USING COUNTER EXAMPLES TO ENHANCE STUDENTS' CONCEPTUAL UNDERSTANDING IN ENGINEERING UNDERGRADUATE MATHEMATICS: A PARALLEL STUDY

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ABSTRACT

This paper addresses a practical issue encountered by many lecturers teaching first-year university engineering mathematics. A big proportion of students seems to be able to find correct solutions to test and exam questions using familiar steps and procedures. Yet they lack deep conceptual understanding of the underlying theorems and sometimes have misconceptions. In order to eliminate misconceptions, and for deeper understanding of the concepts involved, the students were given the incorrect mathematical statements and were asked to construct counter examples to prove that the statements were wrong. They had enough knowledge to do that. However, for most of the students that kind of activity was very challenging and created conflict. 127 students from two universities, in Germany and New Zealand, were questioned regarding their attitudes towards the method of using counter examples for eliminating misconceptions and deeper conceptual understanding. The vast majority of the students (96% in the German group and 84% in the New Zealand group) reported that the method was very effective. Many of the students made positive comments that using counter examples helped them to eliminate misconceptions, prevent mistakes in future, understand concepts better, and develop logical and critical thinking.

Framework

One of the main objectives of the study was to check our assumptions on how effective the usage of counter examples is for eliminating students' misconceptions in engineering mathematics. In this study, practice was selected as the basis for the research framework and, it was decided 'to follow conventional wisdom as understood by the people who are stakeholders in the practice' (Zevenbergen R, Begg A, 1999). Over recent years in some countries, partly due to extensive usage of modern technology, the proof component of the traditional approach in teaching mathematics to engineering students (definition-theorem-proof-example-application) almost disappeared. Students are used to relying on technology and sometimes lack logical thinking and conceptual understanding. 'The rapid increase of information over very short periods of time is a major problem in engineering education that seems worldwide. Misconceptions or unsuitable preconceptions cause many difficulties'. (Kolari S, Savander-Ranner C, 2000). 'The basic knowledge, performance and conceptual understanding of the students in mathematics worsen'. (Gruenwald N, Schott D, 2000). We have more than 50 years experience between us teaching first-year undergraduate mathematics using different pedagogical strategies. The research question arose from our teaching practice.

The theoretical framework was based on Piaget's notion of cognitive conflict (Piaget, 1985). Some studies in mathematics education at school mathematics level (Swan, 1993; Irwin, 1997) found conflict to be more effective than direct instruction. 'Provoking cognitive conflict to help students understand areas of mathematics is often recommended' (Irwin, 1997). Swedosh and Clark (1997) used conflict in their intervention method to help undergraduate students to eliminate their misconceptions. 'The method essentially involved *showing* examples for which the misconception could be seen to lead to a ridiculous conclusion, and, having established a conflict in the minds of the students, the correct concept was taught'. (Swedosh P, Clark J, 1997). Mason and Watson (2001) used a method of so-called boundary examples, which suggested creating by students examples to *correct* statements, theorems, techniques, and questions that satisfied their conditions. 'When students come to apply a theorem or technique, they often fail to check that the conditions for applying it are satisfied. We conjecture that this is usually because they simply do not think of it, and this is because they are not fluent in using appropriate terms, notations, properties, or do not recognise the role of such conditions.'(Mason J, Watson A, 2001). In our study, not the lecturers but *the students* were asked to create and show counter examples to *the incorrect* statements based on their common misconceptions, i.e. the students themselves established a conflict in their minds.

The Study

To enhance students' critical thinking skills, help them understand concepts and theorems' conditions better, eliminate common misconceptions and encourage active participation in class, we were giving our students incorrect statements and asking them to create counter examples to prove that the statements were wrong. The students had to refer to definitions of the basic concepts and to their geometrical illustrations because in most cases the easiest way to prove that the statement was wrong was just to sketch a graph. Often the statements were based on common students' misconceptions. Below are several examples of such statements.

Statement 1. The derivative exists at a point if the graph is smooth and continuous at the point being considered.

Statement 2. If the derivative is zero at a point then the function is neither increasing nor decreasing at this point.

Statement 3. At a maximum point the second derivative is negative and at a minimum positive.

Statement 4. The tangent to a curve at a point is the line which touches the curve at that point but does not cross it there.

After several weeks of using counter examples in teaching Calculus to first-year engineering students, 47 students from a German university and 80 students from a New Zealand university were given the following questionnaire to investigate their attitudes towards the usage of counter examples in learning/teaching.

The Questionnaire

Question 1. Do you feel confident using counter examples?

a) Yes Please give the reasons:

b) No Please give the reasons:

Question 2. Do you find this method effective?

a) Yes Please give the reasons:

b) No Please give the reasons:

Question 3. Would you like this kind of activity to be a part of assessment?

a) Yes Please give the reasons:

b) No Please give the reasons:

Findings from the Questionnaire

The statistics from the questionnaire are presented in the following table:

Number of students	Question 1 Confident?		Question 2 Effective?		Question 3 Part of assessment?	
German group	Yes	No	Yes	No	Yes	No
47	12	35	45	2	19	26
100%	26%	74%	96%	4%	43%	57%
New Zealand group	Yes	No	Yes	No	Yes	No
80	18	62	67	13	15	65
100%	22%	78%	84%	16%	19%	81%

Table 1. Summary of findings from the questionnaire

The majority of the students (74% in the German group and 78% in the New Zealand group) were not familiar with the usage of counter examples as a method of proof. The common comments from the students who answered ‘No’ to question 1 on whether they are confident with using of counter examples or not were as follows:

- I have never done this before;
- I am not familiar with this at all;
- I am not used to this method of proof;
- This method is unknown to me.

The vast majority of the students (96% in the German group and 84% in the New Zealand group) found the method of using counter examples to be very effective. The common comments from the students who answered ‘Yes’ to question 2 on whether the usage of counter examples is effective or not were as follows:

- helps me to think question deeply;

- gives more sound knowledge of the subject;
- we can understand more;
- it makes me think more effectively;
- can prevent mistakes;
- you gain a better understanding;
- it makes you think more in-depth;
- it teaches you to question everything;
- it makes you think carefully about the concepts and how they are applied;
- it makes you think critically;
- it supports self-control;
- it requires logical thinking, not only calculations;
- makes problems more understandable.

The majority of the students (57% in the German group and 81% in the New Zealand group) did not want the questions on creating counter examples to incorrect statements to be part of assessment in contrast to the trends pointing to the effectiveness of the method (96% in the German group and 84% in the New Zealand group). The common comments from the students who answered 'No' to question 3 on whether the questions on creating counter examples be part of assessment or not were as follows:

- it is hard;
- never done this stuff before;
- confusing;
- not trained enough;
- complicated;
- can affect marks.

Most of these students were more concerned about their test results rather than acquiring useful skills.

The students who answered 'Yes' (43% in the German group and 19% in the New Zealand group) provided excellent comments similar to those made on effectiveness of the method. The common comments from the students who answered 'Yes' to question 3 on whether the questions on creating counter examples be part of assessment or not were as follows:

- it provokes generalised thinking about the *nature* of the processes involved, as compared to the detail of the processes;
- better performance test;
- it shows full understanding of topic;
- a good way to test students' insight;
- it is an extremely valuable skill.

Conclusion and Recommendations

The overwhelming statistics of the study and numerous students' comments showed that the students were very positive about the usage of counter examples in first-year undergraduate mathematics. Many of them reported that the method of using counter examples helped them to understand concepts better, prevent mistakes in future, and develop logical and critical thinking. From our experience it also made students' participation in lectures more active. All these give us confidence to recommend this pedagogical strategy to our colleagues to try with their students. There could be different ways of using this strategy: giving the students a mixture of correct and incorrect statements; making a deliberate mistake in the lecture; asking the students to spot an error on a certain page of their textbook or manual; giving the students bonus marks towards their final grade for providing excellent counter examples to hard questions during the lecture and so on.

We are very aware of the limitations of the study. It was not an international comparison. It was intended more as a pilot study to check our assumptions and share the findings with university lecturers and the mathematics education community.

Further Study

We would like to extend the study to measure the effectiveness of this pedagogical strategy on the students' exam performance. We plan to compare the performance of 2 groups of students with similar backgrounds. In one group we will extensively use counter examples, with the other group being the control group. Then we will use statistical methods to establish whether the difference is significant or not. We also would like to extend the study to other countries in order to reduce the effect of differences in cultures, curricula, and education systems and also analyse the data from different perspectives and backgrounds. This co-operation can lead to organising a Research Forum or Discussion Group at an international conference on mathematics education to discuss the issues arising from this collaborative research. Those colleagues who are interested in joining the study group are cordially invited to contact the authors.

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