teach elementary mathematics in a more efficient way than the one we have been reporting, in Portugal.

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