INTEGRATION OF MATHEMATICAL MODELLING AND TECHNOLOGY IN MATHEMATICS TEACHER PREPARATION COURSES

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ABSTRACT

Common themes in requirements for prospective mathematics teachers include mathematical modelling, problem solving, technology, and communicating mathematics. In this presentation we will discuss student presentation of projects to model and communicate mathematics. Technology is used as an integral tool in developing and solving the model as well as a medium for effective presentation.

Key Words and Phrases: teacher preparation, mathematical modelling, technology, computers, calculators, communicating mathematics, problem solving, student presentation of projects.

1. Introduction

It is difficult to correctly and concisely summarize the current state of mathematics education in the United States. However, it is possible to provide a few themes common to many of the contemporary reform movements in mathematics education. For example, mathematical modelling, problem solving, and communicating mathematics are three such themes. Moreover, the role of modern technology in the teaching and learning of mathematics is significant.

In our courses we assign student projects to provide the opportunity for students to communicate mathematics. We stress that mathematical modelling is an ongoing and dynamical process that is useful in daily life.

We require that students work in teams of two or three and report to the class on their projects. Our objectives are twofold: to enable the students to learn about the wide variety of mathematical models and to provide experience in communicating mathematics. Moreover, we require each team to make a formal presentation of their project to the entire class. We encourage the students to use the technology of their choice in the preparation of the paper and for their presentation.

For example, the students can use the technology of the TI-89 computer algebra system to explore differential equations and interpret solutions from three different points of view: graphical, numerical, and analytical. Slope fields and graphs of solutions or direction fields and solution curves in the phase plane contribute to better understanding of long-term behavior of the model. Tables of approximate solutions using Euler or Runge-Kutta methods also provide information. Graphs and tables of exact or approximate solutions can be compared on a split screen. The **deSolve** command of the TI-89 can be used to compute exact symbolic solutions to many 1st- and 2nd-order ordinary differential equations. Matrices, eigenvalues and eigenvectors are also easily handled on the TI-89 to determine the exact solutions to systems of ordinary differential equations.

A variety of computer software packages produce similar results.

Many of our students use a computer with presentation software and/or access to the Internet to present their projects. Others use calculators, posters, and transparencies on overhead projectors.

Some of the topics we have covered are: population models (including several different models of one population as well as models of competing populations from ecology), models of social choice (how groups make decisions), economic models, models of the epidemiology and the immunology of AIDS, and simulation models in planning and development.

2. Illustration

The struggle for existence among species has been studied for centuries. According to the theory of Charles Darwin, the average number of a species of prey depends on how many of the species are consumed by their predators. In the 1920's and 1930's Vito Volterra and Alfred Lotka, independently reduced Darwin's predator-prey interactions to mathematical models. The Lotka-Volterra predator-prey model is the system of first order differential equations:

x' = (- a + by) x = - ax + bxy and

y' = (c - dx) y = cy - dxy, where

a ,b ,c, and d are positive constants and

x = x (t), *y = y*(t) are populations at time t of a predator and a prey, respectively.

This is the simplest predator-prey model. It includes only exponential growth or decay and the predator-prey interaction. All other factors are assumed to be insignificant.

The Italian mathematician Vito Volterra developed the model in response to a problem posed to him by his son-in-law, the Italian biologist Umberto D'Ancona. D'Ancona was researching populations of species of fish that interact with each other in the Adriatic Sea. He had data on percentages of the catch of various species of fish brought into the Mediterranean ports of Trieste, Fiume, and Venice during the years of World War I, a period of reduced fishing from these ports. D'Ancona expected that a period of reduced fishing of food fish would be beneficial to the population of food fish. Yet the data seemed to indicate, in a relative sense at least, that reduced fishing was not beneficial. Instead there was a large increase in the percentage of predator species, selachians (sharks, skates, rays, etc.), which depend on their prey, the food fish.

Let's consider a specific case of Volterra's model, which we will analyze with the TI-89. We choose $a = 1$, $b = 0.1$, $c = 1$, and $d = 0.2$. This predator-prey model is represented by a system of two first order differential equations with constant coefficients:

$x' = -x + 0.1 \, xy$ and

$y' = y - 0.2 \, xy$, where

$x = x(t)$ represents the amount of selachians (predators) and

$y = y(t)$ represents the amount of food fish (prey) at time t.

Let the initial populations be represented by $x(0) = 8$ and $y(0) = 16$. Note: In this setting, it is more realistic to use units of pounds or tons rather than the number of fish i.e. biomass. So "8" might be 8 tons etc.

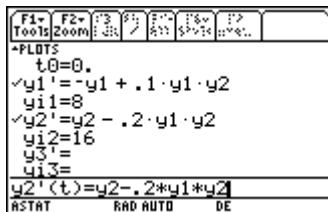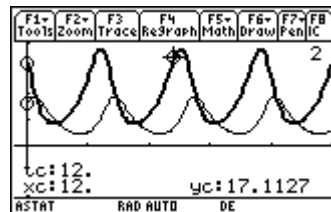To enter the differential equations in the equation editor of the TI-89 Calculator, the built in variables $y1$ and $y2$ are used for x and y, respectively.
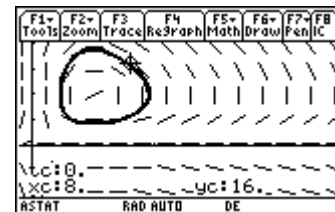

Figure 1
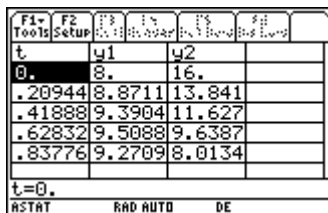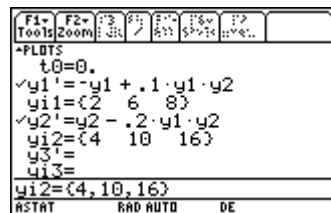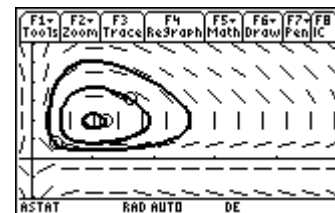

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figures 1-6 illustrate how the students can analyze the model numerically with a table (Figure 4) and graphically with a time graph (Figure 2) and phase portraits (Figures 3 and 6). Figure 2 portrays how the graphs representing the populations of food fish and selachians can be plotted simultaneously with thick and thin lines respectively. The "trace" feature of the TI-89 enables the students to see the values of the coordinates of points on the graph (Figures 2 and 3). Figures 5 and 6 depict how phase portraits with different initial conditions can easily be graphed simultaneously.

The amounts of the food fish (prey) and the selachians (predator) appear to be periodic. Moreover, the trajectories of solutions to the system seem to be closed loops—even closed loops that are "quasi- elliptical". In fact, these specific cases are representative of the general case.

Now return to the Volterra model provided above. The students are now prepared for a serious discussion of such a system of first order differential equations. (In our curriculum, the first course in mathematical modelling does not have systems of differential equations as a prerequisite so many of the students have not had such systems in previous courses). Part of the discussion is the proof that the average value of x (t) is c/d and the average value of y (t) is a/b. (For a proof, see Borelli and Coleman (1998), Chapter 5). What happens to these average values when "fishing is introduced"?

The Volterra model with fishing is

$x' = - ax + bxy - ex = - (a+ e) x + bxy$ and

$y' = cy - dxy - fy = (c - f) y - dxy$, where a, b, c, d, e, and f are positive constants.

Here e is a constant that represents the effect of fishing on the predator and f represents the effect of fishing on the prey. Note that if f is less than c we have the same setting as before since the coefficient of y is positive! The average values are x (t)=$(c-f)/d$ and y (t)= $(a+e)/b$. Thus, a "moderate" amount of fishing will increase the average amount of the prey (food fish)— "moderate" means that the fishing rate on the prey (food fish), f, is less than c (this forces the constant $c - f$ to be positive, so we can use the previous result on the average values). Note that the constant c is directly rated to the growth rate of the prey so "moderate" fishing is a rate less than the growth rate. But, if the fishing rate f is reduced (e.g. no fishing, $f = 0$), the average amount of prey (food fish) will decrease. This was Volterra's resolution of D'Ancona's problem.

There is another nice application of this model, which explains one of the deleterious effects of the pesticide DDT. The scenario is set in the mid 19th century when an insect was accidentally introduced to America from Australia. The insect had no natural predator in America. Its population grew at a rate sufficient to threaten the existence of the citrus industry. A natural predator was imported from Australia. The pest's population was reduced to a level where the citrus industry flourished again. However, the pest was not eliminated. With the introduction of DDT it was assumed that the pest population could be totally eliminated. The DDT is the "fishing agent". Since a "moderate amount of fishing" was "good" for the prey (the pest), as discussed above, the pest population increased rather than decreased — in particular, it was not eliminated!. See Braun's (1993) excellent book for a complete discussion.

The Lotka-Volterra model can be employed in a wide variety of different scenarios. One of the points we stress in our teaching of mathematical modelling is this very principle; namely that the same model can be employed in very different settings. A good project is to have the students investigate other applications of the Lotka- Volterra model.

3. Conclusion

There is evidence that appropriate use of technology does help students to learn mathematics better. Some studies which provide examples of the use and effectiveness of technological pedagogical tools are included in Connors, 1995; Connors & Snook, 2001; Dunham, 1998; and Hurley, Koehn, & Ganter, 1999. It makes sense, therefore, to provide prospective teachers with the opportunity to utilize technology in their mathematics and teacher preparation courses.

Projects provide an extra dimension in the learning process. Students work together on a problem that is not completely laid out for them. In some cases, they have a broad choice in topic selection and, therefore, they acquire a sense of ownership. They are required to analyze the problem, do research, if necessary, make decisions, and find results. Sometimes they are asked to make recommendations based on their findings. This provides an opportunity for them to interpret

their results as it relates to real life. They are encouraged to criticize their work and, in some cases, are asked what else they would have done if they had more time or more resources.

Students often comment that projects helped them to understand better and also to recognize the importance and relevance of the mathematics studied in the course. Some report a sense of accomplishment and personal pride. Most importantly, they are doing mathematics and that is the best way to learn mathematics!

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