

NEW APPROACH TO THE USE OF SOLUTION MANUALS IN THE TEACHING OF HIGHER MATHEMATICS

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ABSTRACT

This paper presents a new approach to the teaching of undergraduate mathematics in which the students are allowed a free access to the complete solutions manual. Our method consists in assigning a very substantial amount of homework problems and allowing the students to consult with the solutions manual while doing it. Our philosophy is that the purpose of the homework is not to test the student's knowledge but to give her/him the opportunity to acquire and experience knowledge. The student is thus being exposed to a very large number of examples in which she/he actively participates, with the comfort that if unsuccessful to do the problem alone, there is a resource which can help. Remarkably, we have so far not experienced blind copying from the solutions manual, which was our fear to start this program. We attribute this fact to making clear early in the semester that the tests are very challenging both in content and level of difficulty. We observe a big jump in the students motivation, interest in the subject and performance on exams. All students in the classes in which we used this method improved their tests scores very significantly. Additional benefit which we observed is a remarkable increase in the students' self-confidence and study-independence. .

1 Introduction

This paper presents a new approach to the teaching of higher mathematics. We describe it in detail and compare it with other approaches. Several statistics which show the advantages of this approach to other methods of teaching are presented and a comparative analysis is done.

The new method, described in this paper, consists in allowing the students completely free access to the solution manual containing the complete solutions to all problems in the textbook and simultaneously assigning an exceptionally large number of homework problems, ranging from easy to the most challenging. Our philosophy is that we use the homework for a learning tool instead of testing acquired abilities. The intellectual challenge level of the exams is raised as high as the most abstract and the most sophisticated problems which the textbook involves. We use these exams to assess the acquired skills and knowledge of the students. The homework is mandatory and the students are being given credit for doing it, but it is used solely as a tool in the learning process and not to test acquired skills. We use the university tutoring center for the place where the solution manual is being made available to the students. The solutions manuals which we use are published by the same company which publishes the textbook in use of the particular course.

The statistics which we have done on the effectiveness of this new approach to the teaching of higher mathematics demonstrate its advantage to the "classical" approach of "rediscovery", in which the solutions of the assigned home work is strictly unavailable to the student before the homework is collected. It was not without an opposition that this new method of teaching established itself in our institution. Our work benefited greatly from it, since it was this opposition that forced us to perform comparative statistics on the effectiveness of the new approach and to analyze the results. These results show consistent and very significant improvement of the performance of the students on exams. To give an example of the comparative statistical analysis which we made - we subject a class (like for example Calculus 3) to this new method of teaching and compare their performance on exams with the performance on exams of *the same group* of students in the previous mathematics course (Calculus 2) taught under the "classical" method, as described above. The statistics show a "jump" in the performance of the students on exams. Under our new method, in which we allow free access to the solution manual, the majority of the students improve their exam grades with 1 - 1.3 letter grades. In addition we observe a jump in the students' motivation, self-confidence and study independence. Surprisingly, we did not observe *any* blind copying from the solution manual, which was our main fear to start this program. Further, we were delighted to be able to raise the challenge level of the tests much higher than we could when using the "classical" method of teaching. Thus, our statistics were done not only on the *same group of students* but also using significantly more difficult exams. Even though this paper concentrates on the remarkable improvement of exam performance resulting from the new method of teaching which we use in our mathematics courses, the jump in the students' motivation, self-confidence and interest in the subject should also be emphasized.

2 Origins and Motivation

The idea for the method of teaching higher mathematics, which this paper presents, has its origins in the Russian educational system.

In the pursue of effective ways of building knowledge in the abstract and challenging subject of higher mathematics we experimented with different approaches to teaching. Here in the United States, historical reasons have firmly established what we call the "classical" method of teaching mathematics, in which revealing any part of the solution to a homework problem, before collecting the homework, is a tabu. This "classical" approach to the teaching of mathematics has its many positive sides, and this paper has no intention of ruling out or raising doubts in its benefits. The feeling of discovery is an important one, and forcing the student to rediscover the solution of a problem by herself/himself by making the solution unavailable, has shown to work with many students. The drawbacks of this method are well known. One significant drawback of this "classical" method is that it lowers the standards of the students toward an assignment- many students submit incomplete homework, when they are unable to find the right solution of a challenging problem. Only the strongest students manage to complete the whole assignment. This is actually not the biggest drawback of the "classical" method of teaching mathematics. The students who submit incomplete assignments have built a criterion on whether a solution is correct or incorrect and it is actually an asset that they do not submit an incorrect solution. These students are a small percentage though. The greatest majority of the students, forced by the fact that the homework carries some weight in calculating final grades, submit wrong solutions. This lowers their standards in the quality of their work, and thus has a very negative effect on their mathematics education and more generally on their growth as professionals and humans. A third drawback of the "classical" method of teaching is that "discovering" takes a lot of time to the student. Even the strongest, most motivated student, needs a lot of time to discover completely by herself/himself a correct solution to a challenging problem. This limits the number of mathematical problems which the student can be exposed to and the types of "situations" which she/he can "experience".

In our efforts to maximize the benefit of a mathematics course we looked into other approaches to teaching and more specifically an approach which will resolve the difficulties which the "classical" way of doing home work in higher mathematics presents. The experience which the Russian educational system provides in the teaching of mathematics was an exceptionally valuable recourse. Even though mathematics is the same everywhere, the philosophies behind teaching it differ drastically from one to another educational system. The task of assessing different approaches to the teaching of mathematics is an enormous one and has been a focal point of our educational research for many years. Much has been written on different approaches and by now we all are convinced that each proposed model has its advantages and its drawbacks. In our exploration of the different methods of teaching higher mathematics we found that the Russian method avoids many of the problems which the "classical" approach described above is subject to. The idea of "**discover by yourself**" is missing from the Russian mathematics teaching philosophy. In its place is the "**strive for performance excellence**" teaching strategy. The students are given everything available and are asked to master the most challenging. All kinds of study resources are made available, like for example books consisting of a collection of problems. These books contain the complete

solutions to all problems with detailed explanations. In the Russian system of teaching mathematics there is absolutely no fear that the students are not sufficiently challenged, because the level of exams can be raised as high as the individual instructor desires. The students are completely aware of the fact that copying the solution of a problem is not improving their abilities to perform on exams. The "discover your self" strategy is replaced by the "who is going to do better on the test" race, which is easily translated to "who is going to learn more" race. As is well known, the Russian approach has proved itself to produce excellent results at all levels of mathematics education. It was an exceptionally valuable resource for ideas in our search for more effective teaching strategies.

3 Description and Statistical Analysis

The method of teaching which we are describing in this paper is based on this Russian approach to teaching mathematics. Our philosophy is that we use the homework as a **tool for learning** and not for assessing skills or knowledge. We assign an **exceptionally large number of homework problems**, much larger than one can assign when the solutions manual is not available. If the "classical" method of teaching was to be used with this amount of required homework, no student is able to do even half of the assignment. The problems range from easy to the most challenging. The large number of problems allows us to expose the student to a much greater variety of problems. The way in which we make it possible for the student to go through this large series of exercises is that we provide the complete solutions to all the assigned problems. For this purpose we use the complete solutions manuals which are published by the same publisher as the one which publishes the textbooks. We use the university tutoring center to provide the access to the complete solutions manual. The students can use the manual freely at their convenience. We provide the solutions manuals in the tutoring center and do not post the solutions on the web and, thus, avoid any possible interference with copyright laws.

The fact that we have used this new method of teaching during the last 5 years we had the opportunity to apply it in classes of predominantly engineering students as well as classes of liberal art students. At Oregon State University the Differential Calculus, Integral Calculus and Vector calculus are about 85% engineering students with the rest being science majors. we have used our method extensively with these classes. Pacific University is a liberal art college and so, the students in the classes in which we have used this new approach are considered liberal art students. For the reader interested in specific data: these classes had about 15% mathematics majors, 20% physics majors, 30% premedical/science majors and 35% humanities/arts majors.

All of the classes in which we have used this approach were required classes. We have not made any specific suggestions on whether or not the students should work together. Our observations were that the students at Oregon State University worked almost exclusively individually, while about 30% of the students at Pacific University worked in groups. These groups seemed to form based on existing friendships. We have noticed no influence of the "working together/working individually" variable on the effect of the solution manuals. The homework is required, collected weekly and graded. The weight of the homework in the final grade is between 10% and 15% with the rest being performance on exams.

At Oregon State University the students had access to the solutions manuals every week day 9 am to 5:30 PM. At Pacific University they have access to it Thursday through Sunday 5:30 PM - 10 PM, which are the working hours of the tutoring center. We work on extending these hours.

At the beginning of each course we make very clear to the students that the exams are very challenging, and that the grades are calculated based on 85 - 90 % exams and only 15 - 10 % homework. A course always includes at least two midterm exams, the first of which is scheduled very early in the semester, so that the students can get a first-hand feed back on the high level of expectations. We often reinforce the testing of acquired skills and knowledge by weekly quizzes. Thus, the homework is a tool in the study process. It is a part of the learning experience. Some of my colleagues view it as forcing the student to being exposed to a very large number of examples in which the student participates actively. Our main fear in starting this program was the anticipation of blind copying from the solution manual. We were afraid that only the most motivated and already advanced students will have the true understanding of how to make use of the provided solutions and how to benefit from them. Encouraged from the success of the Russian system we decided to try our idea and monitor the results. We were surprised and very pleased that we did not observe *any* blind copying from the solutions manual. The performance of the students on exams jumped with comparison with their performance on exams under the "classical" method of teaching. Their performance improved very significantly from the first midterm to the second midterm and from the second midterm to the final exam. We require the students to submit all the assigned homework problems. The excuse "I couldn't solve this problem" is completely eliminated, because of the availability of the solutions manual. The students must consult the manual if they are unable to solve the problem themselves. We observe, that most students do not need much of stimulation in this respect. Knowing that nothing is being hidden from them, they strive to take it. We observed in several classes subjected to our new method of teaching, that a competition is being created between the students in the class. They compete about who is going to "get more" out of the solutions available, who is going to be able to do better on the test. This competition has been a healthy one in every respect in the classes which we have observe it. It makes the class more exciting, more of a race. This method of teaching seems to appeal to the students because of its correlation with their naturally youthful impatience, curiosity and need to compete. Even though the number and level of intellectual challenge may be overwhelming to some of the students, doing the home work is a positive experience, since they do not feel left alone to struggle with the difficulties. They have the help provided by the solutions manual and they approach the long and challenging process of doing all the assignment with the feeling of security. We think that one reason for the success of this method of teaching is the fact that it emphasizes positive, encouraging attitude to the learning process. The fear of "punishment" because of inability to do the problems is completely eliminated. The number of problems to which the student is being exposed is several times larger than in the "classical" method of teaching. This is due to the fact that with the help of the solution manual the students can finish a much larger number of homework problems, and thus we, the teachers, can assign a much larger number of homework problems, than if no solution manual was provided.

We would like to stress that this new method of teaching higher mathematics eliminates the drawbacks of the "classical" method described above. The students do not

quit doing the homework. They consult the manual when they get stuck on a problem. Thus, they submit complete assignments. Especially pleasing is the fact that this method eliminates the difficulty which is inevitably present in the "classical" method of teaching, which allows submitting incorrect solutions and expecting credit for them. With the availability of the correct solutions, the excuse "This is the best I could do", is eliminated. This asset of the method which we propose in this paper and which has established itself in our institution is one of the most valuable. It benefits not only the mathematical education of the student but also his/her growth as a professional and as a person.

In this paper we present the comparative statistical analysis which illuminates the new teaching method described above as opposed to the "classical" method. In the statistical survey shown below we present our observations performed on the same groups of people. Our analysis is based on collecting data on the performance on exams of a class in a given course, say Calculus 3, taught with the new method of teaching, and comparing this data with the performance on exams of the *same* group of people in the previous course, Calculus 2, taught with the "classical" method of teaching.

We have used the method which this paper describes in classes of 25 to 40 students. We hope that this paper will inspire others to continue this line of work and test this approach in classes larger than 40 students as well as on small classes of less than 20. The observations which we have collected are very consistent. Below we show some of these statistics.

In the Calculus 3 course taught in the Fall semester of 2001 at Pacific university there were 28 students enrolled. We subjected the class to our new method of teaching in which we allowed a free access to the solution manual and collected data on the performance of the students on the exams. We then compared this data with the performance of the *same* students on exams in the preceding course, Calculus 2. To keep the statistic accurate we eliminated from our calculations the performance of students in the Calculus 3 course, who did not take Calculus 2. These were only a couple of the students in the Calculus 3 course. The majority of the students improved their letter grades with 1 - 1.3 letter grade from the first midterm to the second midterm in the Calculus 3 course. 61% of the students improved their performance on exams from a test score in the 70-80 range on the Calculus 2 final exam to a score in the 90-99 range on the Calculus 3 final exam. 7% of the students improved their performance from a test score in the 60-70 range on the Calculus 2 final exam to a test score in the 90-99 range on the Calculus 3 final exam. Another 7% of the students improved their performance on these exams from the 60-70 range to the 85-89 range. Another 28% of the students improved their performance from the 89-94 range to the 95-99 range. There was no student who lowered his/her performance on exams under the new method of teaching.

The statistics collected at Oregon State University were made on classes between 35 and 40 students. We compared the performance on exams of the *same group of students* taking Vector Calculus in which free access to the solution manuals was allowed to their performance on exams in the preceding Integral Calculus class. We did the same with Integral Calculus versus the preceding Differential Calculus. The statistics were very close to the ones above, demonstrating consistent and significant improvement of the performance on exams.

In conclusion we would like to share our delight with the benefits of this new ap-

proach to teaching higher mathematics and hope that this paper will serve to encourage other institutions to apply it in their courses.

REFERENCES

- Walter S., ed., 2001, *Changing the Faces of Mathematics*, National Council of Teachers of Mathematics.
- Laughlin C. and Kepner H., 2001, *Guidelines for the Tutor of Mathematics* 2nd ed
- Edward B. and Starbird M., 2000, *The Heart of Mathematics: An Invitation to Effective Thinking* Key College Publishing.